Analysis of circuits containing active elements by using modified T - graphs

DALIBOR BIOLEK *) and VIERA BIOLKOVA**)
Department of Telecommunications *) Radioelectronics **)
Brno University of Technology
Purkynova 118, 612 00 Brno
CZECH REPUBLIC
dalibor.biolek@vabo.cz http://www.vabo.cz/stranky/biolek

Abstract: - The so-called modified transform (T) graphs are described in the paper, which are a graph analogy to well-known voltage and current incidence matrices. The procedure is shown how to pass from T-graphs to the classical Mason-Coates (MC) flow graph. The explanation is illustrated with examples of circuits containing operational amplifiers and current conveyors.

Key-Words: - Signal flow graph, flow graph, T-graph, incidence matrix, linear transformation, operational amplifier, current conveyor.

1 Introduction

Mason's (M-) and Mason-Coates' (MC-) flow graphs as tools of "hand-and-paper" analysis of relatively simple linear circuits have been in existence for many years [1], [2]. However, their basic form is not suitable for a direct solution of circuits containing some active elements, such as today's much promising current conveyors.

Several types of signal flow graph have been developed for circuits containing active elements [3], [4], [5]. Their analysis shows that common utilization of a given graph is conditional on the following factors:

1. Simple rules of graph construction directly from the circuit schematics.
2. Economical graph structure, i.e. the requirement of a relatively simple graph and its simple evaluation.
3. Simple rules of graph evaluation.
4. Possibility of evaluation both voltage & current gains and immittances.

Rules No. 1 and No. 3 are often in contradiction: The way to an economical graph leads mostly through hard-to-remember rules of thumb [4]. Rule No. 3 is fulfilled automatically if the resulting graph is the M- or the MC-graph. Then Mason's well-known general gain formula can be used. Rule No. 4 is not fulfilled for some graphs due to the reduction of some variables as a consequence of applying rule No. 2. However, these variables are necessary, for example, for analyzing immittance relations.

From the point of view of rules No. 1, 3, and 4, the so-called transform (T-) graphs introduced in [5] are interesting. They start from the graph interpretation of voltage and current incidence matrices. For the purpose of fulfilling rule No. 2, we developed the matrix method of so-called twin variables.

The so-called modified T-graphs are introduced in the paper, which represent a synthesis of T-graphs and the method of twin variables. These graphs were developed with the aim to reach a well-balanced compromise while fulfilling rules No. 1 to 4. As a final structure, we obtain the classical MC-graph with its common evaluation.

This paper is organized as follows: In Section 2 we summarize basic knowledge from the theory of T-graphs. The T- to MC-graph conversion is demonstrated on an example of passive linear circuit. In Section 3 we then explain the basic idea of twin variables and its utilization in the construction of economical T-graphs. We use the circuits with OpAmps and current conveyors for explanation. The application Section 4 shows – among others- the problem of searching for permitted twin variables while analyzing circuits comprising several active elements.

2 Incidence matrices, T- and MC-graphs

Consider a passive linear circuit consisting of m one-ports with the admittances Y_k, k = 1,2,.., m. Let the circuit contain n independent nodes to which we assign n node voltages V_i, i = 1,2,…,n, directed from node i toward the reference node, and n node currents I_i, i = 1,2,…,n, which flow into the circuit from outer sources.

We introduce loading orientation of voltages V_k and currents I_k of branch admittances Y_k. Then the equations hold

$$\mathbf{I}^B = \mathbf{Y}^B\mathbf{V}^B \tag{1}$$
$$\mathbf{I}^N = \mathbf{A}^I\mathbf{I}^B \tag{2}$$
$$\mathbf{V}^B = \mathbf{A}^V V^N \tag{3}$$

where $\mathbf{I}^B$ (m,1), $\mathbf{V}^B$ (m,1), $\mathbf{I}^N$ (n,1), $\mathbf{V}^N$ (n,1) are the vectors of branch currents, branch voltages, node currents and node voltages. Here $\mathbf{Y}^B$ (m,m) is a diagonal matrix of branch admittances, $\mathbf{A}^{\mathrm{I}}$ (n,m) and $\mathbf{A}^V$ (m,n) are the current and voltage incidence matrices. For the passive circuits, the incidence matrices are connected through transposition.

