Averaged modeling of switched DC-DC converters based on Spice models of semiconductor switches

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Abstract: A generalization of the model of PWM (Pulse Width Modulation) switch according to Dijk et al. [1] for averaged modeling of switched DC-DC converters is proposed. The original model by Dijk consists of controlled current and voltage sources for modeling the ideal active and passive switch. A new model, proposed in this paper, is based on the same strategy, but it takes into account the nonideal V/I characteristics of semiconductor switches. The rules for determining the parameters of controlled sources from converter state variables remain the same as in [1], but a special procedure provides for the extraction of these parameters directly from PSpice models of concrete transistors or diodes, representing the active and passive switches. The way this model is compiled allows for the influence of parasitic ESRs of the capacitors in the circuit such that the resulting equations are equivalent to the averaged state space equations. The procedure of PSpice modeling and the results of computer simulation of boost converter are shown.

Keywords: - PWM switch, DC-DC converter, switch, averaged model.

1 Introduction

The averaging approach is an effective technique of modeling switched DC-DC converters [1-7]. It enables an easy analysis of DC and AC relations in the converter via conventional SPICE-like circuit analysis programs. The transient analysis is incomparably faster than similar analyses for the switched-mode model of the converter.

In [1], the PWM switch is modeled via controlled sources, whose voltages and currents are derived from the state variables of the converter, namely in the switching phase when the active switch is closed. It is shown that this method generates results that are equivalent to those obtained by the method of state space averaging. The above method is easily implemented in SPICE-like circuit analysis programs. The authors suggest that real parameters of converter components can be included in the switch model. However, these procedures are not described. The on-resistance of the active switch together with the forward voltage drop and the differential resistance of diode-type passive switch belong to significant real parameters of the PWM switch. However, they represent only an approximation of general nonlinear V/I characteristics of semiconductor switches.

The aim of this paper is to show a simple procedure when we use directly the full PSpice models of switches as semiconductor devices, e.g. MOSFETs and diodes, and all the switch parameters which are used for precise averaged modeling are extracted from these models automatically. This technique is then illustrated on an example of SPICE simulation of the boost converter.

2 Generalized model of PWM switch

A PWM switch is shown in Fig. 1 (a). It consists of active and passive switches. In general, they can be separated from each other in the converter topology. The active switch, commonly implemented by a transistor, is closed during the time dT_S, where T_S is the switching period, and d is the duty ratio, which can be modulated by another signal. The passive switch is implemented either by a diode or a transistor. Let us assume that the passive switch is closed within the time interval $d'T_S = (1-d)T_S$, otherwise the converter operates in the continuous current mode (CCM). In other words, the DC-DC converter can be considered a switched network, operating in two non-overlapping switching phases 1 and 2 with the lengths dT_s and $d'T_s$, respectively.

According to [1], a pair of one active and one passive switch can be modeled by an averaged model in the form of a controlled current source

$d < i_a^1 >$ and a controlled voltage source $d < v_p^1 >$, as shown in Fig. 1 (b). The remaining novel controlled source in Fig. 1 (b), modeling the passive switch parameters, will be discussed below. The symbol < > denotes average value within one switching period.

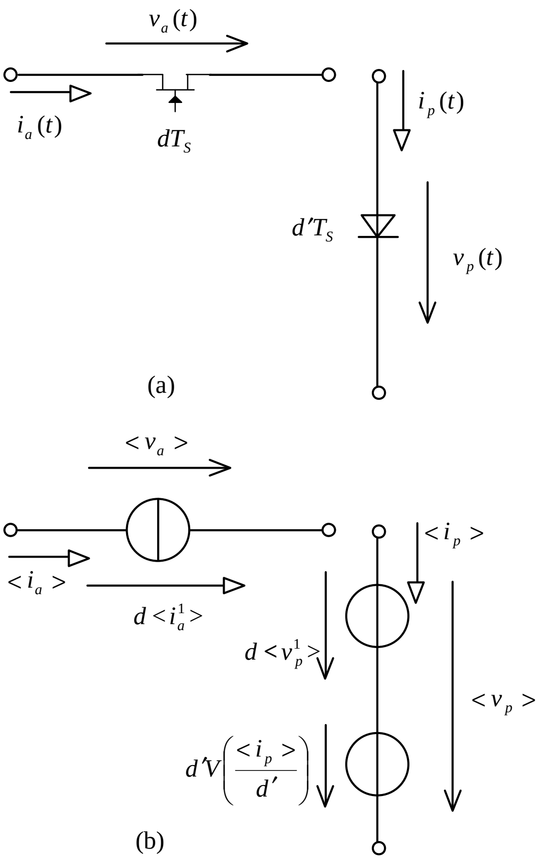

Fig. 1: (a) Pair of one active and one passive switch, (b) the averaged model.

