Flow Graphs for Analysis (not only) Current-Mode Analogue Blocks

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Abstract: - Modified Mason-Coates' flow graphs are introduced in this paper. Utilizing a simple rule, the voltage graph node can be expanded by the current node, which corresponds to the current of a branch oriented out of the incident circuit node. Then an arbitrary type of controlled source can be easily described by means of a flow graph. Applying these procedures, we obtain flow graphs of such circuits that are not commonly utilized, e.g. current conveyors or current feedback amplifiers.

Key-Words: - Mason-Coates' flow graph, node, Current conveyor, Current-Feedback amplifier.

1 Introduction

Signal flow graphs are important tools for "hand-and-paper" analysis of relatively small circuits. Graph methods are of concern not only as a mathematical tool for optimizing the algorithms for computer solution of circuits. They can become an important design tool for the synthesis or a fast verification the required circuit function. Note that in such cases the flow graphs can be of much greater advantage than the modified nodal analysis (MNA). However, this is true on the assumption that the graph method complies at least with the following conditions:

1. Simple rules must exist, on the basis of which we can construct the flow graph directly from the schematic without writing any auxiliary equations.
2. The graph structure must be economical enough, otherwise the analysis of a relatively simple circuit would lead to a complicated and thus hard-to-evaluate graph.
3. Simple rules must exist for graph evaluation, i.e. for extracting the results directly from the graph without any side-computing.
4. The graph must enable the evaluation of voltage & current gains and immittances.

Mason's (M) [1], Coates' (C) [2], or Mason-Coates' (MC) [3] flow graphs belong to the well-known graph techniques. For passive circuits, they match all the conditions given above. However, the layout changes significantly when the model of circuit transformation element is not compatible with the admittance equations of the classical MNA. Then a number of published solutions do not conform to rule No. 1 because of the necessity of compiling the circuit equations prior to constructing the graph.

Several types of signal flow graph have been developed for circuits containing active elements [3], [4], [5]. Their analysis shows the following:

Rules No. 1 and No. 3 are often in contradiction: The way to an economical graph mostly leads through hard-to-remember rules of thumb [4]. Rule No. 3 is fulfilled automatically if the resulting graph is an M-, C-, or MC-graph. Then Mason's well-known general gain formula can be used. Rule No. 4 is not fulfilled for some graphs due to the reduction of some variables in consequence of applying rule No. 2. However, these variables are necessary, for example, for analyzing immittance relations.

A specially modified method of MC graphs with undirected self-loops is described below. This method seems to be optimum as regards fulfilling the above conditions, intended for a quick analysis of both the passive circuits and the active circuits without limitations of the type of circuit element. The limitation of classical MC graphs for controlled current sources is overcome (compiling these graphs was rather complicated). Thus new possibilities of efficient graph analysis of circuits with current elements like current conveyors, current feedback amplifiers (CFAs) and of the analysis of current- or hybrid-mode filters are open.

The paper has the following structure. In Section 2, the necessary knowledge of MC graphs with undirected self-loops and of shortened MC graphs is summarized. The terms *voltage* and *current nodes* of the graph are defined. Section 3 describes the principle of extending the voltage node of the graph to the so-called hybrid node. In Section 4, this principle is applied to the construction of an MC graph of voltage source. In Section 5, such modified MC graphs of all four types of controlled sources are given. Examples of the application to the description of elements like CFA and CCII are shown in Section 6. An illustration of a fast solution of selected circuits is given in Section 7.