Associating equations (1), (2) and (3) yields the nodal admittance equations:

$$\mathbf{I}^N = \mathbf{Y}^N \mathbf{V}^N \tag{4}$$

where

$$\mathbf{Y}^N = \mathbf{A}^I \mathbf{Y}^B \mathbf{A}^V . \tag{5}$$

The T-graphs represent equations (1)-(3). Then equations (4) and (5) give directions on how to pass from the T- to the MC-graphs.

As an example, consider the passive Tchebyshev ladder filter in Fig. 1.

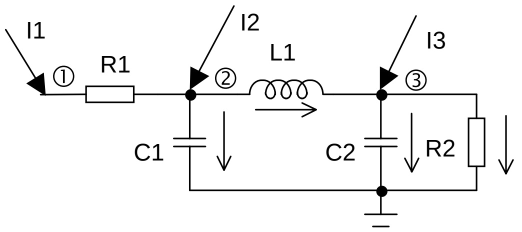

Fig. 1. Passive filter analyzed.

The independent nodes, node currents and specified directions of branch voltages and currents are denoted in the schematic. For the circuit we get

$$\underset{\mathbf{I}^B}{\underset{\uparrow}{\begin{bmatrix} I_{R1} \\ I_{C1} \\ I_{L1} \\ I_{C2} \\ I_{R2} \end{bmatrix}}} = \underset{\mathbf{Y}^B}{\underset{\uparrow}{\begin{bmatrix} G_1 & & & & \\ & sC_1 & & & \\ & & 1/sL_1 & & \\ & & & sC_2 & \\ & & & & G_2 \end{bmatrix}}} \underset{\mathbf{V}^B}{\underset{\uparrow}{\begin{bmatrix} V_{R1} \\ V_{C1} \\ V_{L1} \\ V_{C2} \\ V_{R2} \end{bmatrix}}} \tag{1'}$$

$$\underset{\mathbf{I}^N}{\underset{\uparrow}{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}} = \underset{\mathbf{A}^I}{\underset{\uparrow}{\begin{bmatrix} 1 & & & & \\ -1 & 1 & 1 & & \\ & & -1 & 1 & 1 \end{bmatrix}}} \underset{\mathbf{I}^B}{\underset{\uparrow}{\begin{bmatrix} I_{R1} \\ I_{C1} \\ I_{L1} \\ I_{C2} \\ I_{R2} \end{bmatrix}}} \tag{2'}$$

$$\underset{\mathbf{V}^B}{\underset{\uparrow}{\begin{bmatrix} V_{R1} \\ V_{C1} \\ V_{L1} \\ V_{C2} \\ V_{R2} \end{bmatrix}}} = \underset{\mathbf{A}^V}{\underset{\uparrow}{\begin{bmatrix} 1 & -1 & \\ & 1 & \\ & 1 & -1 \\ & & 1 \\ & & 1 \end{bmatrix}}} \underset{\mathbf{V}^N}{\underset{\uparrow}{\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}}} \tag{3'}$$

$$\mathbf{Y}^N = \mathbf{A}^I \mathbf{Y}^B \mathbf{A}^V = \begin{bmatrix} G_1 & -G_1 & \\ -G_1 & G_1+sC_1+1/sL_1 & -1/sL_1 \\ & -1/sL_1 & G_2+sC_2+1/sL_1 \end{bmatrix} \tag{5'}$$

Equations (1)'-(3)' can be represented by the T-graph in Fig. 2.

The following structures can be recognized in the transform graph:

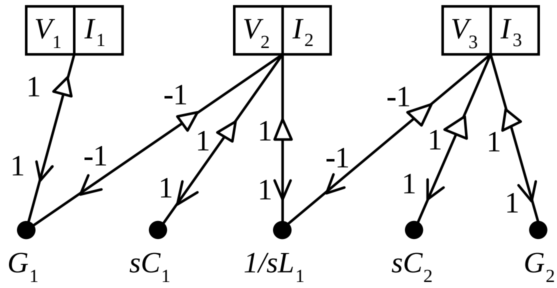

Fig. 2. Transform graph of the circuit in Fig. 1.

- The twins “node voltage/node current”.
- The branch admittances.
- The voltage and current branches of the T-graph, marked by the open and contoured arrows; the transform coefficients corresponding to the elements of incidence matrices are placed near the arrows.

As a consequence of the circuit passivity and the symmetry of voltage and current incidence matrices, the voltage and current gains of the corresponding branches of the T-graph are equal.

