VIRTUAL DYNAMICS OF RESISTIVE CIRCUITS

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Abstract. The thermodynamical principle of minimum entropy production is used to discuss the stability testing of the operating points of nonlinear circuits. The physical background of the theory of virtual dynamics of the resistive circuits is explained. The proof of the general stability theorem is given.

1. Introduction

It is shown in [1] that the stability testing of linearized circuits with the controlled sources using well-known criteria based on the testing of poles location can lead to the results which do not correspond to the reality. In some cases, all poles can lie on the left half-plane in spite of the evident circuit instability. Green showed in [2] that the cause can consists in the wrong modeling of the inertia of the controlled sources and the absence of prospective parasitic reactance elements which play important role in the stability of investigated circuit. For this reason, he divided whole problem to two parts: We investigate so-called potential stability of dc operating point of resistive circuit and the stability of corresponding dynamical circuit after augmenting of resistive circuit by reactance elements. Dc operating point can be either potentially stable or unstable. Unstable operating point cannot be stabilized by augmenting resistive circuit by the reactance elements. The potential stability is the necessary but not sufficient condition of the stability of corresponding dynamical circuit.

In [2], Green described so-called „Γ-test“ as an algorithm of identifying of unstable DC operating points. However, this algorithm is based on the necessary bur not sufficient condition of instability. As a result, some of unstable operating points cannot be identified. Therefore we published in [3-6] the theory of so-called virtual dynamics of resistive circuits which leads to the general criterion of potential stability of DC operating point. We briefly summarize main results.

In accordance with Green, the nonlinear circuit is linearized around investigated operating point. The linear model containing dependent sources is obtained. But as a novelty, we complete outputs of voltage (current) controlled sources by so-called virtual sources δV (δI) with the aim to deflect operating point from its equilibrium state. The system motion (reaction) is then investigated using so-called virtual trajectories in the space of virtual shifting (virtual space). The stability of the operating point can be considered from the character of the virtual trajectories around the operating point. Some virtual eigenvalues correspond to the virtual trajectories. The necessary and sufficient condition of operating point potential stability (the stability theorem) is formulated in [1, 3] without the proof:

All virtual eigenvalues have to lie on the *complex right half-plane*.

The principle how to compute virtual eigenvalues from the linear model with the controlled sources is also indicated in [1].

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In other words, this method is based on the idea that the virtual (thought) deflection of the operating point results in its attraction back, or on the contrary, its displacement along the corresponding virtual trajectory. The set of these trajectories is given by the solution of following matrix equations which can be compiled using the modified nodal approach [1]:

$$\delta \mathbf{V} = \mathbf{R}\delta \mathbf{I} \text{ or } \delta \mathbf{I} = \mathbf{G}\delta \mathbf{V} \tag{1}$$

Then the virtual eigenvalues are the eigenvalues of matrix **R** or **G**, respectively.

In following part, we investigate aforementioned approach from the thermodynamical point of view.

Thermodynamical aspects

Ohm's law which is the basis of calculations in DC resistive circuit is the phenomenology law about the relationship between the average voltage and average flow intensity of large amount of charge carriers across given cross-section. Both quantities fluctuate round their average values. These fluctuations round the operating point are real and they can be assessed for given circuit and temperature. Obviously, only stable operating points, e.g. points with the „attraction ability" can be kept in the ideal DC resistive circuit. The „attraction time" or the relaxation constant is given by the thermodynamic characteristic of circuit. In the sense mentioned above, the virtual trajectories round the operating point are real trajectories along which the fluctuation is damped or propagated, depending on the attraction ability of the operating point.

The total power inside the circuit is given by quadratic form for arbitrary fluctuation

$$\delta\mathbf{V}^T \cdot \delta\mathbf{I} = \delta\mathbf{I}^T\mathbf{R}^T\delta\mathbf{I} = \delta\mathbf{V}^T\mathbf{G}\delta\mathbf{V}\,. \tag{2}$$

The power will be positive if all eigenvalues of matrix **R** or **G** are positive. Thus, each fluctuation is utilized to increase total heat energy of the system. This knowledge is important especially on the field of thermodynamics.