The current $< i_a^1 >$ and the voltage $< v_p^1 >$ can be obtained from the inspection of the converter according to the following rules [1]:

Rule 1: $< i_a^1 >$ equals the current through the closed active switch during a switching phase 1, written as a function of the state and input variables.

Rule 2: $< v_p^1 >$ equals the voltage across the open passive switch during a switching phase 1, written as a function of the state and input variables.

Note that the symbols $< v_p^1 >$ and $< i_a^1 >$ represent averaged values of voltage $v_p(t)$ and current $i_a(t)$ within the switching period T_S.

In Fig. 1 (b), this model is complemented with the influence of nonlinear V/I characteristic of the passive switch $V_p=V(I_p)$. The corresponding derivation is given in the Appendix.

The information about the nonlinear V/I characteristic of the active switch is seemingly not present in the model. Actually, it is included in the parameters of the above controlled sources. The following section demonstrates a procedure of compiling the averaged model of boost converter.

3 Demonstration of modeling the boost converter

The boost converter, analyzed in [1], but now with concrete semiconductor switches, is shown in Fig. 2 (a). Fig. 2 (b) demonstrates its modeling according to the algorithm from Fig. 1 (b). The coil and capacitor ESRs and the V/A characteristics of the active and the passive switches are included in the model. The duty ratio is determined by the voltage-controlled source E3. Its DC attribute defines the bias value of the duty ratio, whereas its AC value serves to model the control-to-output frequency response.

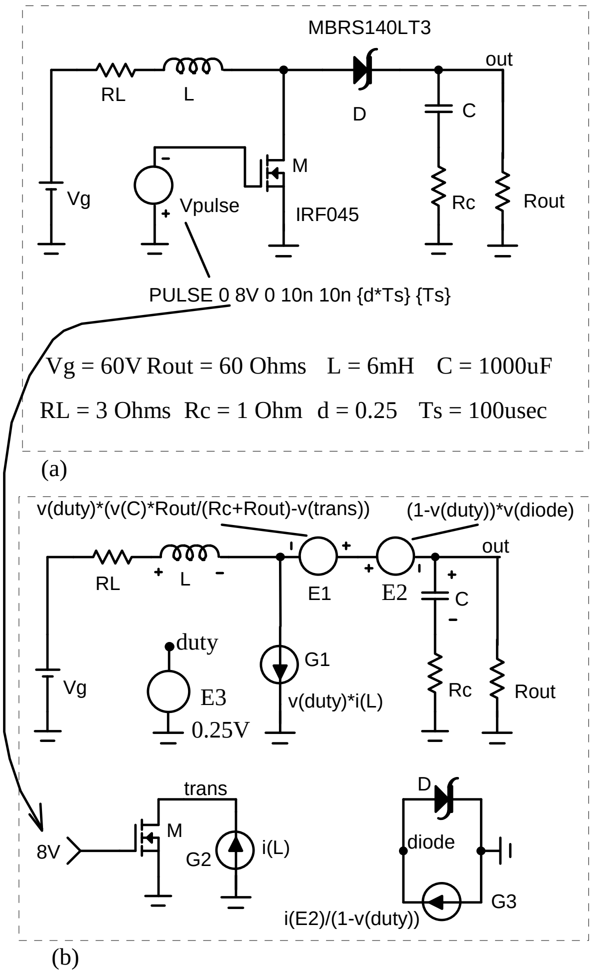

Fig. 2: (a) The boost converter, adopted from [1], (b) its averaged model, including serial resistances of L and C and full PSpice modeling of semiconductor switches.

The controlled current source G1 is defined in conformity with **Rule 1:** When the active switch is closed, the diode is in the off-state and the total inductance current $i(L)$ flows through the active switch. This current is multiplied by the duty ratio.