Passing from the T- to MC-graphs can be done according to the following algorithm:

- The self-loop gain of node voltage V_i is obtained as a sum of products

$$\sum_{k=1}^{m} a_{ik}^V Y_k a_{ki}^I , \tag{6}$$

where a_{ik}^V and a_{ki}^I are the voltage and the current gain of the branch that directly connects twins $Vi|Ii$ with admittance Y_k.

- The gain of path directed from node V_i to node V_j is obtained as a negative sum of products

$$-\sum_{k=1}^{m} a_{ik}^V Y_k a_{kj}^I . \tag{7}$$

- The path with a gain of 1 is directed from node I_i to node V_i.

The application of the procedure given is in Fig. 3. The MC-graph can be constructed directly above the T-graph. The resulting MC-graph fully conforms to equation (4), where Y^N is given by equation (5‘).

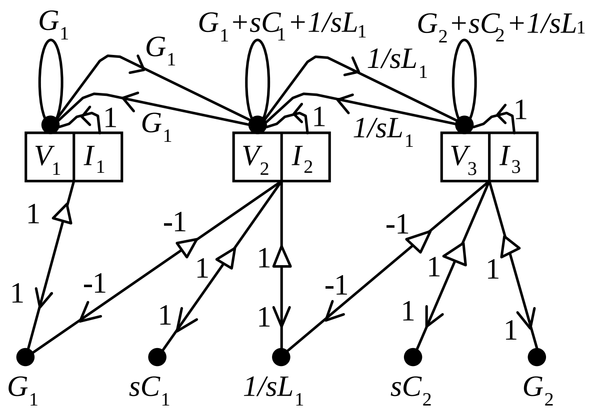

Fig. 3. T- to MC-graph construction (circuit from Fig. 1).

It should be noted that the corresponding unity-gain paths from the node currents to the node voltages are constructed only if it is advisable to consider the given node current.

The T-graph can be constructed directly from the schematic, following this procedure:

- In the schematic, we number the nodes, we draw the branches of outer sources and direct them inside the circuit, and we direct (in an arbitrary way) each internal branch comprising the one-port admittance.
- We draw symbols of twin variables, ordering them as “unknown variable|known variable” (for passive circuits we have “node voltage|node current”). Below them we draw graph nodes, labelled by the corresponding branch admittances.
- Utilizing Kirchhoff’s voltage law, we draw branches of the voltage T-graph. We add gains (+1 or –1) to individual branches.
- We complete the T-graph branches symmetrically (for passive circuits) by the current gains.

After that, the MC-graph is constructed and evaluated as described above.

3. Method of twin variables and modified T-graph

3.1 Circuits with ideal OpAmps

Consider an ideal operational amplifier (OPA) connected to the circuit as shown in Fig. 4.

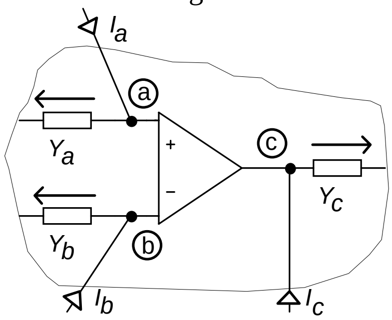

Fig. 4. Example of OPA connection.

The OPA maintains zero voltage between its inputs, such that the equality $V_a = V_b$ is true. This will be utilized for the reduction of unknown variables. If we are not interested in the OPA output current (it can be calculated post facto from the voltages), we need not compile the equation of Kirchhoff’s current law for node *c*.

Be aware that the number of node voltages and currents must be the same in the set of circuit equations. After the reduction described, these variables are [V_a=V_b, V_c] and [I_a, I_b].

The circuit in Fig. 4 gives the equations

$$I_a = V_a Y_a - \dots,\ I_b = V_b Y_b - \dots .$$

The dots indicate other possible elements which are generated by the floating admittances Y_a and Y_b. In the matrix form,

$$\begin{bmatrix} I_a \\ I_b \\ \dots \end{bmatrix} = \begin{bmatrix} Y_a & 0 & \dots \\ Y_b & 0 & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} V_a = V_b \\ V_c \\ \dots \end{bmatrix} . \qquad (8)$$

There are two corresponding MC-graphs in Fig. 5.

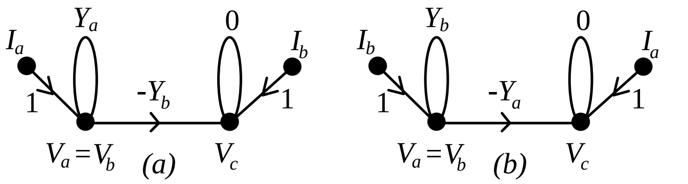

Fig. 5. Two versions of MC-graphs corresponding to equation (8).