Let us try to derive stability condition of DC resistive circuits from the thermodynamics point of view.

First law of thermodynamics is the law of energy conservation: the inside energy of isolated system is constant during the motion:

$$U = A + Q = \text{const}, \tag{3}$$

where the energy is redistributed between the heat component Q and the work A.

Second law of thermodynamics: *Clausius* has invented that following equation is true for infinity slow changes of the trajectory:

$$\oint_\Gamma \frac{dQ}{T} = \oint_\Gamma dS = 0\,. \tag{4}$$

The new quantity - entropy S is preserved during the reversible processes. T is the absolute temperature.

Generally speaking, the entropy can arise in the system and in the same time it can be exported outside the system. We can write equation of the entropy balance

$$dS/dt = P(t) + J(t) \tag{5}$$

where $P(t)$ is so-called entropy production or the amount of entropy arisen during the time unit as a consequence of irreversible processes inside the system. The second element $J(t)$ represents flow of entropy into the system from the environs across its surface.

During the motion, the isolated system is controlled by the principle which has been formulated by *Prigogine* [4]:

During the evolution, the system tends to the state with the minimum entropy production.

After system being deflected from its stable DC operating point, such motion is done that the entropy production will fall in time. The minimum production of the entropy will be reached at the operating point. Thus, the entropy production represents scalar potential which characterizes system evolution round the operating point (see Fig.1).

For stable operating point, fluctuations will always decrease the entropy ($J(t) < 0$ - discharge) and thus the system has to generate it:

$$P(t) \geq 0. \tag{6}$$

In this way, the evolution will be directed back to the operating point.

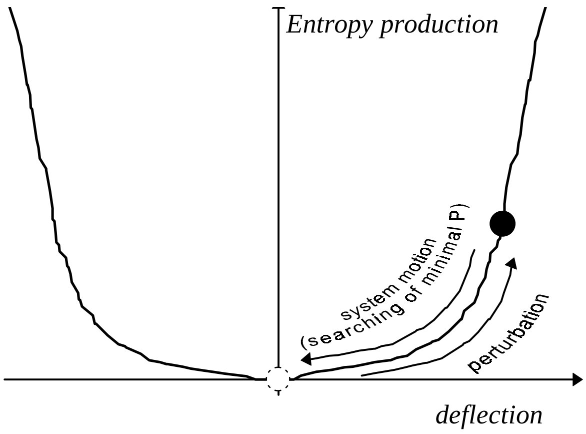


Fig.1 - Entropy production as a function of the deflection from operating point.

In the opposite case, the system can be moved either to another operating point or to the state with the unlimited degree of chaos. Second possibility is common in case of circuits with ideal controlled sources without the nonlinearities etc.

Entropy defined by (5) can be well observable for the thermodynamic processes, which is divided to the mutually influenced flows J_i and forces X_i. Such partial processes are always influenced according to so-called constitutive equations

$$\begin{bmatrix} X_1 \\ \cdot \\ \cdot \\ \cdot \\ X_N \end{bmatrix} = \begin{bmatrix} R_{11} & \cdot & \cdot & \cdot & R_{1N} \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ R_{N1} & \cdot & \cdot & \cdot & R_{NN} \end{bmatrix} \begin{bmatrix} J_1 \\ \cdot \\ \cdot \\ \cdot \\ J_N \end{bmatrix}, \tag{7}$$

The flows and forces can be of arbitrary physical origin. It is important that computations based on the macroscopic description of such process lead always to the formal same result for the entropy production:

$$P(t) \approx \sum_i X_i J_i, \tag{8}$$

i.e. the entropy production is proportional to the power of whole process.

