OAHU -OBJECT ANALYSIS HAKE UTILITY

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Abstract - In the paper, 9 rules are given on which the operation of so-called Object Analysis Hake Utility is based. The basic resource is represented by VERTEX graphs, which describe a set of nonlinear equations between the parameters of circuit elements and the coefficients of circuit functions. Some feasible techniques of transforming these graphs into simpler structures are shown. As a result, the couplings among variations of individual parameters are made obvious that hold the monitored circuit features.

I. INTRODUCTION

The principles of so-called VERTEX and causal graphs are given in [1] and [2]. Formulating the coefficients of the operator circuit functions of a linear dynamic system as a function of element parameters, we have generally the sum or subtraction of products of these parameters. From the mathematical point of view, this is a set of nonlinear equations of a special form, where the number of equations does not correspond to the number of unknowns. These equations can be described by the VERTEX graphs, first published in [1] and [2]. As shown in [3], in some cases the VERTEX graphs can be transformed into the so-called causal graphs, which will enable monitoring the effects of modification of individual parameters around their nominal values, and the possible ways of compensating these modifications by other parameters as well.

In this paper, we outline the principle of so-called Object Analysis Hake Utility (OAHU). OAHU is a computer algorithm, based on the theory of VERTEX and causal graphs, enabling an efficient simplification of a set of nonlinear equations, elucidating the causal relations among the individual element parameters of the circuit examined.

II. BASIC RELATIONS AMONG THE ELEMENT PARAMETERS AND THEIR GRAPH REPRESENTATION

Hereafter we will denote element parameters by symbols P_i, and their reciprocal values by symbols Q_i=1/P_i. The constants, which are given by the numerical system characteristics, will be denoted by symbols α_i, or α if their indexing will not be necessary. Their reciprocal values will then be β_i =1/α_i, or β =1/α. A quantity, where it does not matter if it is a constant or parameter, will be denoted by symbol C.

Examples of common couplings among the parameters are as follows:

$$P_1P_2 = \alpha \Rightarrow Q_1 = P_2\beta,\, Q_2 = P_1\beta,\, Q_1Q_2 = \beta \quad (1)$$

$$P_1P_2P_3..P_N = \alpha \Rightarrow Q_1 = P_2P_3..P_N\beta,\, ...,\, Q_1Q_2Q_3...Q_N = \beta \quad (2)$$

$$P_1 + P_2 = \alpha \quad (3)$$

$$P_1P_2 + P_3P_4 = \alpha \quad (4)$$

$$(P_1 + P_2)P_3 = \alpha \Rightarrow P_1P_3 + P_2P_3 = \alpha,\, Q_3 = (P_1 + P_2)\beta \quad (5)$$

$$(P_1 + P_2)P_3 = \alpha_1,\, P_2 = \alpha_2P_1 \Rightarrow P_1P_3 = \frac{\alpha_1}{1+\alpha_2},\, P_2P_3 = \frac{\alpha_1\alpha_2}{1+\alpha_2} \quad (6)$$

$$P_1C = \alpha_1,\, P_2C = \alpha_2 \Rightarrow P_1Q_2 = \alpha_1\beta_2,\, Q_1P_2 = \alpha_2\beta_1 \quad (7)$$

$$P_1P_2...P_N\alpha_1 = \alpha_2 \Rightarrow P_1P_2...P_N = \alpha_2\beta_1 \quad (8)$$

$$P_1P_2P_3P_4 = \alpha_1,\, P_3P_4 = \alpha_2 \Rightarrow P_1P_2 = \alpha_1\beta_2 \quad (9)$$

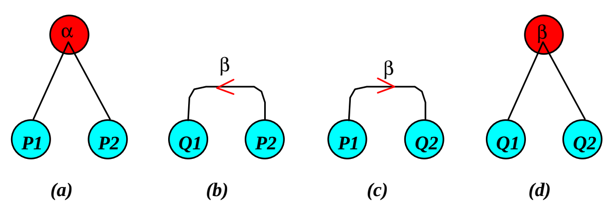

Fig. 1 (a) VERTEX, (b), (c) causal, (d) VERTEX (with inversion base) graph accordant with Eq. (1).

