Fast Analysis of Blocks with Current Conveyors

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Abstract: - On the basis of the well-known modified nodal analysis, universal matrices-stamps are defined for modeling more types of current conveyors. From these stamps, modified Mason-Coates‘ graphs are derived for fast analysis of circuits with current conveyors CCI and CCII.

Key-words: - Modified Nodal Analysis, Matrix-Stamp, Mason-Coates‘ Flow Graph, Current Conveyor.

1 Introduction

Current conveyors CCI and CCII in particular belong to important components of modern blocks for analog signal processing [1]. Although up-to-date analysis and simulation of electronic circuits is coupled with the utilization of computer programs, hand-and-paper analysis of simple blocks is also meaningful. The reasons are several. To verify the principal circuit functionality, the analyzed circuit function, e.g. the *s*-domain transfer or imittance function, is usually required in the form of a mathematical formula. It is possible to use some not commonly available programs for symbolic analysis [2]. Since today there are of more types of current conveyor, and further new types will appear, the problem of updating the current model libraries of these programs is urgent. Another reason for hand-and-paper analysis is the so-called method of signal flow graphs, which can be utilized not only for the actual analysis but also to make the opposite process – circuit synthesis based on its required features – more efficient.

Current conveyors (CCs) are elements whose presence in the circuit complicates its intuitive analysis. They have no admittance matrix so that their inclusion in the generally-used method of modified nodal analysis (MNA) is doubtful. In the past, a number of attempts were made to find simple algorithms of compiling equations for circuits with CCs [3], [4], [5], especially in the form of flow graphs. In most cases, they focused on the second - generation conveyors. The common shortcoming of these approaches consists in the rather complicated rules of compiling a model of the circuit (i.e. flow graph) directly from the circuit schematics. Due to the transformation of circuit variables, caused by CC operation, the models of the conveyor and of the circuit remainder blended into each other. This is in conflict with the basic advantage of MNA, where every circuit element has its own description, which is modularly inserted into the pseudoadmittance matrix or into the oriented graph of the entire circuit. In addition, the transformation of circuit variables depends on the type of CC. That is why the method of building the circuit model is not universal. Owing to the transformation, the linkage to the circuit is not clearly evident from the flow graph, and such flow graphs are not appropriate for circuit synthesis and optimization.

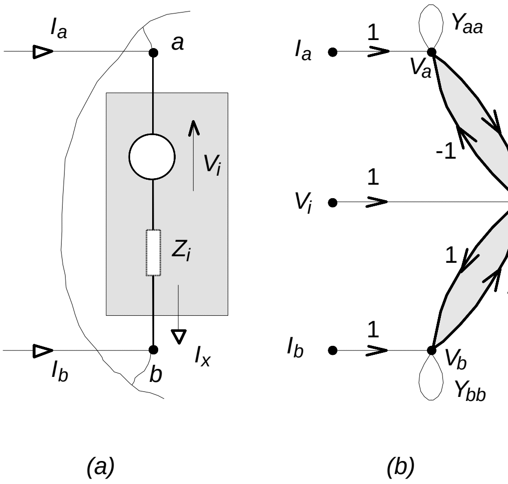
An additional disadvantage of the present approaches consists in the absence of conveyor output current in the equations formed. However, this current is often the required quantity, especially for circuits operating in the current or mixed mode.

In the paper, one method of algorithmic matrix analysis of circuits with CCs is shown. The method is based on the well-known “matrix-stamp” approach, which overcomes the above problems. We define a universal stamp, which can be used for all the basic types of CC of the first and second generation. This matrix-stamp then leads to Mason-Coates’ (M-C) graph [6] of CC with undirected self-loops. Two examples demonstrate the convenience of these graphs in the analysis and synthesis of active filters.

2 Matrix-stamp and M-C graph- stamp of Thévenin model of reciprocal two-port

Consider a circuit that is described by equations of Kirchhoff’s current law (KCL) of the classical nodal analysis. The number of equations is equal to the number of independent nodes in the circuit. The equations contain the same number of unknown nodal voltages.

A two-port described by its Thévenin model according to Fig. 1 (a) is additionally connected between nodes *a* and *b*. Then both the voltage and the current relations in the circuit will be changed. A current I_x will flow through the two-port and current relations at nodes *a* and *b* will be violated. The original nodal voltages will also be changed.



(a) (b)

Fig. 1 (a) Thévenin model of included two-port, (b) corresponding M-C graph-stamp.