3. Utilization for the electric circuits

Because electrical circuits are the thermodynamic systems, equation (1) can be considered as the mathematical description of system fluctuation round the DC operating point. Here the variable $\delta\mathbf{V}$ represents voltage fluctuations of controlled sources and $\delta\mathbf{I}$ corresponding current fluctuations supplied by these sources. Then equations (1) are the constitutive equations of thermodynamic system. We can apply the principle of minimum entropy production and to formulate requirements to the matrix **R** or **G**. However, following problem has to be overcome.

Originally, the principle of minimum entropy production has formulated for the systems with the symmetrical matrix in the constitutive equation (6). In other words, so-called Onsager reciprocity relations would be true:

$$R_{ij} = R_{ji} \,. \tag{9}$$

Onsager showed that in the linear region the generalized forces X_i and flows J_i can be chosen so that conditions (8) will be true.

In general, matrices **R** and **G** in the equation (1) are not symmetrical. Therefore we must find such another set of voltage and current fluctuations $\delta\mathbf{U}$ and $\delta\mathbf{J}$ for which the Onsager reciprocity relations will be fulfilled [5]. Even one can find such linear transformation which transfers the fluctuation matrix **R** or **G** to the diagonal form. For example,

$$\delta\mathbf{J} = \Lambda \cdot \delta\mathbf{U} \,,$$

where $\mathbf{\Lambda}$ is the diagonal matrix with the same eigenvalues as for the matrix **G**. The entropy production is proportional to the expression

$$P(t) \approx \delta\mathbf{U}^T \cdot \underline{\Lambda} \cdot \delta\mathbf{U} = \sum_i \lambda_i \delta U_i^2 \,. \tag{10}$$

The stable operating point has to be located in the local minimum of the entropy production P. It is possible only if

$$\mathrm{Re}(\lambda_i) > 0 \quad \forall i \tag{11}$$

where λ_i are eigenvalues of the matrices $\mathbf{\Lambda}$ and **G**. At the same time, there are virtual eigenvalues of corresponding resistive circuit with the controlled sources. We proved the necessary and sufficient condition of the DC operating point stability.

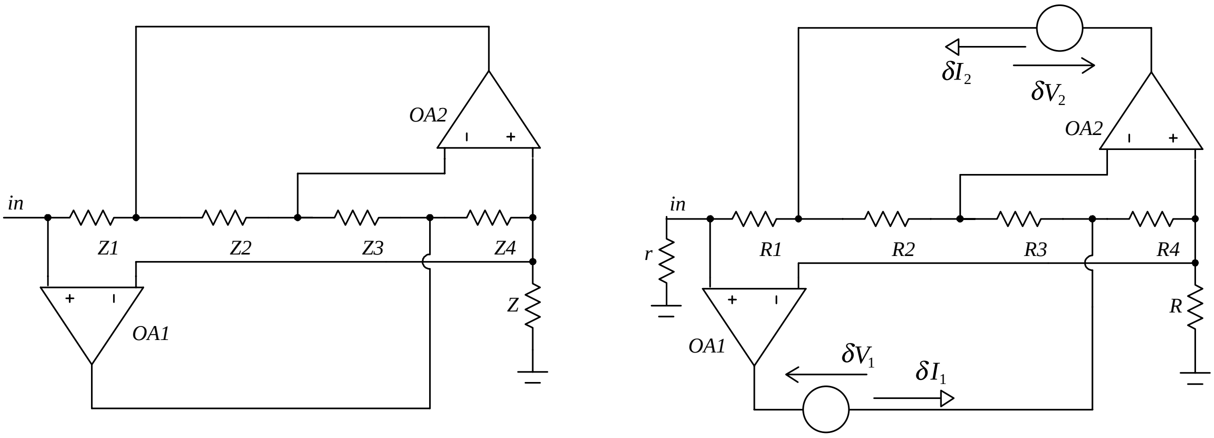
4. Example

Consider circuit modification of well-known immitance converter on Fig. 2 (a). Linearized analysis of the input impedance leads to the known result