The example of *VERTEX* graph in Fig. 1 (a) corresponds to the first product in Eq. (1). The parameters P_1 and P_2 in the product are given in the so-called *graph base*. The product itself is represented by undirected paths, which converge in the graph vertex. The *vertex value* is equal to the product value.
The so-called *causal graphs* are shown in Figs. 1 (b) and (c). They arose from the original VERTEX graph after inverting one of the base elements. The causal graph is oriented and it models a linear equation. The rule for transforming the VERTEX graph to a causal graph is obvious: we perform the inversion of one base element, and then we draw an oriented path from the

second element to the inverted element. The path gain is the reciprocal value of the vertex value.

The rule for the inversion of the whole base is demonstrated in Fig. 1 (d). This inversion must be accompanied by the inversion of the vertex value.

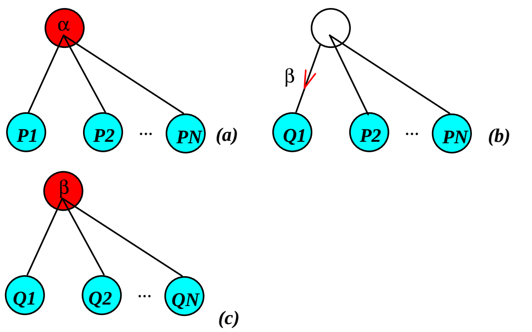

Fig. 2 (a) VERTEX, (b), hybride, (c) VERTEX (with the inverted base) graph accordant with Eq. (2).

The generalization to a multielement base is shown in Fig. 2. From Fig. 2 (b), which corresponds to the second equation in (2), it is obvious that the Q_1 element is expressed by the path directed from the vertex, representing the product of the other base elements. However, this vertex value is not given because it is not a fixed value. We speak about a *free vertex*. The gain of the directed path is the reciprocal value of the original vertex.

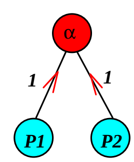

Fig. 3 Summing graph accordant with Eq. (3).

The graphical representation in Fig. 3 probably does not need any special comment.

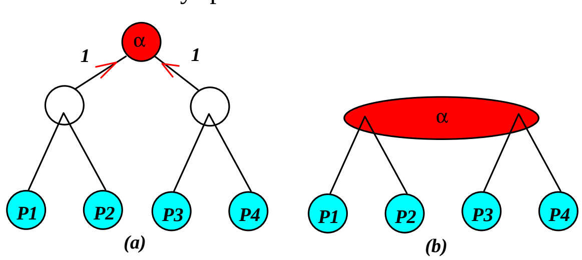

Fig. 4 VERTEX graph with summing vertex accordant with Eq. (4), (a) general representation, (b) simplified form.

The graphical representation of Eq. 4 is given in Fig. 4 (a). If the gains of all directed paths in this graph are equal to one, we can pass onto the simplified graph in Fig. 4 (b). The given vertex will be denoted as *summing vertex*.

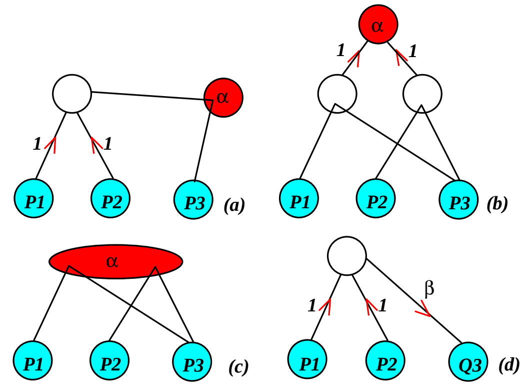

Fig. 5 (a), (b), (d) hybride, (c), VERTEX graph accordant with Eq. (5).

A frequent case is described by Eq. (5) when the couplings among the parameters can be modelled by the product of one parameter and the sum of other parameters. The graphs in Fig. 5 (a), (b), and (c) can be compiled in terms of the above rules. The base transformation in order to determine the Q_3 element is obvious from Fig. 5 (d).