The original equation describing the current equilibrium at node *a* has to be completed by current I_x, flowing outside the node, and at node *b* by current I_x with negative sign because it flows into node *b*. In addition, nodal voltages V_a and V_b are now bound by the following condition:

$$Z_i I_x + V_b = V_i + V_a \text{, or } V_i = Z_i I_x + V_b - V_a .$$

All these modifications can be included in a new set of equations of MNA:

$$\begin{array}{cccccccc}
 & & & V_a & V_b & & I_x & \\
a & I_a & & Y_{aa} & \dots & \dots & +1 & V_a \\
b & I_b & & \dots & Y_{bb} & \dots & -1 & V_b \\
 & . & = & . & . & . & . & . \\
 & . & & . & . & . & . & . \\
 & V_i & & -1 & 1 & & Z_i & I_x
\end{array}$$

stamp

The vector of unknown nodal voltages is extended by another variable – current I_x. The number of equations is also increased by 1 by the above condition between nodal voltages V_a and V_b. The voltage V_i is included in the left-side vector of the known exciting quantities. The modification of KCL equations for nodes *a* and *b* is implemented by entering the numbers +1 and –1 into column „ I_x “.

The original circuit admittance matrix together with the stamp of the Thévenin model of included two-port represent directions how to analyze – for instance – circuits with voltage sources by means of MNA. Impedance Z_i can be zero, in which case the ideal voltage source is modelled. For $V_i = 0$ and simultaneously $Z_i = 0$ we model a short connection between the nodes and the short-circuited current can be computed. This procedure can be used for the analysis of circuits with current controlled sources.

The set of MNA equations will be transformed into another form for a direct construction of the flow graph in Fig. 1 b).

$$\begin{aligned}
&Y_{aa} V_a = I_a - I_x - \dots \\
&Y_{bb} V_b = I_b + I_x - \dots \\
&\dots \\
&Z_i I_x = V_i + V_a - V_b .
\end{aligned}$$

Symbol Y_{aa} (Y_{bb}) specifies the sum of admittances connected to node *a* (*b*). The corresponding self-loops in the graph are drawn as a dotted curve because the given admittances are not part of the included circuit in Fig. 1 (a).

The flow graph-stamp of included two-port has a thick-line shape of “dragonfly”. It is the graph representation of its matrix-stamp. New graph nodes are input node V_i and node of the unknown circuit variable I_x. Each “dragonfly wing” is formed by a couple of paths with gains +1 and –1. The “outwards” directed paths are current paths (they are directed out of current node I_x). The path with the + sign follows the wing towards the node, to which the current I_x flows. The second path has the – sign. Voltage paths follow the opposite wing margins. They have both the directions and signs opposite to those of the current paths.

The “dragonfly head” is formed by a self-loop, the gain of which is given by the inner impedance of the Thévenin model. The “dragonfly tail” contains the node of inner voltage of the Thévenin model.

The above “dragonfly” model includes all existing models of reciprocal two-ports: for passive impedance, the “tail” will vanish. For ideal voltage source the “head” gain will be zero. In case of grounded two-port, only one “wing” will appear in the graph.

The dragonfly graph-stamp can theoretically be utilized in the analysis of circuits with both current and voltage sources. However, its practical importance appears when solving circuits with elements like CCs, where current relations are under consideration.

In case of nonreciprocal circuits – for instance OpAmps and all the types of CC – their flow graphs will contain “dragonflies” or structures after their transformations.

3 Circuits with CCs

The symbol of the general three-port CC, which is included into a circuit at nodes *a*, *b* and *c*, is shown in Fig. 2 (a). This conveyor is described by general hybrid equations in the following form:

$$\begin{bmatrix} I_y \\ V_x \\ I_z \end{bmatrix} = \begin{bmatrix} & a & \\ b & & \\ & g & \end{bmatrix} \begin{bmatrix} V_y \\ I_x \\ V_z \end{bmatrix}$$

The values of α, β, and γ coefficients depend on the type of CC as shown in Tab. 1. In addition, we can use them to model basic conveyor inaccuracies. For example, deviating the value of γ coefficient from 1 models the inaccuracy of the gain of internal current mirror of CCII+. Similarly, the deviation of β models the inaccuracy of voltage follower transfer between the *x* and *y* inputs of a noninverting CCII.