$$Z_{inp} = \frac{Z_1 Z_3}{Z_2 Z_4} Z \,.$$

However, it is necessary to find out if the polarity of the OpAmps and the whole circuit configuration ensure stable operating point. Outputs of the controlled sources are completed by the virtual sources according to [1], see Fig. 2(b), to model fluctuations of operating point. Impedances are replaced by their resistive parts. The input resistance of the circuit connected to the node *in* is modeled by the resistor r.

For example, consider DC gain of OpAmps *OA1* and *OA2* as $A_{01} = A_{02} = 200.000$. For the resistances $R_1 = R_2 = R_3 = R_4 = R = r = 1\mathrm{k}\Omega$, the constitution equations are as follows:



(a) (b)

Fig. 2. (a) Modification of well-known impedance converter, (b) resistive model with the virtual sources.

$$\begin{bmatrix} \delta I_1 \\ \delta I_2 \end{bmatrix} = 10^8 \begin{bmatrix} 0,66668 & -0,66666 \\ 0,666673 & 1,333346 \end{bmatrix} \begin{bmatrix} \delta V_1 \\ \delta V_2 \end{bmatrix}$$

and the virtual eigenvalues of matrix **G**

$$\lambda_1 = 10^8 (1 + j0,577)s, \quad \lambda_2 = 10^8 (1 - j0,577)s$$

Thus, operating point is stable. To interchange inputs of *OA1* or *OA2*, we obtain

$$\lambda_1 = 1,5352.10^8 s, \quad \lambda_2 = -8,685.10^7 s \text{ or } \lambda_1 = -1,5352.10^8 s, \quad \lambda_2 = 8,685.10^7 s .$$

In both cases, operating point is unstable.

Interchanging inputs of both OpAmps yields

$$\lambda_1 = 10^8 (-1 + j0,577)s, \quad \lambda_2 = 10^8 (-1 - j0,577)s$$

and the operating point is again unstable. That is why the circuit on Fig. 2 (a) represents sole stable configuration.

The virtual trajectories in virtual space [δV_1 δV_2] for all four configurations are on Fig. 3.

5. Conclusion

DC circuits which arise from real circuits after neglecting reactance elements have own dynamics. This so-called virtual dynamics results from the thermodynamic essence of these circuits. For these circuits, the principle of minimum entropy production is true. The system deflected from its operating point searches another state in which the entropy production would be minimal of all possible states. For stable operating point, the curve of entropy production has to have local minimum in this point.

It is shown that the principle of minimum entropy production offers the necessary and sufficient stability condition: all virtual eigenvalues of the fluctuation matrix (**R** or **G**) have to lie on the right-half complex plane. The virtual eigenvalues have the strict physical interpretation: they are complex loads on which all fluctuations round the operating point are dissipated. From the location of the virtual eigenvalues, we can conclude how the system „flashly" balances the permanent fluctuations (focus, node, saddle etc.).

In case of resistive circuit, the time of settlement of the fluctuation is slight. These processes are isothermal and according to definition (5) the entropy is proportional to the heat. Thus, in the electrical circuits, the principle of minimum entropy production is reduced to another principle: electrical currents flowing through the material are spread in such way to arise minimum heat. In other words, in each time instance, the system searches the state in which the instantaneous power (i.e. the speed of energy releasing) is minimal. Therefore the method of virtual trajectories can be accepted without the considerations about the entropy which is not commonly used quantity in the circuit theory.

Avoiding the physical background of presented method, the philosophy point of view remains for its understanding: the virtual trajectories represent trends of the system which defends oneself from the virtual fluctuation.