The graph in Fig. 6 is a generalization of the case in Fig. 5, when some proportional coupling exists between the P_1 and P_2 parameters. This coupling will enable canceling of the summing subgraph, utilizing a simple procedure from Fig. 6 (b) and (c). The corresponding rule can be referred to as *the rule of dependent base*.

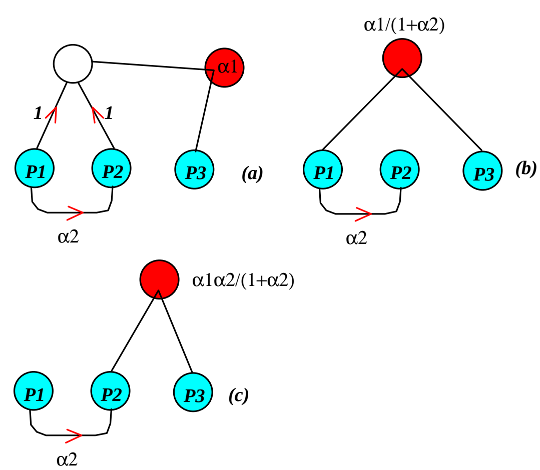

Fig. 6 Graphs corresponding to Eq. (6).

Fig. 7 shows various possibilities of graph modification in case of multiplicative dependence of two parameters on a single quantity where it is not specified if this quantity is a parameter or some system constant. This dependence will enable determining the causal relationship between both parameters. We speak about *the rule of the third quantity*.

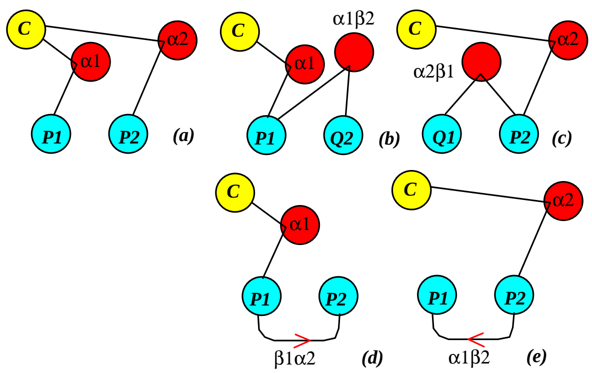

Fig. 7 Graphs corresponding to Eq. (7).

Fig. 8 shows the *absorption of the constant* in the VERTEX graph according to Eq. (8).

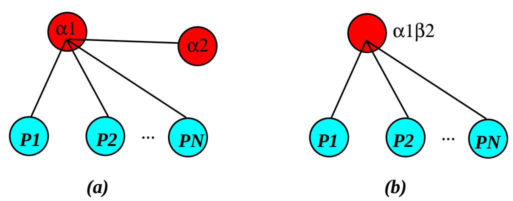

Fig. 8 VERTEX graphs corresponding to Eq. (8).

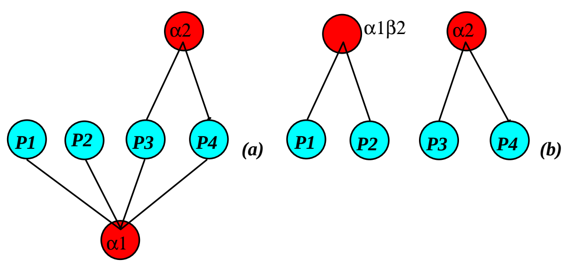

Fig. 9 VERTEX graphs corresponding to Eq. (9).

If the base elements form part of more VERTEX graphs, it is useful to perform the reduction to vertices with minimum number of base elements, as shown in Fig. 9. We speak about *the rule of reduction of the long-base vertices*.

The utilization of the above rules will be demonstrated on the following example.

