(SEMI)SYMBOLIC MODELING OF LARGE LINEAR SYSTEMS: PENDING ISSUES

D. Biolek, V. Biolková), and J. Dobeš**)*

Department of Telecommunications, Radioelectronics*)
Brno University of Technology
Purkyňova 118, 612 00 Brno, Czech Republic
dalibor.biolek@vabo.cz

Department of Radioelectronics **)
Czech Technical University in Prague
Zikova 4, 166 27 Prague, Czech Republic
dobes@feld.cvut.cz

ABSTRACT

The paper deals with some open problems from the area of symbolic and semisymbolic modeling of linear systems, focusing on combining the symbolic, semisymbolic and numeric computation.

1. INTRODUCTION

Evolution of ever more efficient computers in recent years has enabled a greater utilization of the symbolic methods of circuit analysis in the process of design, testing, and optimization, as well as a combination of these methods and the classical numerical and semisymbolic approaches [1-7].

The symbolic algorithms generate results in the form of analytic formulae. The most frequent utilization is in the area of generation of *s*-domain circuit functions of linearized circuits with lumped parameters. The result is then in the form of a rational fraction function. The coefficients belonging to the individual powers of the *s* (or *z*) operator are then generated as the sums of products of symbols, which specify the parameters of circuit elements. The semisymbolic result means a formula, in which the numerical equivalents of symbolic coefficients appear. Then the only symbol in the equation is the *s* (or *z*) operator.

The symbolic and semisymbolic analyses may be applied also beyond the operator circuit functions. For example, combining semisymbolic and numerical algorithms yields time waveforms of circuit signals as equations.

Today, the symbolic and semisymbolic algorithms are invaluable in numerous applications. A single formula may concentrate a large amount of numerical data, printouts and diagrams. Moreover, the formula offers important connections between the circuit and its behavior. The symbolic algorithms are also used to verify the circuit principle, especially in the area of synthetic elements, frequency filters, linear circuits containing *OpAmps*, current conveyors, and other active elements. It is possible to investigate more effectively the influences of parasitic elements and real properties of components on circuit parameters. Placing symbolic algorithms in optimization loops can achieve such results which are unrealizable when using classical numerical methods. A combination of the symbolic and numerical methods can increase the accuracy of analysis results.

2. SYMBOLIC ALGORITHMS

The symbolic algorithms are disproportionately more time expensive than the numerical algorithms. That is one reason why their utilization is limited to a set of smaller-sized circuits. The complexity and non-transparency of symbolic result are the second limiting factor. Due to the exponential growth of the numbers of symbolic result, the classical symbolic analysis of circuits used in praxis (tens of nodes) leads to extensive formulae which are either non-interpretable or they even exceed the potential of present-day computers.

That is why since the mid-eighties till this time a considerable effort is exerted to find methods which would enable decreasing the result complexity by means of an approximation [1]. The approximation is arranged in such a way that only some terms are retained in the original formula according to a certain error criterion. This procedure can be performed only if the numerical values of symbolic parameters are known at least approximately. Three basic methods of simplification have been subsequently developed: Simplification After Generation (SAG), Simplification Before Generation (SBG), and Simplification During Generation (SDG). For more details see [2], [3].

The mathematical model of a linearized circuit is represented by a set of linear algebraic equations in the operator form. For the symbolic solution, the Cramer rule turned out to be most advantageous, when the generation of symbolic formula is reduced to the symbolic computation of the matrix determinant [4]. Two basic approaches exist: algebraic and topological.

The algebraic method utilizes algebraic operations above the circuit matrix. Then the Laplace determinant expansion follows. Because of the huge number of terms of this expansion even for relatively small circuits, it is necessary to use special algorithms.

For instance, the procedure by Prof. Čajka [8] starts from the fact that each element of the analyzed circuit has its special code. It is described by a special matrix – so called “stamp”. The nonzero stamp elements are called *parameters*. The parameters are written in the circuit matrix **H** by a special procedure. However, this matrix is not compiled by the classical method but as elements (*atoms*) in the form of a table. If the computed algebraic minor has n rows and n columns, then the symbolic result will contain a sum of several products of just n parameters that occur in the circuit. The algorithm finds all combinations of n parameters from the total number of m parameters in the **H** matrix and explores the existence of every product. It utilizes a special determinant expansion according to all the parameters in the product. In so doing there can be easily identified the existence/nonexistence of the examined product. Fast identification of the vanishing terms of the Laplace expansion of the determinant contributes to the considerable acceleration of the symbolic computation.

The second method, so-called topological method, uses the *Signal Flow Graph* (SFG) theory for the indirect computation of determinants. Numerous methods belong to this category, for instance "Tree Enumeration Methods", „Flow-Graph Methods", etc.

The utilization of both the algebraic and topological methods requires the solution of two basic problems:

- ✓ For the analysis of larger circuits, the method of generating symbolic formula has to enable the SBG or SDG approximation.
- ✓ The Cramer rule enables the solution of a general set of linear equations, while the equations describing an electrical circuit are subject to additional regularities, e.g. they have to fulfill Kirchhoff's laws. This fact leads to the occurrence of such terms in the symbolic result which are mutually subtracted. The number of these reversible terms can be decreased by a proper design of the computing method. Thus we also decrease the memory and time requirements.