2 MC graphs with undirected self-loops

The basic conception of these graphs is described in [3]. Consider the simple circuit in Fig. 1 (a), which is described by the set of equations (1) or (2) according to the MNA:

$$\begin{aligned} Y_1V_1 + Y_3(V_1 - V_2) &= I_1 \\ Y_2V_2 + Y_3(V_2 - V_1) &= I_2 \end{aligned} \quad (1) \qquad \begin{aligned} (Y_1 + Y_3)V_1 &= I_1 + Y_3V_2 \\ (Y_2 + Y_3)V_2 &= I_2 + Y_3V_1 \end{aligned} \quad (2)$$

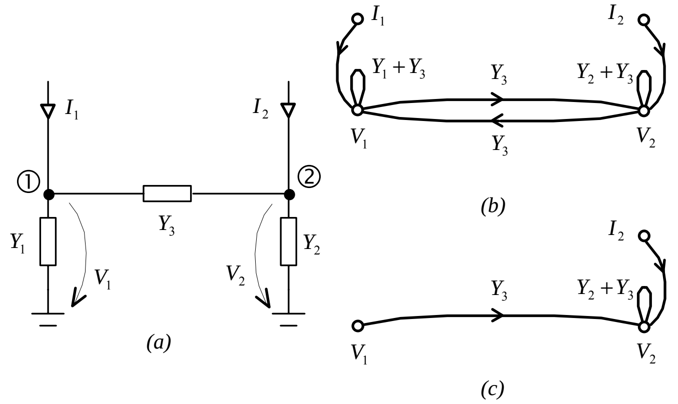


Fig. 1 (a) Example of analyzed circuit, (b) MC graph, (c) shortened MC graph.

In the MC graph in Fig. 1(b), which is an interpretation of equations (2), voltage and current nodes V_1, I_1, V_2, and I_2 correspond to circuit nodes 1 and 2. Each of the two graph voltage nodes, denoted V_1 and V_2, along with the incoming branches, represents one of equations (2). The self-loop gain of a given voltage node is equal to the sum of all admittances connected to a corresponding circuit node. The path gain between two voltage nodes is equal to the admittance connected between the corresponding circuit nodes.

If the circuit in Fig. 1 (a) is driven from a voltage source V_1 whose voltage is known, and if we are not interested in the input current I_1 flowing from this source, then we do not need the first equation in (2) for the analysis. The corresponding reduction of paths and the self-loop in the MC graph are given in Fig. 1 (c).

The shortened graph can be of advantage in cases of calculating the voltage gains. A complete graph is necessary, for example, when solving immittance relations.

The evaluation of the MC graph is done using the well-known Mason-Coates' formula described, for example, in [3]. That is why we will not give it here.

3 Description of current branch by modified MC graph

Consider the circuit in Fig. 2 (a). Current I_A flows through the short connection between nodes 1 and 2. This current will be included in a set of unknown variables. A typical example is represented by the input gate of current-controlled source, connected between nodes 1 and 2.

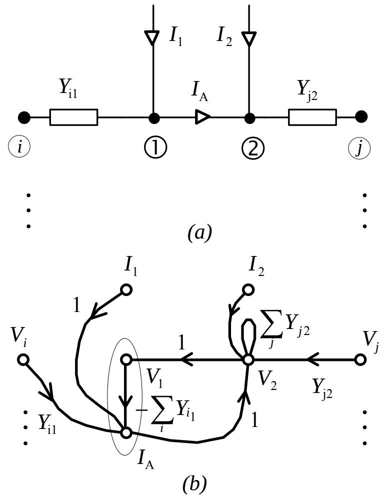


Fig. 2. (a) Circuit with current branch, (b) MC graph corresponding to equations (3).

If current I_A were outside our interest, both nodes could be reduced to a single one, and the classical MC graph would be used for the description. However, in our case it is advisable to consider the following equations, which model the influence of current I_A on the current conditions in all nodes:

$$\begin{gathered} V_1 = V_2 \\ I_A = I_1 - \sum_i Y_{i1}V_1 + \sum_i Y_{i1}V_i \\ \sum_j Y_{j2}V_2 = I_2 + I_A + \sum_j Y_{j2}V_j \end{gathered} \quad (3)$$

The special MC graph in Fig. 2 (b) corresponds to these equations. It is designed to maximally respect the rules of compiling the MC graph directly from the circuit diagram. Note a new structure – a couple of voltage and current nodes V_1 and I_A instead of the original voltage node V_1. This couple is locked inside an auxiliary pattern, which emphasizes a "hypernode". Let us call it the *hybrid node*. The following new rules are true for the hybrid node:

- The self-loop gains of the internal voltage and current nodes are equal to one and we do not need to draw them.
- Only one path is directed into the voltage node, which describes a coupling condition between the voltage of the circuit node and the voltage of the interconnected node. In this case, the path gain is 1, and this path is directed from voltage node V_2.
- The path directed from the internal voltage node into the internal current node has a gain that is equal to the negative sum of all the admittances connected to circuit node 1.

