

LDI matrix for discrete-time filter design

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Abstract: The LDI matrix is introduced in its full and shortened forms. As an analogy of the Pascal matrix for the bilinear transform, the LDI matrix serves to transform the coefficients of the transfer functions of continuous- and discrete-time linear circuits which are connected by the LDI (Lossless Discrete Integration) transform. The basic properties of the LDI matrix are presented and a procedure of its algorithmic compilation is proposed.

Keywords: - LDI matrix, s-z transformation, discrete-time system.

1 Introduction

The conception of s-z transforms is useful in designing linear discrete-time (DT) or digital systems from their continuous-time (CT) counterparts [1-3]. The most noted s-z transforms are the bilinear (BL), forward-difference (FD), backward-difference (BD), lossless discrete integration (LDI), and some parametric s-z transformations. Such transforms represent one-to-one correspondences between the coefficients of transfer functions of CT and DT systems.

It is useful, particularly for large systems, to have an algorithm for computing the z-domain coefficients from the s-domain coefficients and vice versa. More historical attempts can be traced in the literature, for example [4, 5]. For the well-known bilinear transform, an original tool has been found in the form of the Pascal matrix (PM) [5], which transforms a vector of s-domain coefficients to a vector of z-domain coefficients. The inverse transform is easy, utilizing the inverted Pascal matrix.

In [6, 7], the so-called generalized Pascal matrix (GPM) is proposed, which includes the conventional PM as a special case. The GPM provides a one-to-one correspondence between the s- and z- domain coefficients of CT and DT systems for the so-called general first-order s-z transform, which covers all the known transforms such as the BL, BD, FD, and parametric transforms [6], but not the LDI transform.

During the last several years, a lot of papers have appeared on the conventional and the generalized Pascal matrices from the area of digital filter design [8-15], numerical mathematics [16-20], and signal processing [21-23]. The reason is that the discrete polynomial transforms, which are related by the Pascal matrix, have numerous applications in signal processing, image

processing, control systems, communications, engineering, and physics [21].

The LDI transform [24] finds application in the low-sensitivity digital ladder filter design, where the conventional bilinear transform cannot be smoothly used in direct ladder simulation [25]. Low-sensitive switched-capacitor ladder structures are also preferably designed via the LDI transform [26] or through combining the LDI and the BL transforms [27]. The LDI transform is also useful in designing the switched-current (SI) filters [28, 29], in SC Sigma-Delta Converters [30], etc.

However, the LDI transform has rather a different character from the above transforms due to the non-integer power of z-operator in its definition. That is why the concept of a generalized Pascal matrix cannot be used here for algorithmic utilization of this transform. The only paper dealing with the so-called LDI matrix, which is an analogy of the well-known Pascal matrix, is [31]. However, only a brief mathematical definition is given here without any analysis of the properties of this matrix and without any algorithm for building the LDI matrix for transforming large systems.

In this paper, the original concept of the LDI matrix from [31] is extended. A shortened LDI matrix is defined for faster transformation. Some basic properties of this matrix are discussed, and an effective algorithm of its compilation is proposed.

2 LDI transform

The LDI transform is defined as follows [19]:

$$s = \frac{1}{T} (z^{1/2} - z^{-1/2}), \text{ or } z = 1 + \frac{(sT)^2}{2} \pm \frac{sT}{2} \sqrt{4 + (sT)^2}, \quad (1)$$

where T is the sampling period used for a discrete-time system.

The exponents $\pm 1/2$ in (1) have their physical interpretation in the switched system, which consists in dividing the sampling period into two switching phases. The substitutions

$$z = e^{j\omega T}, \quad z^{\pm 1/2} = e^{\pm j\omega T/2}$$

show that the discrete-time system, which is designed using the LDI transform, can be considered a system with half the sampling period (double sampling frequency). As shown below, the $N+1$ coefficients of CT system are transformed into $2N+1$ coefficients of DT system.