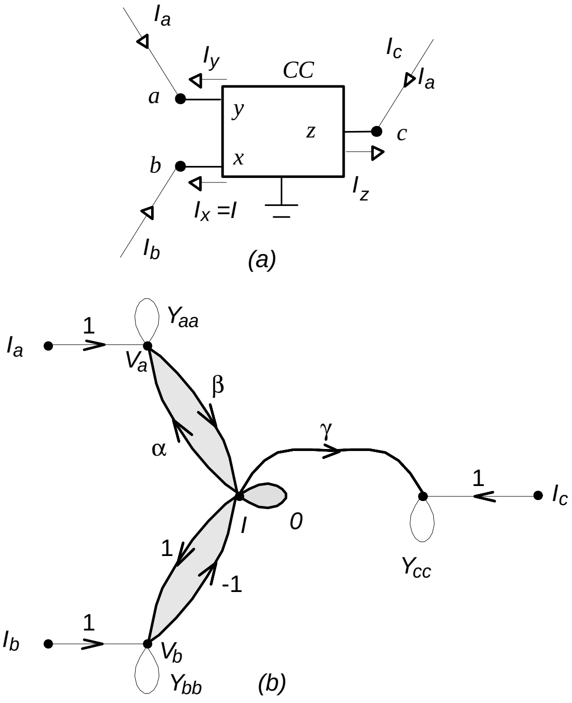


Fig. 2. (a) three-port CC, (b) its M-C graph - stamp.

coefficient	*a*	*b*	*g*
CCI	1		
CCII	0		
noninverting		1	
inverting		-1	
positive			1
negative			-1

Tab. 1. Coefficients for various types of CC.

The corresponding matrix-stamp can be derived from hybrid equations and from Fig. 2 (a):

$$\begin{array}{cc} \\ a & \\ b & \\ c & \\ & \\ & \end{array} \begin{array}{c} \\ \boxed{I_a} \\ \boxed{I_b} \\ \boxed{I_c} \\ \boxed{..} \\ \boxed{} \end{array} = \begin{array}{ccccc} V_a & V_b & V_c & & I \\ \hline Y_{aa} & \dots & \dots & \dots & -a \\ \dots & Y_{bb} & \dots & \dots & -1 \\ \dots & \dots & Y_{cc} & \dots & -g \\ .. & .. & .. & .. & .. \\ b & -1 & & & \end{array} \begin{array}{c} \\ \boxed{V_a} \\ \boxed{V_b} \\ \boxed{V_c} \\ \boxed{..} \\ \boxed{I} \end{array}$$

The symbol *I* indicates independent current $I = I_x$.

The M-C graph of CC in Fig. 2 (b) is derived by a similar procedure as for the above Thévenin model. Comparing with the original “dragonfly” in Fig. 1 (b), the path gains on the upper wing are modified. One additional part models the current transfer into the CC output. The self-loop gain of node *I* is now 0. In the case of the well-known effect of nonzero resistance R_x for CCII, this gain will be just R_x.

It should be noted that for the commonly used type of CCII, the part with gain *a* disappears from its flow graph. This fact contributes to its simplification.

4 Examples of analysis

The 2nd -order high-input impedance insensitive filter in Fig. 3 (a) was published in [7]. Assume that the β and γ parameters of both CCII+ are not exactly 1. Let us find the influence of these inaccuracies on the filter features.