- The paths directed from without into the current node are the same as the paths that would be directed into the voltage node in the classical MC graph.
- The path with gain 1 is directed from the inner current node outside the hybrid node into voltage node V_2, which corresponds to the circuit node interconnected with node 1.

The above procedure will be used for the description of both the classical and the controlled ideal voltage sources.

4 Description of voltage sources by modified MC graph

A generalization of the above case is in Fig. 3 (a). Instead of short connection, an ideal voltage source V is connected between nodes 1 and 2. Now the nodal voltage V_1 is given by the sum of nodal voltage V_2 and source voltage V. The corresponding MC graph is in Fig. 3 (b).

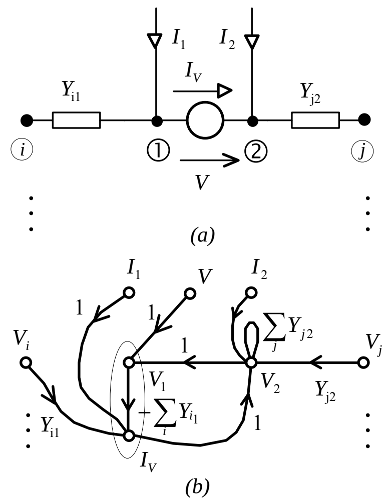


Fig. 3. (a) Circuit with a voltage source, (b) modified MC graph.

If we are not interested in the source current, we can achieve a substantial simplification of the MC graph for grounded voltage sources (see Fig. 4 a). Then the equation evaluating the current I_V and thus the inner current node in the MC graph need not be considered. The shortened graph in Fig. (c) results from the original graph in Fig. (b). It is advisable to retain the boundary of voltage node V_1 – we indicate that no further paths may pass into this node. However, paths can go out of this node. For instance, from node V_1 into node V_i there will be an oriented path with gain Y_{i1} (if no voltage source is connected to circuit node i).

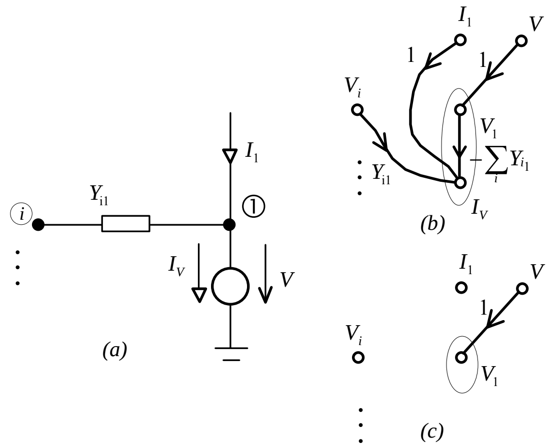
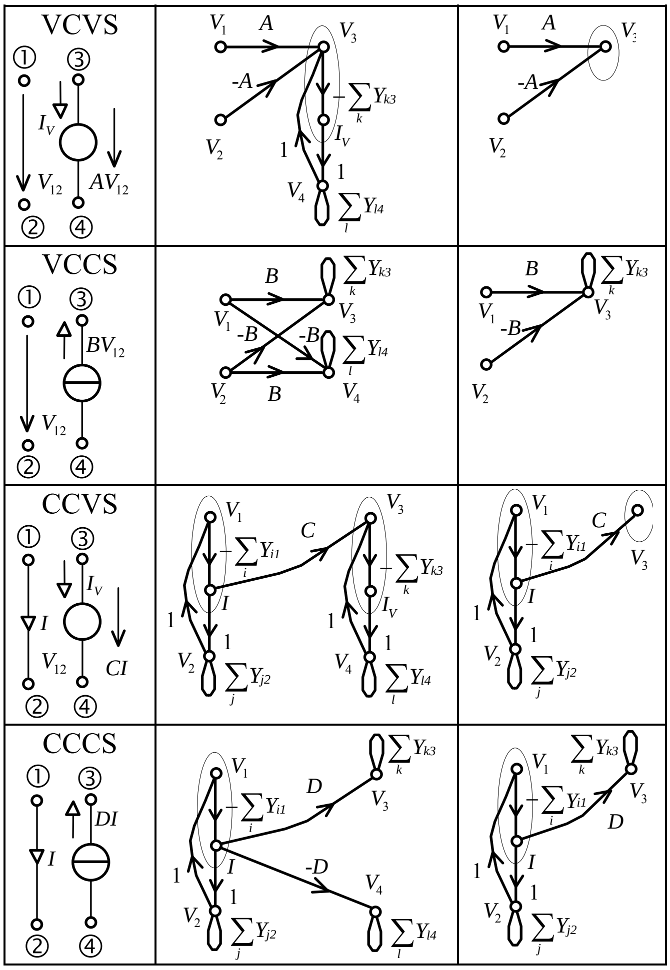