The controlled voltage source E1 is determined according to **Rule 2:** When the active switch is closed, the voltage across the non-conducting diode is equal to the difference between the output voltage of the converter and the voltage across the active switch. Since the diode is in off-state, the output voltage is determined by the voltage across the capacitor, which is transformed into the output via the voltage divider R_{out}, R_c (the first part of the expression for the voltage of source E1 in parentheses). The voltage across the active switch $v(trans)$ is computed by PSpice as a voltage drop between the D and S terminals of IRF045 when making a copy of $i(L)$ flow to the drain.

The converter model is complemented with the source E2 according to the procedure of modeling the passive switch in Fig. 1 (b). The diode voltage drop is computed on the assumption of diode current $<i_p>/d'$ (see Fig. 1 (b)).

4 Demonstration of PSpice analyses

The converter model in Fig. 2 (b) has been implemented in OrCADPSpice 16 [8]. To enable a comparative analysis, the switch-level model of the converter has been simultaneously analyzed. Results of transient analysis in Fig. 3 confirm a very good correspondence between both models. The averaging-based analysis takes only 0.3 seconds while the switching-level analysis time was 35 seconds on the AMD Athlon ™ Processor, 2.4GHz, 2GB RAM. The reason for such a large simulation time was the low step ceiling of 100 nseconds applied in order to increase the precision of the analysis.

In the steady state, the following quantities were measured in Probe: an average output voltage of 72.654V with a peak-to-peak ripple of 1.974V, an average inductor current of 1.622A with a peak-to-peak ripple of 229mA. This is in good conformity with the outputs of proposed averaged modeling: an output voltage of 72.662V, and an inductor current of 1.617A. Note that the averaged values of switched waveforms were computed via special Probe functions, employing curve integration. Also the DC analysis of the averaged model confirms the values from the steady state analysis.

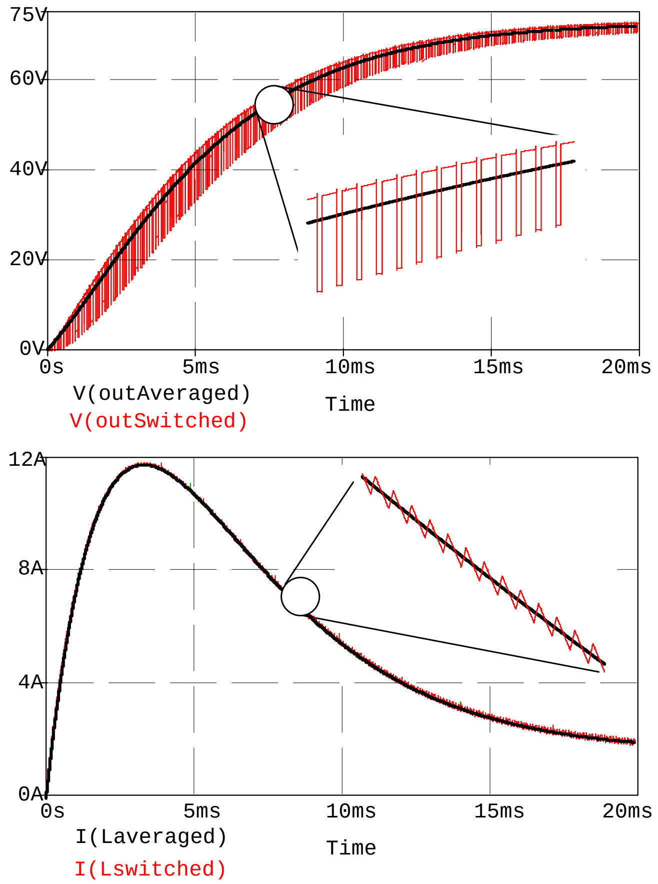

Fig. 3: Comparison of results of switched and averaged modeling: The input-step response.