Two possible variants of twin variables result form the graphs:

a) V_a=V_b|I_a, V_c|I_b, b) V_a=V_b|I_b, V_c|I_a.

Conclusion: In the case of ideal OpAmp, we only consider unknown variables $V_a = V_b$ and V_c. Either current I_a or current I_b is allocated to the first unknown voltage variable (OpAmp input). The second current will be allocated to the OpAmp output voltage.

Both variants of the T- and the MC-graphs of the circuit in Fig. 4 are shown in Fig. 6. Note that the graphs are equal to those in Fig. 5.

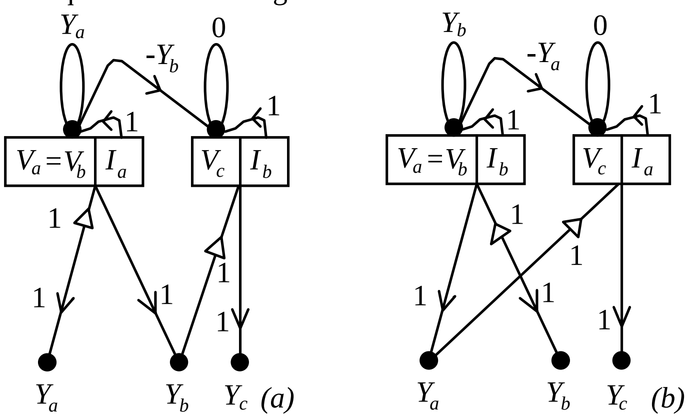

Fig. 6. Construction of graphs of circuit in Fig. 4.

3.2 Circuits with current conveyors CCII±

Shown in Fig. 7 is a current conveyor with the well-known internal structure, connected in a general network. The circuit can be described by the following equations:

$$I_X = Y_X V_X - I_0 - \dots$$
$$I_Y = Y_Y V_Y - \dots$$
$$I_Z = Y_Z V_Z \mp I_0 - \dots$$

At the same time the equality V_X=V_Y holds, which will be used to reduce the variable.

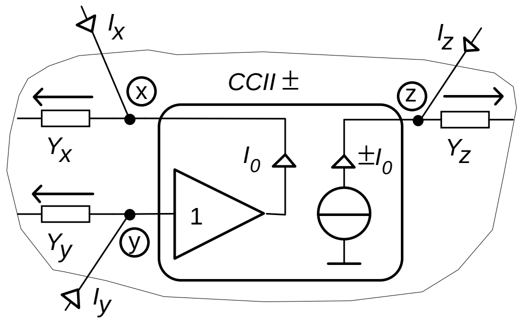

Fig. 7. Subcircuit with current conveyor CCII±.

Two graph variants in Fig. 8 correspond to the above equations:

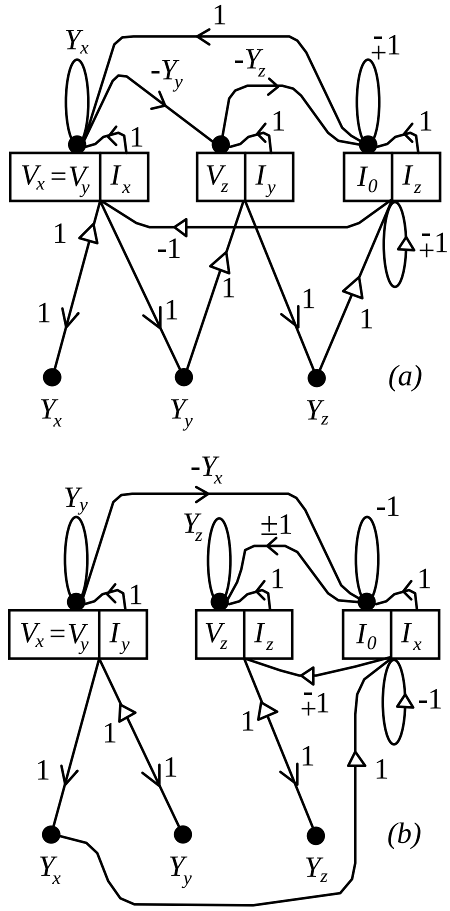

Fig. 8. Graph representation of circuit in Fig. 7.