Fig. 4. (a) Circuit with grounded voltage source, (b) modified MC graph, (c) shortened MC graph.

5 MC graphs of controlled sources

Applying the above procedures leads to the graphs of controlled sources in Table 1. In the last column, there is a graph version for grounded sources. For voltage sources, the modification is chosen when we are not interested in the source current.



Tab. 1. Controlled sources (column 1), their MC graphs (column 2), shortened MC graphs (column 3).

6 MC graphs of VFA, CCII and CFA elements

The ideal voltage feedback amplifier (VFA) in Fig. 5 (a), whose gain A converges to infinity, is described by the following equations:

$$V_3 = \lim_{\substack{A\to\infty \\ V_1\to V_2}} A(V_1 - V_2)\text{, or}$$

$$0.V_3 = V_1 - V_2 \qquad (4)$$

The VFA output behaves as an ideal voltage source, and the input conductance is zero. The corresponding MC graph is in Fig. (b). If the output current need not be computed, we can choose the simple graph in Fig. 5 (c).

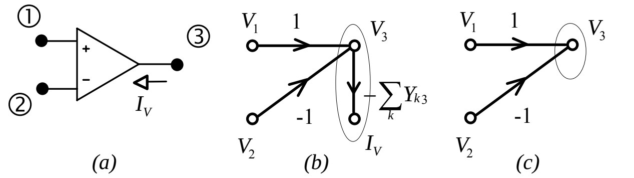


Fig. 5. Ideal OpAmp – VFA type, (a) circuit diagram, (b), MC graph, (c) shortened MC graph.

The circuit diagram of ideal 2nd generation positive/negative current conveyor and its behavioral model are in Figs. 6 (a) and (b). The MC graph in Fig. (c) is simple. However, note that for the positive/negative CCII the path gain from node I to node V_3 is –1/+1.

It is obvious from Fig. 7 (b) that the OpAmp -CFA (Current Feedback Amplifier) type (Fig. 7 a) is based on a CCII completed by a unity-gain buffer. The corresponding MC graph is in Fig. 7 (c) (or its simplified version in Fig. 7 (d) if the output current is not requested). The appropriate zero-gain self-loop is due to the zero transadmittance at the input terminal of the buffer.

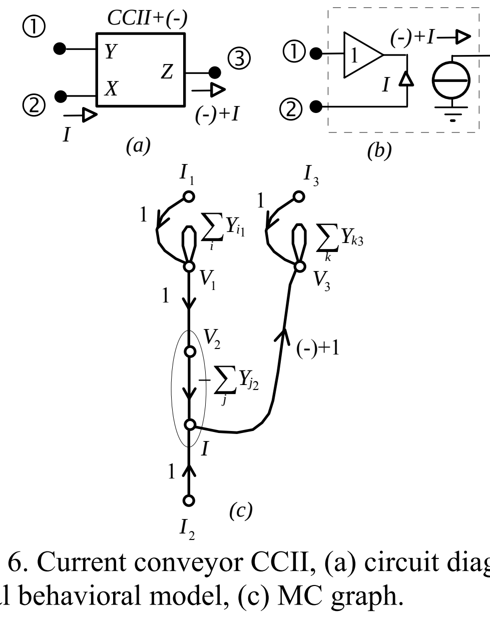


Fig. 6. Current conveyor CCII, (a) circuit diagram, (b) its ideal behavioral model, (c) MC graph.