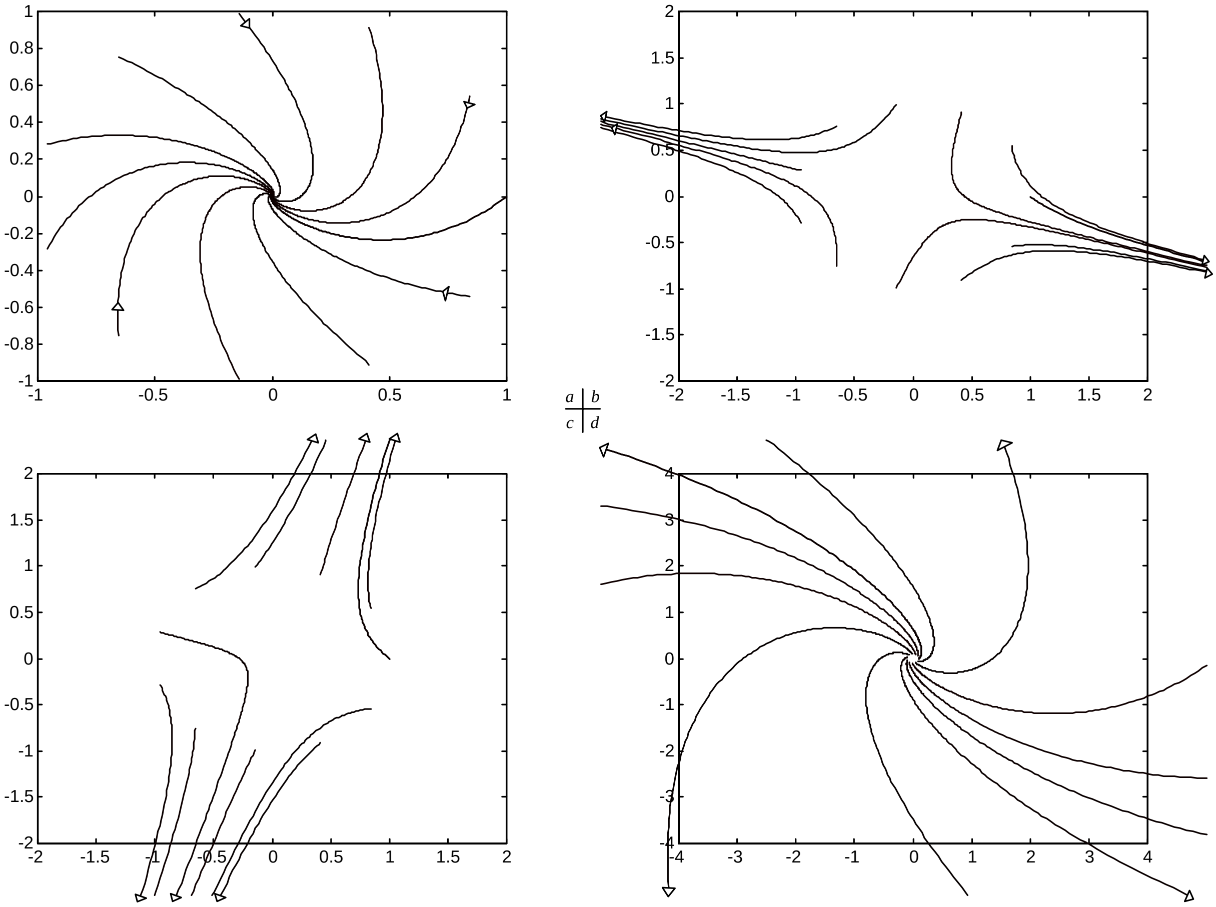


Fig.3. Virtual trajectories of the circuit on Fig. 2a) round the DC operating point [0 0]. The virtual voltage δV_1 (δV_2) is on the horizontal (vertical) axis; a) original circuit, b) interchanged inputs of *OA1*, c) interchanged inputs of *OA2*, d) interchanged both inputs of *OA1* and *OA2*.

References

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