To simplify further considerations, let us introduce the substitutions

$$x = z^{1/2}, \quad a = \frac{1}{T}. \quad (2), (3)$$

Then the LDI transform can be rewritten in a more compact form [26]:

$$s = a \left(x - \frac{1}{x} \right) = a \frac{x^2 - 1}{x}, \quad x \neq 0. \quad (4)$$

3 LDI matrix [26]

Consider an N -th order CT linear system with the transfer function

$$K_{CT}(s) = \frac{\sum_{k=0}^N a_k s^k}{\sum_{k=0}^N b_k s^k}. \quad (5)$$

We apply LDI transform (4):

$$\begin{aligned} K_{DT}(x) &= K_{CT} \left(s = a \frac{x^2 - 1}{x} \right) = \frac{\sum_{k=0}^N a_k \left(a \frac{x^2 - 1}{x} \right)^k}{\sum_{k=0}^N b_k \left(a \frac{x^2 - 1}{x} \right)^k} = \\ &= \frac{\sum_{k=0}^N a_k a^k (x^2 - 1)^k x^{N-k}}{\sum_{k=0}^N b_k a^k (x^2 - 1)^k x^{N-k}}. \end{aligned} \quad (6)$$

After the power expansion and some algebraic arrangements, we get the general structure of the transformed transfer function:

$$K_{DT}(x) = \frac{\sum_{k=0}^{2N} c_k x^k}{\sum_{k=0}^{2N} d_k x^k} \quad (7)$$

The numerator (denominator) coefficients c_k (d_k) of the DT system can be derived from the set of the numerator (denominator) coefficients a_k (b_k) of the CT system. Because of the formal identity of the numerator and the denominator in (6), it is sufficient to derive transform relations only for the numerator.

Using the binomial rule and making laborious simplifications, we obtain the following equations:

$$c_{N-k} = \sum_{i=0}^{\left[\frac{N-k}{2} \right]} (-1)^{k+i} \binom{k+2i}{i} \tilde{a}_{k+2i}, \quad 0 \leq k \leq N, \quad (8)$$

$$c_{N+k} = \sum_{i=0}^{\left[\frac{N-k}{2} \right]} (-1)^i \binom{k+2i}{i} \tilde{a}_{k+2i}, \quad 0 \leq k \leq N, \quad (9)$$

where $[]$ is the integer of the argument, and

$$\tilde{a}_{k+2i} = a^{k+2i} a_{k+2i}. \quad (10)$$

The proof is given in the Appendix. Equations (8) and (9) represent an algorithm for computing the numerator coefficients of DT systems from the numerator coefficients of CT systems. For better transparency, let us set the coefficients a_k of the CT system and the coefficients c_k of the DT system to the vectors

$$\mathbf{A} = [a_0 \ a_1 \ a_2 \ \dots \ a_N]^T, \quad \mathbf{C} = [c_0 \ c_1 \ c_2 \ \dots \ c_{2N}]^T. \quad (11)$$

Now we define an auxiliary vector

$$\tilde{\mathbf{A}} = [\tilde{a}_0 \ \tilde{a}_1 \ \tilde{a}_2 \ \dots \ \tilde{a}_N]^T. \quad (12)$$

Equations (8) and (9) can then be rewritten in the matrix form

$$\mathbf{C} = \mathbf{L} \tilde{\mathbf{A}}. \quad (13)$$

Here $\mathbf{L}$ is the $(2N+1) \times (N+1)$ matrix, whose elements are given by equations (8) and (9). Let us call it the *LDI matrix* [26].

As an example, with the utilization of (13), (8) and (9), we can concretize the LDI matrix for $N = 5$ as follows:

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 & -4 & 0 \\ 0 & -1 & 0 & 3 & 0 & -10 \\ 1 & 0 & -2 & 0 & 6 & 0 \\ 0 & 1 & 0 & -3 & 0 & 10 \\ 0 & 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{a}_0 \\ \tilde{a}_1 \\ \tilde{a}_2 \\ \tilde{a}_3 \\ \tilde{a}_4 \\ \tilde{a}_5 \end{bmatrix} \quad (14)$$