Concerning the topological methods, the most important progress has been achieved in the mid- nineties. By means of the matroid theory, a method based on the analysis of so-called voltage and current circuit graph has been formulated [5]. This method:

- does not generate reversible terms,
- enables generation of the terms in the symbolic formula in descending order according to their dimension; this leads to a useful utilization of the SDG technique.

In the area of matrix methods, attention was given in the eighties to the minimization of the reversible terms and to a subsequent application of the SAG method. A rather successful implementation is in the ISAAC program [7] by the group of W.Sansen. In the mid-nineties, the SBG techniques were developed also for some algebraic methods of symbolic analysis [8]. The *Analog Insydes* system by R.Sommer [9] works on this principle.

It is apparent that the potential of algebraic methods has not been fully exploited. In practical experiments, the authors frequently stated that the different methods of compilation of circuit equations result in different efficiency of the approximation algorithms. The efficiency is measured by the number of remaining terms with the defined declination from the nominal course preserved [9]. This reality evidently results from the fact that the process of equation simplification has not been analyzed systematically from the circuit or topological point of view.

3. OPEN PROBLEMS IN COMBINING (SEMI)SYMBOLIC AND NUMERIC ALGORITHMS

It is often necessary to complete symbolic algorithms with numerical ones. For example, for numerical stability, the preferred computation of zeros and poles of a circuit function is numerical from the circuit matrix rather than from the coefficients of the semisymbolic result [10-16]. For the purposes of simplifications using the SDG technique, it is advisable to obtain the semisymbolic result by another way than the symbolic analysis.

The computation of zeros and poles is a weak point of contemporary numerical analysis of large linearized circuits. The computation is usually done in two steps: First the deflation of circuit matrix is performed and then the algorithm of finding eigenvalues follows. The currently utilized QR and QZ algorithms [10-13] achieve satisfactory accuracy only while solving certain circuits. They mostly break down for the stuff systems, in the case of multiple roots, and for higher-order dynamic systems (around the order 50 and higher) [15]. Recently the authors of this contribution have worked on the so-called *reverse generalized eigenvalue problem*, which could bring certain improvements.

Another ill-conditioned problem is finding the roots of polynomials, i.e. the computation of zeros and poles directly from the semisymbolic form of circuit function. It should be noted that the commonly preferred algorithms, implemented for example in MATLAB, in some cases offer significantly worse results than the less-known procedures [17,18]. The word length representing numerical values in the MATLAB language is fixed and limited to approximately 16 significant places. Thus the accuracy of computation of polynomial roots is limited. This drawback could however be overcome by using other languages, for instance MAPLE or MATHEMATICA.

The next ill-conditioned problem – partial fraction expansion – is also connected to the semisymbolic analysis. Numerous methods have been published. The algorithm by CHIN and STEIGLITZ [19] shows good results. The question is whether the numerical precision could be increased by a combination of numerical, symbolic and other new approaches. Some of our present results indicate this possibility.

The partial fraction decomposition is also connected with the problem of finding the greatest common polynomial divisor for the numerator and denominator of the simulated network function, i.e. the problem of finding the zeros and poles that are mutually cancelled. This procedure can suppress the influence of idling (dormant) parts of the network on the network function in question. At the same time, the numerator and denominator powers are decreased and finding the roots is simplified.

4. SOLVING THE OPEN PROBLEMS

Solving the open problems above involves a detailed analysis of the contemporary methods of both classical and approximate symbolic analyses. Consequently, these methods have to be optimized with the utilization of specific features of mathematical models which are given by the system and topological regularities: Finding the optimum form of the initial system equations, determination of the procedures and criteria of simplification of symbolic terms regarding the consequences of this simplification in the frequency, time and *s*-domains.

The prerequisite is the improvement of contemporary and the design of new numerical methods, which would support and complete the symbolic methods: generalized eigenvalue problem of large systems, finding the roots of characteristic equation, finding the greatest common polynomial divisor, methods of the secondary roots polishing [20], etc.

The progressive approach is represented by combined symbolic-numerical methods with enhanced accuracy. Let us mention highly precise methods of finding zeros and poles based on the numerical principle with the utilization of so-called *rational arithmetic*. Their utilization in the algorithms of approximate symbolic analysis seems to be promising. Hard work is expected in the area of modification of the algorithms of generalized eigenvalue problem with the utilization of sparse techniques.

5. CONCLUSION

The main pending issues of (semi)symbolic/numeric analysis of large linear circuits are outlined. Their solution requires cooperation of workers from the areas of numerical mathematics and system/circuit theory. Authors of this contribution already have developed some special algorithms, which are integrated step by step in the SNAP (Symbolic and Numeric Analysis Program) [21-23] and CIA (Circuit Interactive Analyzer) [15] software tools.

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