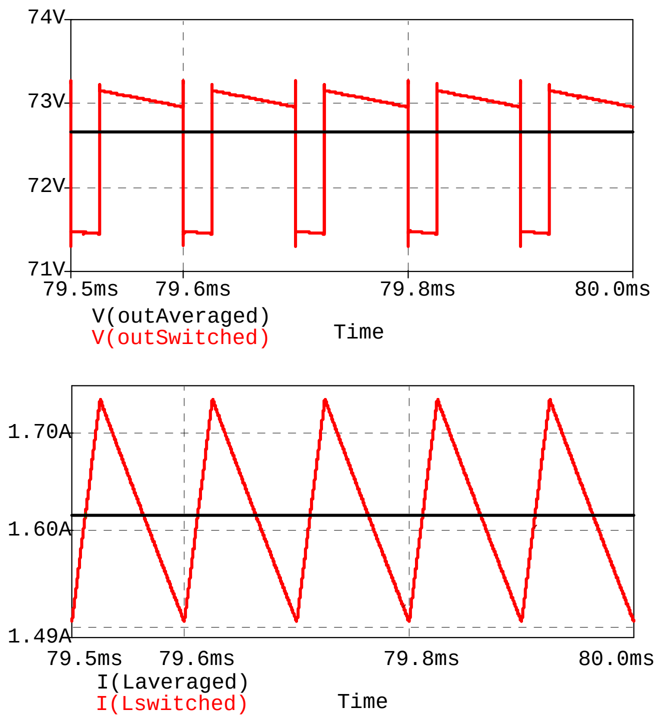

Fig. 4: Comparison of results of switched and averaged modeling: The steady state.

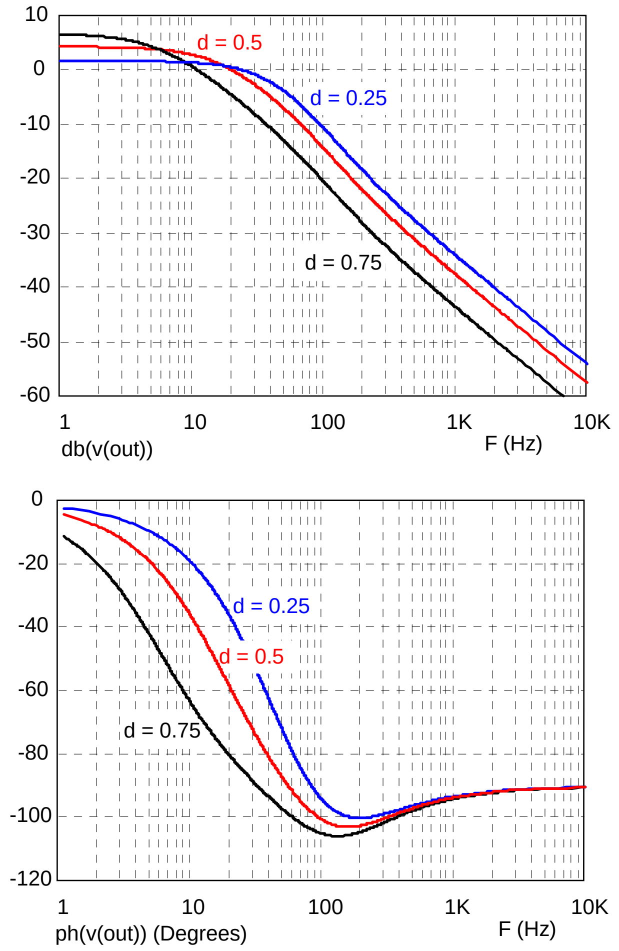

Fig. 5: Line-to-output frequency responses for different duty ratios.

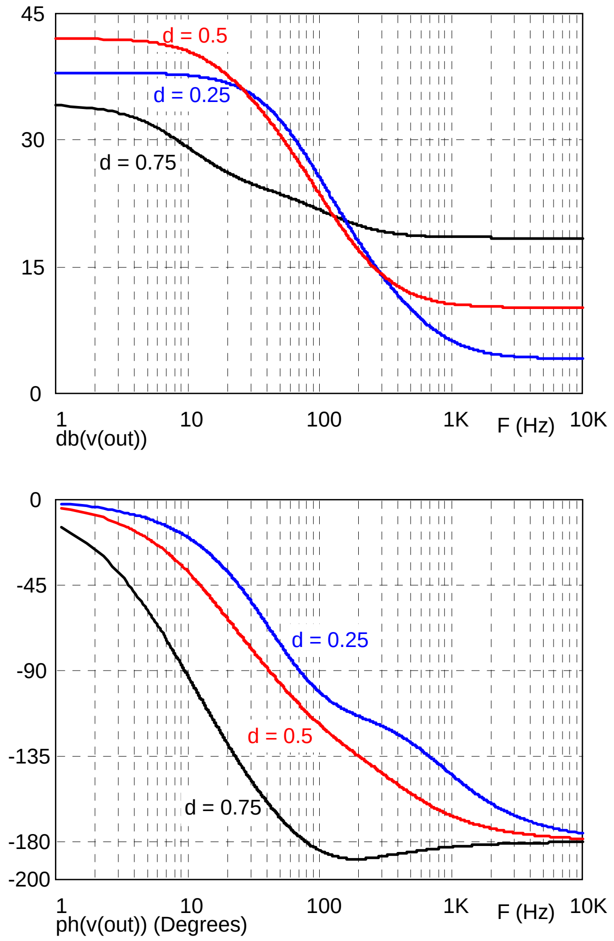

Fig. 6: Control-to-output frequency responses for different duty ratios.