Conclusion: In the case of CCII± elements, we only consider the unknown variables V_X=V_Y, V_Z, and I_0. Either current I_x or current I_y is allocated to the first unknown voltage variable (V_X=V_Y). Either current I_y or current I_z will be allocated to the second unknown voltage variable (V_Z). Either current I_z or current I_x will be allocated to variable I_0. Considering the sequence of unknown variables V_X=V_Y, V_Z, I_0 , the corresponding sequence of twin currents is either I_X , I_Y , I_Z or the sequence I_Y, I_Z , I_X, which is its cyclic-shifted equivalent.

4 Illustrative examples

4.1 Circuits with operational amplifiers

Let us consider the 2nd -order bandpass filter in Fig. 9 [6] and the corresponding MC-graph constructed from the T-graph in Fig. 10.

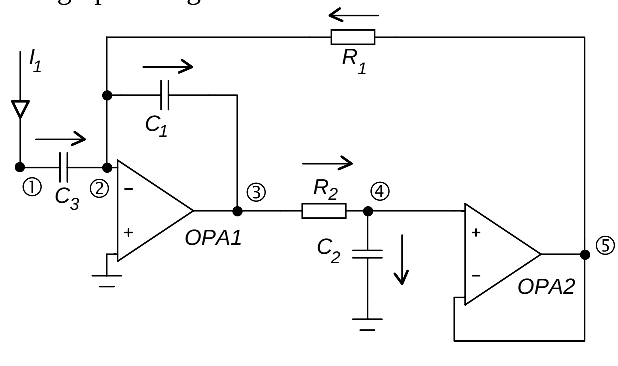

Fig. 9. 2nd -order bandpass filter.

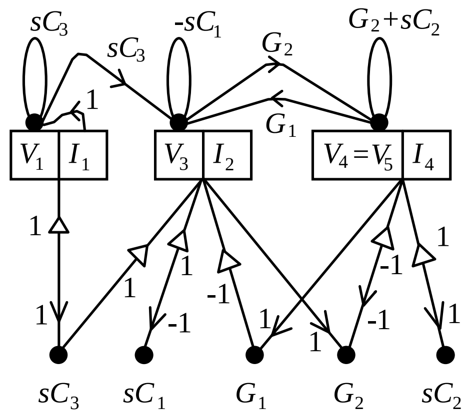

Fig. 10. Graph representation of filter in Fig. 9.

Evaluating the MC-graph, we obtain the voltage transfer function

$$\frac{V_5}{V_1} = C_3 G_2 \frac{s}{-sC_1(G_2 + sC_2) - G_1 G_2} = -\frac{sR_1C_3}{s^2 R_1 R_2 C_1 C_2 + sR_1 C_1 + 1}$$

4.2 Circuits with current conveyors

Shown in Fig. 11 is a simple circuit containing CCII+. The voltage gain from node 1 to node 3 is as follows:

$$\frac{V_3}{V_1} = 1 + \frac{R_2}{2R_1}.$$

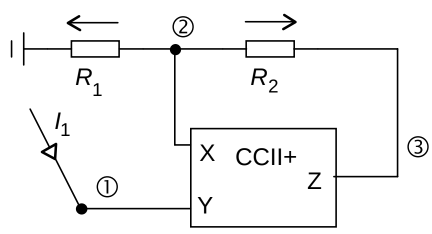

Fig. 11. Circuit containing CCII+.

The corresponding graphs are in Fig. 12. Evaluating the MC-graph yields the above transfer function.

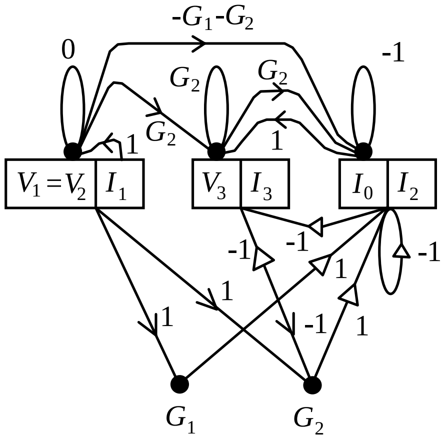

Fig. 12. Graph representation of circuit in Fig. 10.

An impedance converter with both positive and negative current conveyors is shown in Fig. 13 [7]. The unknown variables are as follows:

V_1=V_2, V_3=V_4, I_0^+ (CCII+ current), I_0^-(CCII- current).