III. EXAMPLE OF UTILIZATION

The filter in Fig. 10 has three different transfer functions, depending on the choice of outputs HP, BP, or LP. The symbolic analysis leads to the following results:

The denominator of all transfer functions is the same:

$$b_2 s^2 + b_1 s + b_0 , \tag{10}$$

$$b_2 = R_2 R_3 R_4 C_1 C_2 (R_5 + R_6),\; b_1 = R_4 R_6 C_2 (R_1 + R_2),$$
$$b_0 = R_1 (R_5 + R_6). \tag{11}$$

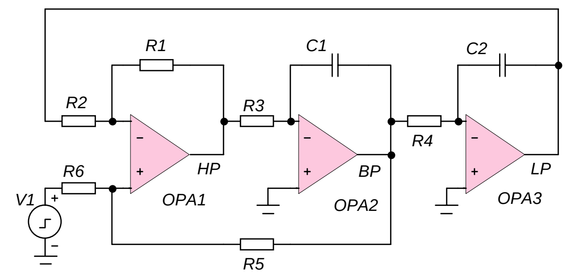

Fig. 10 Example of the KHN-type filter [4].

The numerators for HP/BP/LP outputs are as follows:

$$a_0 ,\; - a_1 s ,\; a_2 s^2 , \tag{12}$$

$$a_0 = R_5 (R_1 + R_2),\; a_1 = R_4 R_5 (R_1 + R_2) C_2 ,$$
$$a_2 = R_3 R_4 R_5 C_1 C_2 (R_1 + R_2). \tag{13}$$

The purpose of the following analysis is finding a law of varying the parameters

$$R_1, R_2, R_3, R_4, R_5, R_6, C_1, C_2$$

such that all the above transfer functions are preserved.

Relations (11) and (13) form a set of 6 nonlinear equations with 8 unknowns. The graphical interpretation is given in Fig. 11. In this graph, use is already made of the fact that the a_1 coefficient can be expressed by means of the a_0 coefficient, and a_2 by means of a_1.

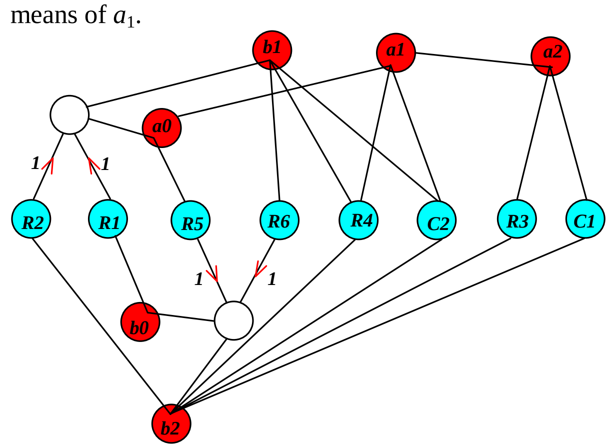

Fig. 11 The original graph corresponding to the set of equations (11) and (13).

The first simplifying step can be seen in Fig. 12. The rule of *the absorption of the constant* according to Fig. 8 is used.

In the second step, the *rule of reduction of the long-base vertices* from Fig. 9 is used for further simplification. The result is given in Fig. 13. It is obvious already in this stage that the individual sets R_4-C_2 and R_3-C_1 are set off by the total set of parameters, which have neither mutual couplings nor couplings to the third set R_2-R_1-R_5-R_6.

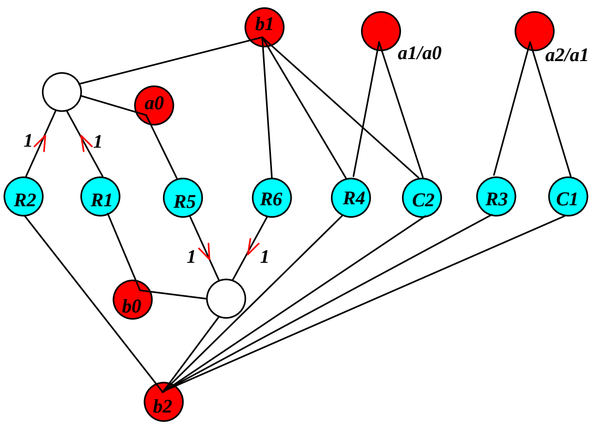

Fig. 12 Graph after the first simplifying step.

So far it is not clear whether the last mentioned set will be decomposed into some subsets without mutual couplings.