7 Examples of analysis by means of novel graphs

The demonstrations are focused on circuits with CFA and CCII elements.

A simple noninverting amplifier based on the CFA element is shown in Fig. 8 (a). Evaluating the graph in Fig. 8 (b) yields the well-known voltage gain formula:

$$\frac{V_2}{V_1} = \frac{-G_1 - G_2}{0 - G_1} = 1 + \frac{R_1}{R_2}$$

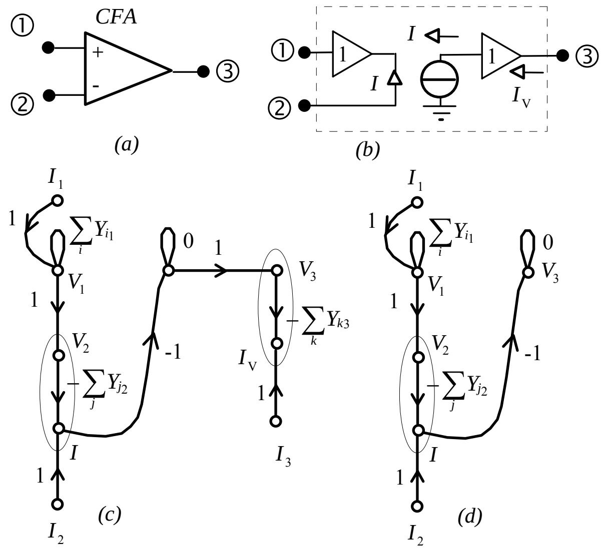


Fig. 7. OpAmp - CFA, (a) circuit diagram, (b) its ideal behavioral model, (c) MC graph, (d) shortened MC graph.

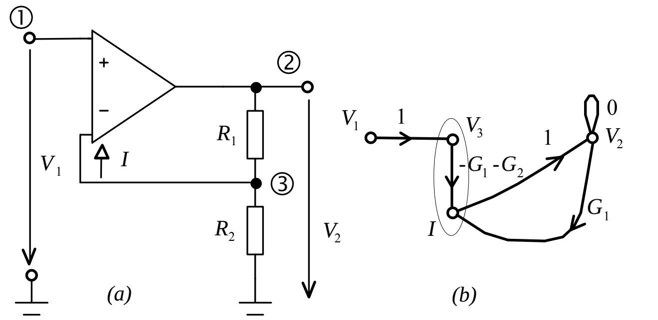


Fig. 8. Non-inverting amplifier with CFA element, (b) MC graph.

An active filter with two CCII+ elements operating in the voltage mode [6] is given in Fig. 9 (a). It is necessary to find voltage gain V_3/V_1.

The corresponding MC graph is in Fig. 9 (b). Evaluating this graph yields

$$\frac{V_3}{V_1} = \frac{G_3(G_3 + sC_2) - G_3^2}{sC_4(G_3 + sC_2) + G_1G_3} = \frac{sR_1C_2}{s^2R_1R_3C_2C_4 + sR_1C_4 + 1},$$

which is the result from [6]. Substituting numerical values leads to the conclusions that this 2nd order bandpass filter has frequency f_0 = 1.1MHz and quality factor Q = 11.

An illustration of the negative impedance converter is given in Fig. 10 (a) [7]. Let us determine how the input impedance depends on working impedance Z, on gain A of the internal buffer (ideally A = 1), and on gain B of the internal current mirror (ideally B = 1) of the current conveyor.

