On the Validity of SSA-based Models of DC-DC Converters

VIERA BIOLKOVA, ZDENEK KOLKA, DALIBOR BIOLEK
Depts. of Radio Electronics and Microelectronics, Brno University of Technology
Dept. of EE, University of Defence, Brno, Czech Republic
biolkova@feec.vutbr.cz, kolka@feec.vutbr.cz, dalibor.biolek@unob.cz

Abstract: State-space averaging (SSA) belongs to commonly used techniques of modeling switched DC-DC converters. In the paper, an analysis of the accuracy of line-to-output (LTO) frequency responses, obtained by applying the SSA technique to switched converters, is performed. A method of simple computation of the Fourier components of signals of the converter with single-tone excitation and operating in the continuous current mode with unmodulated duty ratio is described. This method is used for determining the frequency dependence of the spectral term at the frequency of the input signal. This dependence is then compared with the LTO frequency response which is obtained from the SSA method. These comparisons are made for Buck-, Boost-, and Buck-Boost- type converters. Conclusions about the accuracy of the SSA outputs are drawn within the context of the character of the state matrices of the converter.

Key-Words: switched DC-DC converter, averaging, SSA, spectrum, LTO frequency response

1 Introduction

Averaging belongs to the most popular tools for effective modeling and computer simulation of switched DC-DC converters, both in the time and the frequency domains [1-15]. Instead of a complicated switched-level model of the converter, a simplified model is employed, which smoothes the switching effects and replaces the complicated waveforms by their low-frequency envelopes. The State-Space Averaging method (SSA) was published in 1977 [2], and circuit-oriented tools for simple building of the averaged models were subsequently designed, particularly the method of converter canonical models [3] and the method of PWM switch [4, 5]. Problems associated with the impact of ESR (Equivalent Series Resistances) modeling of filtering capacitors are also discussed in the last two references when the method of PWM switches can provide different results from those by the SSA approach. The final solution of the above problem is described in [6]. A special way of implementing the model of the PWM switch guarantees the same results as those of the SSA method. This procedure is then generalized in [7] for behavioral modeling of the actual influence of switch parameters, and another generalization is described in [8], providing automatic extraction of switch parameters from complex SPICE models of the transistor and the diode.

Since the average models enable a simple small-signal AC analysis, they play an irreplaceable role in the analysis of small-signal frequency responses of switched converters. However, since the averaging techniques represent a certain simplification of the original switched-level model, the question can arise whether such a simplification introduces errors in the AC analysis, with a negative impact, for instance, on the results of the stability testing of the switched regulator. Back in the original work [2], the accuracy of the SSA method is determined by a condition that the natural frequency of the filter inside the switched converter must be much lower than the switching frequency. An additional condition must be also fulfilled that the frequency of the signal which modulates the switching duty ratio must be also substantially below the switching frequency [9], [10]. A statement is given in [11] that SSA provides exact results only for zero-frequency signals, and when this frequency approaches the switching frequency, the error becomes ill-defined. In fact, the switching frequency does not appear in the classical SSA method, but it represents an important real parameter of switched converter. In [12], two assumptions for a correct operation of the SSA method are defined: 1) The switching frequency is much higher than the highest natural frequency of the converters in each switching phase, and 2) the input of the converters in each switching phase must be time-independent or a slow time-varying variable in comparison with the switching frequency. In [13], the conditions for the justification of state-space averaging have been characterized as follows: a „small ripple" condition, a „linear ripple" approximation, and „the degree to which certain vector fields commute". As a consequence of these limitations, several modifications of the conventional averaging techniques have been developed, enabling, for example, an analysis of resonant-type converters [12, 13, 14] or predicting the switching instabilities in peak-current PWM converters [15].

The above facts can raise doubts whether the conventional SSA technique provides credible outputs of the analysis of frequency responses, particularly near and above the Nyquist frequency. When perturbing the input of the converter by a small-signal harmonic excitation with the frequency ω_i, there appears a rich spectrum at the converter output. If we consider the single spectral term at frequency ω_i, then the converter transfer function can be defined as a ratio of the phasor, which represents this spectral term, to the phasor of the input signal. In this paper, we find an answer to the following question: Does the SSA method provide a frequency response, also known as the LTO (Line-To-Output) response, that precisely corresponds to the above transfer function?

The paper has the following structure: Section 2, which follows this Introduction, specifies the assumptions for the subsequent analysis, and introduces the model of switched converter. Based on the Fourier analysis, a structure of general equations of the converter is found, which enables numerical computation of the spectral terms of the output signal of the converter on the assumption of its harmonic excitation. Also, a connection between the conventional SSA equations and the above general equations is found, and an error term is identified which represents the disagreement between the output of the SSA method and the actual behavior of the converter. In Section 3, the results of numerical computations for Buck, Boost, and Buck-Boost converters are presented, and connections between the SSA inaccuracies and the forms of the state matrices of the converters are discussed.

2 Mathematical model

Let us consider a DC-DC converter consisting of linear passive elements and ideal switches that have r different configurations. Normally, $r = 2$ for converters operating in the continuous current mode (CCM), and $r = 3$ for converters operating in the discontinuous current mode (DCM).