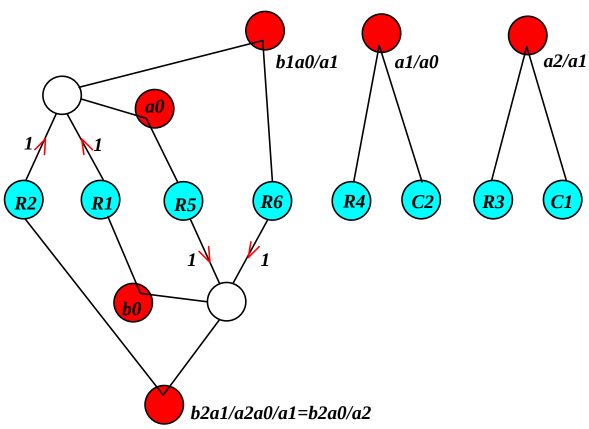

Fig. 13 Graph after the second simplifying step.

In the third step, the *rule of the third quantity* from Fig. 7 is used. The result is shown in Fig. 14.

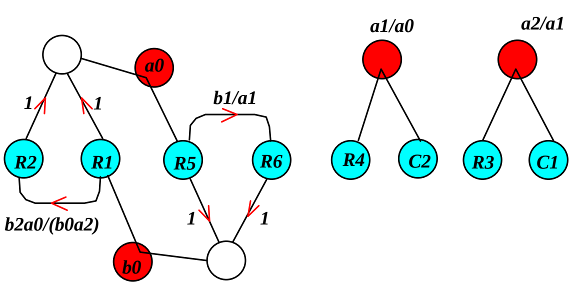

Fig. 14 Graph after the third simplifying step.

In the next step, the *rule of dependent base* from Fig. 6 comes into play. The result is shown in Fig. 15. It can be proved that both vertices above the R_1-R_5 base have identical values.

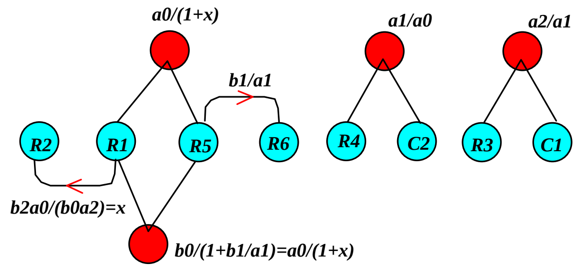

Fig. 15 Graph after the fourth simplifying step.

The results of the final graph modifications can be seen in Fig. 16. It should be noted that the original transfer functions will remain unchanged while varying the parameters inside three mutually independent sets R_1-R_2-R_5-R_6, R_4-C_2, and R_3-C_1. This knowledge can be utilized, for example, for the optimization of ratios of element parameters in the KHN filter.

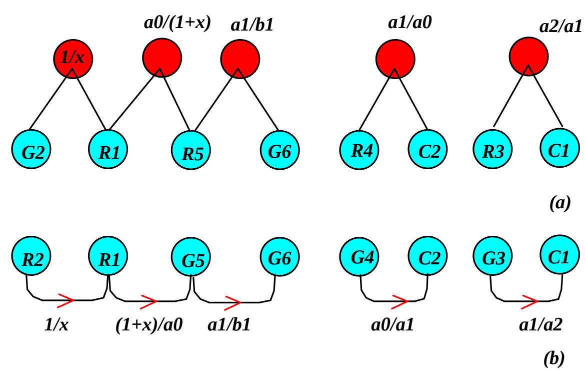

Fig. 16 Graph after the fifth simplifying step, (a) VERTEX, (b) causal.

IV. CONCLUSIONS

In the paper, 9 basic rules are described, enabling a simplification of complicated description of nonlinear couplings among the individual parameters of components of linear circuits. To put this technique into practice, it is necessary to develop algorithms of automatic compilation of VERTEX graphs, and, above all, algorithms of their successive simplification. The VERTEX graphs can be generated from the symbolic results while analyzing the system functions. The SNAP program [5] can be used for these purposes.

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