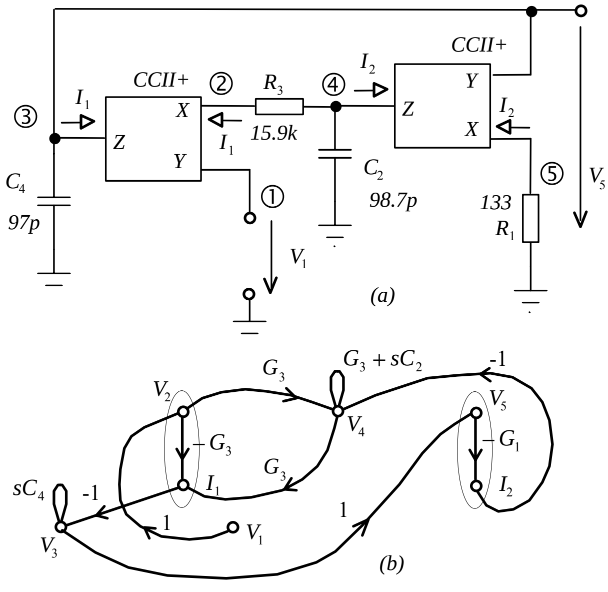


Fig. 9. Bandpass filter with two CCII+, (b) MC graph.

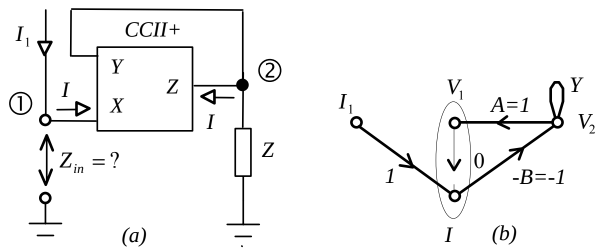


Fig. 10. (a) Negative impedance converter with CCII+, (b) MC graph.

In the graph in Fig. 10 (b), the path gain between internal nodes V_1 and I is zero because no admittances are connected to node 1. Evaluating the graph yields immediately

$$Z_{in} = \frac{V_1}{I_1} = \frac{-AB}{Y} = -ABZ = -Z \; .$$

Fig. 11 (a) represents the Sallen-Key current-mode filter [8]. The relation V_1=0 holds because of the grounded Y-terminal of the current conveyor. An analysis of the graph leads to the transfer function

$$\frac{I_{out}}{I_{in}} = \frac{G_1 G_2}{(G_1 + G_2 + sC_1)(G_2 + sC_2) - G_2^2 - sC_1 G_2} =$$

$$= \frac{1}{s^2 R_1 R_2 C_1 C_2 + sC_2(R_1 + R_2) + 1} .$$

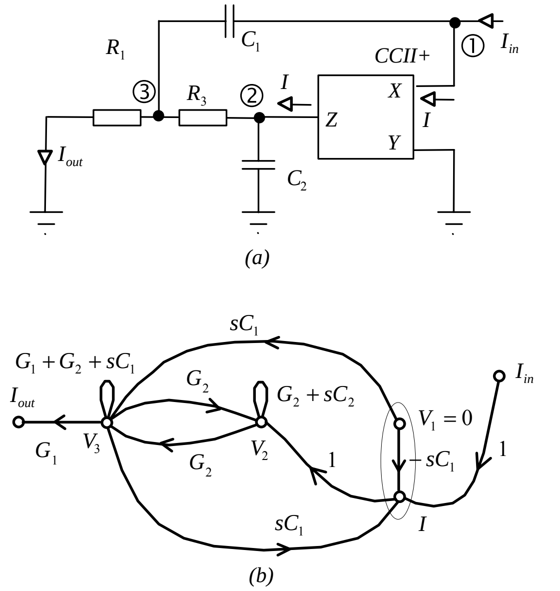


Fig. 11. (a) Curent-mode Sallen-Key filter, (b) MC graph.

8 Conclusion

The described modification of MC graphs, when so-called hybrid nodes are introduced, enables compiling in a simple way the graph of a circuit with arbitrary active elements without any limitation. This fact can be utilized in the analysis of circuits with transformation elements, such as the CCII current conveyors, where compiling the classical MC graphs is complicated.

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