Appending the attribute AC=1 to the voltage source Vg, one can easily determine the line-to-output frequency response via the AC analysis of averaged model. Similarly, the control-to-output frequency response can be analyzed after modeling the AC component of source E3, which determines the instantaneous duty ratio.

The analysis was run for different values of duty ratios, see Figs 5 and 6.

When applying the SPICE Transfer Function (TF) analysis to the averaged model in Fig. 2 (b), the transfer function from the source Vg to the output must correspond for the given duty ratio to the DC value of line-to-output frequency response in Fig. 5. Similarly, the transfer function from the source E3 to the output must give the DC value of control-to-output frequency response in Fig. 6. In our case, the results of TF and AC analyses are identical with an accuracy to 4 digits.

5 Conclusion

A generalization of the averaged model of PWM switch from [1] is presented. In contrast to [1], which has considered only ideal switches, this modeling now includes also the general V/I characteristics of active and passive switches. A practical procedure is described which can directly utilize the SPICE models of switches as semiconductor devices, i.e. transistors and diodes. During the computer analysis of DC-DC converter, the corresponding parameters of switches, which are necessary for averaged modeling, are automatically extracted from these models.

All the comparisons of the results of this averaged modeling with the corresponding results of the analysis of boost converter on the switched level of modeling, and with the results of SPICE TF analysis indicate the correctness of the proposed method, which is equivalent to the method of state space averaging.

Acknowledgments

This work has been supported by the Grant Agency of the Czech Republic under grant No. 102/08/0784, 102/08/0851, and by the research programmes of BUT MSM0021630503, MSM0021630513, and UD Brno MO FVT0000403.

Appendix – Proof of the model in Fig. 1 (b)

Consider the active switch in Fig. A1 (a), periodically closed within the phases 1, i.e. within the time intervals $<kT_S, kT_S+dT_S)$, k = ..1, 0,1, … . In the on-state, the current $i_a(t)$ through this switch is a part of the current $i_x(t)$. This current depends on the converter state variables and on the input voltage. For the boost converter in Fig. 2 (a), i_x is directly the inductor current.

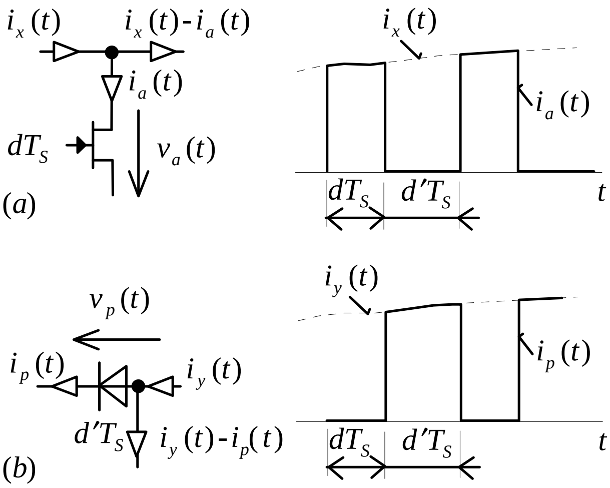

Fig. A1: Signal relations of the active and the passive switches.

In the off-state of the active switch, the switch current is practically zero, see Fig. A1 (a).

The relation between the currents $i_x(t)$ and $i_a(t)$ is as follows:

$$i_a(t) = i_x(t)\, f_d(t)\,, \qquad \text{(A1)}$$

where $f_d(t)$ is a periodic function which is 1 for time instants when the active switch is closed, and 0 for other times.

In the first step, consider a simplification which is postulated in many references, e.g. [6], i.e. neglecting the variations of i_x within one switching period. Then $i_x(t) = <i_x>$ and

$$< i_a >= d < i_x >. \qquad \text{(A2)}$$

During the switching phases 1, $i_x = i_a$. Thus, according to the notation from Rule 1,

$$< i_a >= d < i_a^1 >. \qquad \text{(A3)}$$

This equation is a mathematical model of the controlled current source which models the active switch in Fig. 1 (b).