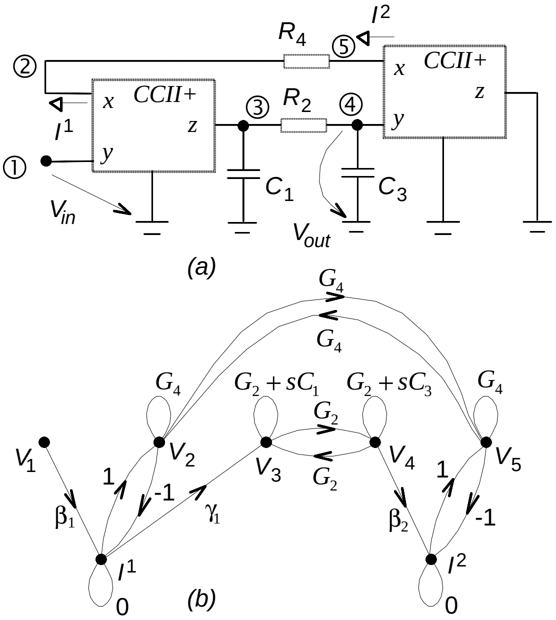


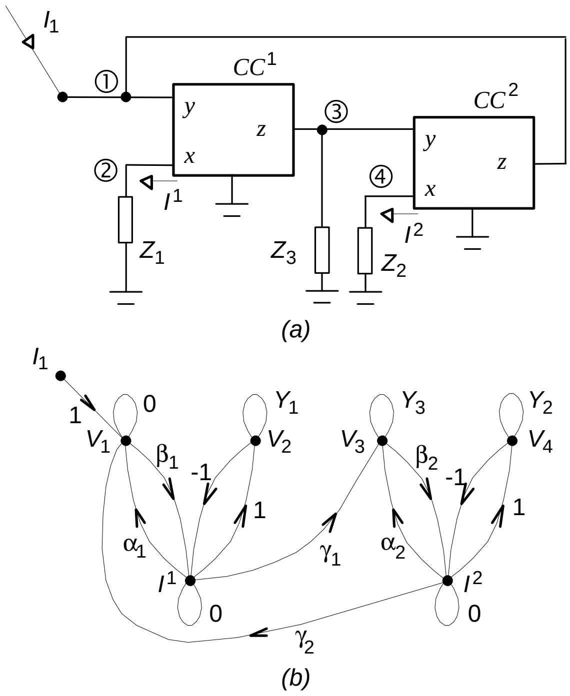
Fig. 3. (a) 2nd -order filter, (b) its M-C graph.

To determine filter transfer function V_4/V_1, let us construct the shortened M-C graph in Fig. 3 (b), considering general parameters β_1, β_2, γ_1 and γ_2. The graph evaluation is considerably simplified by the presence of three zero-gain self-loops. Applying the generalized Mason’s gain formula [8] yields the result

$$\frac{V_4}{V_1} = \frac{\dfrac{b_1}{b_2}}{1 + \dfrac{R_4(C_1 + C_3)}{b_2 g_1} s + \dfrac{R_2 R_4 C_1 C_3}{b_2 g_1} s^2},$$

which is in accordance with [7]. The resulting formula enables a simple determination of the influence of β and γ coefficients on filter parameters ω_0 and *Q*.

As a second demonstration, a general impedance converter (GIC) with two CCs is given in Fig. 4 (a) [1]. As noted in [1], this circuit operates as a negative GIC, if both conveyors are of equal polarity, that is both are CCII+ or CCII-. Conversely, if the two conveyors are of opposite polarity, then a positive GIC is obtained. The possible operation for other types of CC is not mentioned.



(a)

(b)

Fig. 4. (a) GIC, (b) its M-C graph.

Now consider general CCs with their graphs-stamps according to Fig. 2 (b). The resulting M-C graph, constructed from the schematics, is in Fig. 4 (b). Evaluating it by inspection yields the formula for input impedance:

$$Z_{in} = \frac{V_1}{I_1} = \frac{Y_3 - \alpha_2 \beta_2 Y_2}{-\beta_1 \gamma_1 \beta_2 \gamma_2 Y_1 Y_2 + \alpha_1 \beta_1 \alpha_2 \beta_2 Y_1 Y_2 - \alpha_1 \beta_1 Y_1 Y_3}$$

After arrangement we have

$$Z_{in} = \frac{Y_3 - \alpha_2 \beta_2 Y_2}{\beta_1 Y_1 [\beta_2 Y_2 (\alpha_1 \alpha_2 - \gamma_1 \gamma_2) - \alpha_1 Y_3]} . \qquad (1)$$

Utilizing two CCIIs, we set $\alpha_1 = \alpha_2 = 0$. Then equation (1) yields:

$$Z_{in} = -\frac{1}{\beta_1 \gamma_1 \beta_2 \gamma_2} \frac{Z_1 Z_2}{Z_3} . \qquad (2)$$

This result confirms the above conclusions from [1] and extends them the possibility of utilizing both the classical and inverting CCs (for inverting CCs, coefficients β are negative).

Analysing equation (1), we conclude that both conveyors must be of the CCII type to achieve the simple impedance conversion according to equation (2).

5 Conclusions

Novel modified M-C graphs are described in the paper. They enable simple modeling of current conveyors CCI and CCII regardless of whether they are positive, negative, noninverting or inverting. Selecting the values of α, β, and γ coefficients and the self-loop gain of graph current node, we can also model some basic CC nonidealities such as inaccuracy of internal voltage follower between *y* and *x* terminals, influence of nonzero R_x resistance, inaccuracies of internal current mirror, etc., without any modification of the flow graph topology.

By reason of its universality, the corresponding matrix-stamp has been incorporated in the SNAP program [2] for symbolic analysis of linear networks.

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