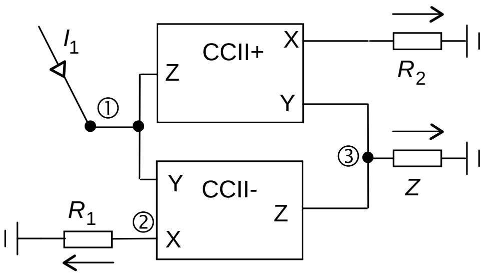

Fig. 13. Impedance converter with CCII+ and CCII- [7].

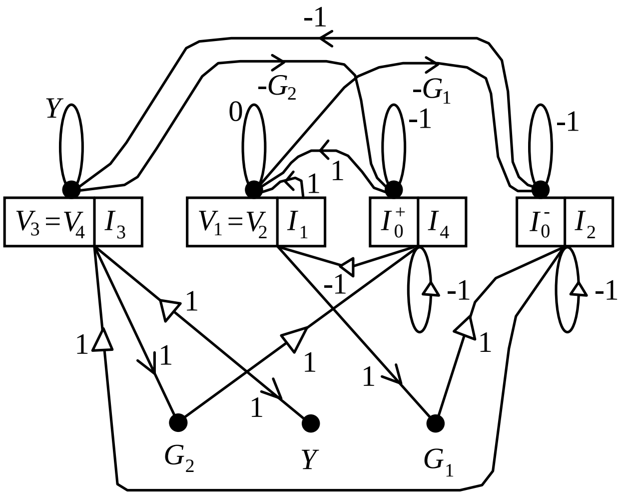

Fig. 14. Graph representation of circuit in Fig. 13.

From the viewpoint of CCII+, the following twin variables could be possible:

a)	V_3=$V_4 \mid I_4$	or	V_3=$V_4 \mid I_3$
b)	$V_1 \mid I_3$		$V_1 \mid I_1$
c)	$I_0^+ \mid I_1$		$I_0^+ \mid I_4$

Now we complete them from the viewpoint of CCII-:

V_1=$V_2 \mid I_2$ conflict with b) or V_1=$V_2 \mid I_1$ see b)
$V_3 \mid I_3$ see a)

The only possible pairing is given in Fig. 14. Evaluating the MC-graph, we get the input impedance:

$$Z_{inp} = \frac{V_1}{I_1} = \frac{1.Z.(-1).(-1)}{0 + G_1 G_2} = \frac{R_1 R_2}{Z} .$$

5 Conclusions

The described method of graph solution of linear systems combines the so-called transform graphs and the method of twin variables. As a result, the algorithm of graph construction is relatively simple, and the size of the resulting MC-graph is reduced enough. As shown in the final example, the selection of permitted twin variables must be performed carefully if there is a number of transform elements in the circuit analyzed. This method can be used to solve general linear problems, not only in the analysis of electrical circuits.

References:

[1] Mason, S.J., Feedback Theory: Further Properties of Signal Flow Graphs. *Proc. IRE*, Vol. 44, No. 7, pp. 920-926, 1956.

[2] Coates, C.L., Flow-graph Solution of Linear Algebraic Equations. *IRE Trans. Circuit Theory*, CT-6, pp. 170-187, 1959.

[3] Mikula, J., Signal-Flow-Graph-gain with Respect to the General Node of a Graph. *Electronics Letters*, August 1969, No. 16, pp. 382-383.

[4] Biolek,D., Novel Signal Flow Graphs of Current Conveyors. *38th MWSCAS*, Rio de Janeiro, Brazil August 13-16, 1995, pp. 1058-1061.

[5] Mikula, J., Pospisil,J., Analysis of circuits containing active transform elements. *Electronic Horizon*, Vol. 35, No. 4, pp. 161-167, 1974 (in Czech).

[6] BIOLEK,D., BIOLKOVA,V., Optimization of Linear Systems using Symbolic Modelling. *MS2000*, Las Palmas de Gran Canaria (Spain), pp. 701-707.

[7] Tomazou,C., Lidgey,F.J, Haigh,D.G., Analogue IC Design: the Current-Mode Approach. *IEE Circuits and Systems Series 2*, Peter Peregrinus Ltd, London 1993,UK.

Acknowledgement: *This work is supported by the Grant Agency of the Czech Republic under grants No. 102/01/0432 and 102/00/0907, and by the research programme of BUT „Research of electronic communication systems and technologies".*