Note that such proofs are based on the above simplifications that i_x is held constant within the switching period. This fact indicates that one cannot be sure whether the well-known averaging techniques are precise enough in a general case when the variations of circuit variables cannot be neglected.

Consider the DC-DC converter in its periodic steady state. Then all the circuit variables, including the current $i_x(t)$, are periodic with the repetition period T_S, and they can be expanded to the following Fourier series:

$$i_x(t) = I_{x0} + \sum_{k=1}^{\infty} I_{xk} \cos(k\Omega_s t + \varphi_{xk})\,, \qquad \text{(A4)}$$

$$f_d(t) = F_{d0} + \sum_{k=1}^{\infty} F_{dk} \cos(k\Omega_s t + \varphi_{dk})\,, \qquad \text{(A5)}$$

where I_{xk}, F_{dk}, φ_{xk}, and φ_{dk} are magnitudes and initial phases of the harmonic components of signals i_x and f_d, and $\Omega_S=2\pi/T_S$. Since f_d is a rectangular signal, it is well-known that

$F_{d0} = d$,

$$F_{dk} = 2d\,\frac{\sin(k\Omega_s dT_s/2)}{k\Omega_s dT_s/2} = 2\,\frac{\sin(k\pi d)}{k\pi},\; k \neq 0\,. \qquad \text{(A6)}$$

Also note that

$$I_{x0} = < i_x(t) >,\, F_{d0} = < f_d(t) >. \qquad \text{(A7)}$$

Substituting (A4) and (A5) into (A1), short re-arrangenment, taking (A6) into account, yields

$$\begin{aligned} i_a(t) &= dI_{x0} + I_{x0}\sum_{k=1}^{\infty} F_{dk}\cos(k\Omega_s t + \varphi_{dk}) + \\ &+ d\sum_{k=1}^{\infty} I_{xk}\cos(k\Omega_s t + \varphi_{xk}) + \\ &+ \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} I_{xk}F_{dl}\cos(k\Omega_s t + \varphi_{xk})\cos(l\Omega_s t + \varphi_{dl})\,. \end{aligned} \qquad \text{(A8)}$$

Integrating both sides of Eq. (A8) within one switching period and dividing the result by T_s, the average values are as follows:

$$< i_a >= d < i_x > + \frac{1}{2}\sum_{k=1}^{\infty} I_{xk}F_{dk}\cos(\varphi_{xk} - \varphi_{dl})\,. \qquad \text{(A9)}$$

Comparing (A9) and (A2), one can conclude that the second term in (A9) represents a difference between the real average value of current i_a in the converter steady state, and the value evaluated by conventional averaged model of active switch in Fig. 1 (b).

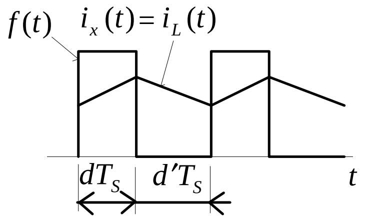

Fig. A2: Waveforms typical of DC-DC converter.

Fig. A2 shows a typical triangular waveform of inductance current in the converter steady state. A comparison of initial phases φ_{xk} and φ_{dk} of *k*-th harmonic components of both signals in Fig. A2 for different duty ratios *d* leads to the conclusion that their difference, which appears in Eq. (A9), is always 90 degrees. That is why the second term in Eq. (A9) is zero and thus Eq. (A2) is sufficiently precise for the converter steady state.

Similarly, Fig. A1 (b) shows that the following equality is true for the passive switch:

$$<i_p>=d'<i_y>=d'<i_p^2>, \qquad (A10)$$

where i_p^2 is the current flowing through the passive switch during switching phase 2.

The average voltage across the passive switch is given as follows:

$$<v_p>=d<v_p^1>+d'<v_p^2>, \qquad (A11)$$

where v_p^1 and v_p^1 are voltages in phases 1 and 2, respectively. During phase 2, the passive switch is closed, and the dependence of its voltage on current is given by its nonlinear V/A characteristics:

$$v_p^2=V(i_p^2). \qquad (A12)$$

Equations (A12), (A11), and (A10) yield

$$<v_p>=d<v_p^1>+d'V\left(\frac{<i_p>}{d'}\right). \qquad (A13)$$

This equality can be modeled by two voltage sources in series connection as shown in Fig. 1 (b). Thus the correctness of this model of passive switch is also confirmed.

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