4 Shortened LDI matrix

Some symmetrical property of the LDI matrix is obvious from Eq. (14). Comparing Eqs. (8) and (9) and considering the basic properties of binomial coefficients, one can write the equality

$$c_{N+k} = (-1)^k c_{N-k}, \quad 0 \leq k \leq N. \quad (15)$$

For example, the coefficients of the LDI matrix in (14) in row “c6” are negative values of the corresponding coefficients in row “c4”. Similarly, the coefficients in row “c7” are equal to the coefficients in row “c3”, etc. That is why a shortened version of the LDI matrix, labeled **Ls**, can be used for a faster determination of only the coefficients c_i , $i = 0..N$. The remaining coefficients can be easily computed, applying rule (15). The general form of the **Ls** matrix can be obtained from (8):

$$\begin{bmatrix} 0 & \dots & 0 & 0 & 0 & (-1)^N \binom{N}{0} \\ 0 & \dots & 0 & 0 & (-1)^{N-1} \binom{N-1}{0} & 0 \\ 0 & \dots & 0 & (-1)^{N-2} \binom{N-2}{0} & 0 & (-1)^{N-1} \binom{N}{1} \\ 0 & \dots & (-1)^{N-3} \binom{N-3}{0} & 0 & (-1)^{N-2} \binom{N-1}{1} & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \quad (16)$$

The form of the last row results from the evolution of the matrix coefficients given by Eq. (8); thus it depends on whether N is odd or even:

N odd-

$$\left[1 \ 0 \ -\binom{2}{1} \ 0 \ \binom{4}{2} \ 0 \ \dots \ (-1)^{\frac{N-1}{2}} \binom{N-1}{(N-1)/2} \ 0 \right]. \quad (17)$$

N even-

$$\left[1 \ 0 \ -\binom{2}{1} \ 0 \ \binom{4}{2} \ 0 \ \dots \ 0 \ (-1)^{\frac{N}{2}} \binom{N}{N/2} \right]. \quad (18)$$

5 Properties of the LDI matrix

The rules how to compile the **LDI** matrix, either the full or the shortened one, can be investigated from equations (8) and (9), with the aid of examples (14) and (16-18). The summarization of the rules for the shortened matrix is as follows:

- 1] **Ls** is a $(N+1) \times (N+1)$ square matrix.
- 2] The diagonal matrix elements, starting from the top-right to the bottom-left corner, are elements with alternating signs. The sign of the first one is plus/minus if N is even/odd, respectively. Let us call this diagonal the “TRBL diagonal”.
- 3] The matrix elements above the TRBL diagonal are zero.
- 4] Each nonzero matrix element in the rows or columns below the TRBL diagonal is accompanied by one zero element.
- 5] The signs + and - of nonzero elements in each row and column alternate.

6] The nonzero elements in the last column of **Ls** form an ordered set of binomial coefficients with alternating signs according to (16).

Other important properties can be deduced from an example of **Ls** matrix for $N = 10$:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -8 & 0 & 45 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 7 & 0 & -36 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -6 & 0 & 28 & 0 & -120 \\ 0 & 0 & 0 & -1 & 0 & 5 & 0 & -21 & 0 & 84 & 0 \\ 0 & 0 & 1 & 0 & -4 & 0 & 15 & 0 & -56 & 0 & 210 \\ 0 & -1 & 0 & 3 & 0 & -10 & 0 & 35 & 0 & -126 & 0 \\ 1 & 0 & -2 & 0 & 6 & 0 & -20 & 0 & 70 & 0 & -252 \end{bmatrix} \quad (19)$$

$N = 5 \qquad N = 8$

One can check that the sum of selected elements -1, -6, -21, -56, and 84 is zero. This fact can be generalized as follows:

Consider an element l_{rr} of **Ls** matrix in the r th row and r th column, thus lying on the TRBL diagonal. In (19), such an element is $l_{66} = -1$.

Consider m other elements, l_{xx} , $x=r+1, r+2, \dots, r+m$, and one element l_{yz} , $y=r+m+1, z=r+m-1$. In (17), such elements are -6, -21, -56, and 84, and $m = 3$.

From the graphical point of view, all the above elements lie on a broken line, depicted in (19). Let us abbreviate this object as “BL” (Broken Line). It is obvious that more BLs can be constructed within the **Ls** matrix. For example, matrix (19) for $N = 10$ contains 20 BLs.

In addition to Rules 1] to 6], Rule 7] holds:

7] The sum of elements, lying on an arbitrary BL of the **Ls** matrix, is zero.

The proof of this rule is beyond the scope of this paper.

Similarly, the following Rule 8] is a consequence of Rule 7]:

8] The sum of absolute values of the nearest two nonzero elements in column No. c is equal to the absolute value of the nonzero element in column $c+1$, which is nearest to these elements.

In (19), the above elements are, for example, -4, 6, and -10.

6 Compilation of L_s matrix

The Rules from Section V can be used for automated compilation of the L_s matrix of arbitrary order. Probably the simplest algorithm is as follows:

- 1) Zeroing all the elements of L_s .
- 2) Filling the elements of TRBL diagonal according to Rule 2].
- 3) Filling the elements of the last column of L_s according to Rule 6].
- 4) Computing the values of the remaining nonzero elements, starting from the top-right position of L_s , combining Rules 8] and 5].

Step No. 4 can be clarified on the example of Eq. (19): According to Rule 8], element 9 is computed as a difference between the absolute values of elements -10 and -1. Its sign (plus) results from Rule 5]. Then element -36 is determined from elements 45 and 9, etc. After identifying all the elements in column No. 10, the procedure is repeated for column No. 9, etc.

7 Example of algorithmic LDI transform

Let us apply the algorithm to the design of DT lowpass filter from the CT prototype, which has the following specifications: the 3 dB cutoff frequency is 3.4 kHz, the attenuation is at least 40 dB at a frequency of 6 kHz. The inverse Chebyshev approximation leads to 5th - order CT filter with the coefficients from Table 1, columns a_i , b_i .

Table 1: Coefficients of CT and DT filters. The second halves of the c_i and d_i coefficients are given by mirror property (15).

i	a_i	b_i
0	$7.85974 \cdot 10^{21}$	$7.85974 \cdot 10^{21}$
1	0	$9.34917 \cdot 10^{17}$
2	$8.23781 \cdot 10^{12}$	$6.38420 \cdot 10^{13}$
3	0	$2.75293 \cdot 10^9$
4	$1.72681 \cdot 10^3$	$7.42216 \cdot 10^4$
5	0	1
$DT, f_s = 200\text{kHz}$		
i	c_i	d_i
0	0	$1.021021341474 \cdot 10^{-1}$
1	$-8.815549313352 \cdot 10^{-4}$	$-3.789091879917 \cdot 10^{-1}$
2	0	$-5.034836700329 \cdot 10^{-1}$
3	$3.421082477628 \cdot 10^{-3}$	$1.507488746406 \cdot 10^{-1}$
4	0	1
5	$-5.081562893298 \cdot 10^{-3}$	$-2.25718451391 \cdot 10^{-1}$

The coefficients of DT filter are designed for the switching frequency $f_s = a = 200\text{kHz}$ via the LDI transform. For $N = 5$, we use transformation (14) for computing 11 coefficients of the numerator and also 11 coefficients of the denominator of transfer function (7) of DT filter. Table 1 summarizes the results for the first 6 coefficients. Behavioral modeling of CT and DT filters, based on their transfer functions, has been performed in PSpice. The results of the AC analysis are shown in Fig. 1. The conformity of both frequency responses is excellent up to half the switching frequency.

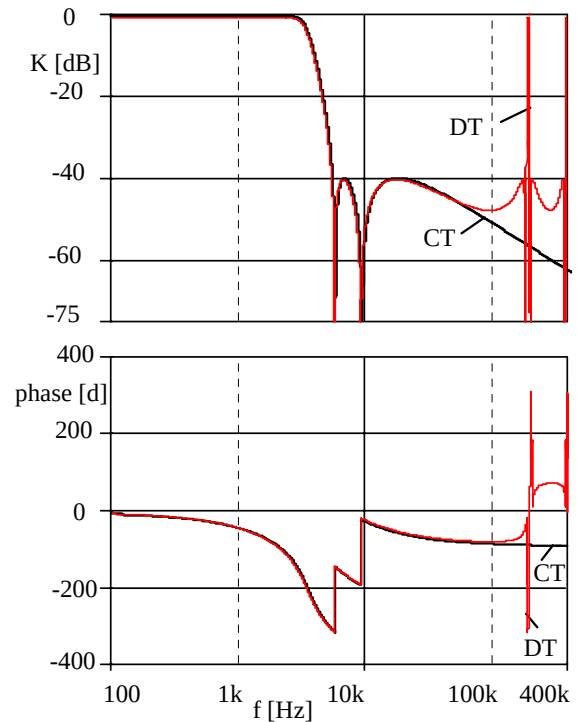


Fig. 1: Comparison of frequency responses of designed DT filter and its CT counterpart.

8 Conclusions

A shortened LDI matrix, introduced in this paper, can be regarded as an analogy of the well-known Pascal matrix for the bilinear s - z transform. Several of its properties, as described in the paper, enable an easy algorithmic compilation of this matrix for an arbitrary order.

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Appendix –Proof of Eqs. (8) and (9)

Considering Eq. (10) and the binomial rule, the numerator on the right side of (6) can be expanded as follows:

$$\sum_{k=0}^N \tilde{\alpha}_k (x^2 - 1)^k x^{N-k} = x^N \sum_{k=0}^N \sum_{i=0}^k \tilde{\alpha}_k (-1)^i \binom{k}{i} x^{k-2i}. \quad (A1)$$

Let us apply the substitution

$$m = k - 2i \Rightarrow k = m + 2i.$$

Note that m is the power of x in the double sum in Eq. (A1). Table A1 represents the values of m in the space of summation indices $[i, k]$ used in Eq. (A1). Let us compute the number of appearances of concrete m in Table A1 for given N . A simple analysis leads to the value

$$1 + \left\lfloor \frac{N - |m|}{2} \right\rfloor, \quad (A2)$$

where the function $\lfloor \cdot \rfloor$ returns the integer value of the argument if the argument is positive, and 0 for the negative argument.

For $m \geq 0$ and a concrete row No. i in Table A1, the number m is on the position $k = m + 2i$. That is why the elements of the double sum in Eq. (A1), corresponding to nonnegative powers m can be written in the form

$$\sum_{i=0}^{\lfloor (N-m)/2 \rfloor} \tilde{\alpha}_{m+2i} (-1)^i \binom{m+2i}{i} x^{N+m}, \quad m \geq 0.$$

After formal replacement $m \rightarrow k$, we can check that Eq. (9) is correct.

Table A1: Values of m (Eq. A2) as a function of summation indices k and i in Eq. (A1).

	$\xrightarrow{k}$							
	0	1	2	3	4	5	6	...N
$\downarrow i$	0	0	-1	-2	-3	-4	-5	-6 ...
	1		-1	0	1	2	3	4 ...
	2			-2	-1	0	1	2 ...
	3				-3	-2	-1	0 ...
	4					-4	-3	-2 ...
	5						-5	-4 ...
	6							-6 ...
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
N								

For $m \leq 0$, let us define the variable $n = -m \geq 0$. It is obvious from Table A1 that the first row where the number m appears is row No. n . The number of appearances of m is given by Eq. (A2), thus $1 + \lfloor (N-n)/2 \rfloor$.

For concrete row No. i in Table A1, the non-positive number m is on the position $k = 2i + m = 2i - n$. The elements of the double sum in (A1) for non-positive powers

$m = -n$ are as follows:

$$x^N \sum_{i=n}^{\lfloor (N-n)/2 \rfloor + n} \tilde{\alpha}_{2i-n} (-1)^i \binom{2i-n}{i} x^{-n}.$$

The substitution $j = i - n$ and a short re-arrangement yield

$$\sum_{j=0}^{\lfloor (N-n)/2 \rfloor} \tilde{\alpha}_{n+2j} (-1)^{j+n} \binom{n+2j}{j} x^{N-n}, \quad n \geq 0.$$

This result is formally identical to the term before coefficient $\tilde{\alpha}_{k+2i}$ in Eq. (8). It proves the correctness of this formula.

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