Let us restrict the subsequent analysis to converters operating in CCM, where the switching instants are determined only by the external control signal, i.e. $r = 2$. In each switch configuration the converter represents a linear system

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}_i\mathbf{x} + \mathbf{B}_i v\,,\; i = 1, 2, \tag{1}$$

where $\mathbf{x}$ is the state vector, and v is the input voltage. Switch states periodically repeat with the period $T_s = 1/f_s = 2\pi/\omega_s$, where f_s (ω_s) is the switching frequency.

Let the converter operate with a constant ratio of durations of switching phases T_1 and T_2, where $T_1+T_2 = T_S$, i.e. with a constant duty ratio $d = T_1/T_S$. Thus the results obtained by this analysis will be useful in assessing the applicability of the SSA method for converters operating with constant duty ratio and for the evaluation of accuracy of „line-to-output“ responses.

With respect to the above assumptions the converter can be regarded as a linear system with time-varying structure having state equations

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)v\,, \tag{2}$$

where $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are periodic matrix (vector) functions with repeating period T_S.

With respect to periodicity, matrix $\mathbf{A}$ and vector $\mathbf{B}$ can be represented by the Fourier series:

$$\mathbf{A}(t) = \sum_{k=-\infty}^{+\infty}\mathbf{A}^{(k)}e^{jk\omega_s t}\,,\; \mathbf{B}(t) = \sum_{k=-\infty}^{+\infty}\mathbf{B}^{(k)}e^{jk\omega_s t}\,. \tag{3}$$

In the case of two-state switching, where the converter is described by two state matrices $\mathbf{A}_1$, $\mathbf{A}_2$ and vectors $\mathbf{B}_1$, $\mathbf{B}_2$ corresponding to each phase, the spectral components $\mathbf{A}^{(k)}$ and $\mathbf{B}^{(k)}$ are given by the well-known formulae:

$$\mathbf{A}^{(0)} = d\mathbf{A}_1 + (1-d)\mathbf{A}_2 = \mathbf{A}_2 + d(\mathbf{A}_1 - \mathbf{A}_2)\,,$$
$$\mathbf{A}^{(k)} = d\,\frac{\sin(k\pi d)}{k\pi d}e^{-jk\pi d}(\mathbf{A}_1 - \mathbf{A}_2)\,,\; k\neq 0, \tag{4a}$$

$$\mathbf{B}^{(0)} = d\mathbf{B}_1 + (1-d)\mathbf{B}_2 = \mathbf{B}_2 + d(\mathbf{B}_1 - \mathbf{B}_2)$$
$$\mathbf{B}^{(k)} = d\,\frac{\sin(k\pi d)}{k\pi d}e^{-jk\pi d}(\mathbf{B}_1 - \mathbf{B}_2)\,,\; k\neq 0. \tag{4b}$$

As system (2) is linear, a “one-sided” excitation of the converter can be considered in the form

$$v(t) = e^{j\omega_i t} \tag{5}$$

with frequency f_i (or $\omega_i = 2\pi f_i$).

Then the spectrum of the state vector $\mathbf{x}$ of the converter in the steady state contains only components with combination frequencies $k\omega_s+m\omega_i$, $k=$ …-2, -1, 0, 1, 2, …, m = -1, 0, 1. The spectral term of the state vector on frequency $k\omega_s+m\omega_i$ will be denoted $\mathbf{x}^{(k,m)}$ or simply (k, m).

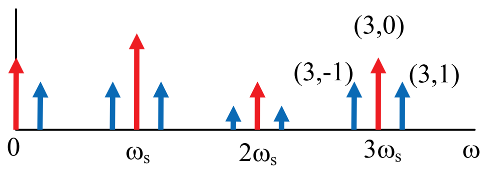


Fig. 1 Spectrum of state vector.

The Fourier series of the state vector can be written as

$$\mathbf{x}(t) = \sum_{k=-\infty}^{+\infty}\left[\mathbf{x}^{(k,0)} e^{jk\omega_s t} + \mathbf{x}^{(k,1)} e^{j(k\omega_s+\omega_i)t} + \mathbf{x}^{(k,-1)} e^{j(k\omega_s-\omega_i)t}\right] =$$

$$= \sum_{k=-\infty}^{+\infty}\left[\mathbf{x}^{(k,0)} + \mathbf{x}^{(k,1)} e^{j\omega_i t} + \mathbf{x}^{(k,-1)} e^{-j\omega_i t}\right] e^{jk\omega_s t}. \qquad (6)$$

and its time-derivative will be

$$\frac{d}{dt}\mathbf{x}(t) = \sum_{k=-\infty}^{+\infty}[jk\omega_s \mathbf{x}^{(k,0)} + j(k\omega_s+\omega_i)\mathbf{x}^{(k,1)} e^{j\omega_i t} +$$

$$+ j(k\omega_s-\omega_i)\mathbf{x}^{(k,-1)} e^{-j\omega_i t}] e^{jk\omega_i t}. \qquad (7)$$

With respect to (3), (5)-(7), state equation (2) can be rewritten to

$$\sum_{k=-\infty}^{+\infty}[jk\omega_s \mathbf{x}^{(k,0)} + j(k\omega_s+\omega_i)\mathbf{x}^{(k,1)} e^{j\omega_i t} +$$

$$+ j(k\omega_s-\omega_i)\mathbf{x}^{(k,-1)} e^{-j\omega_i t}] e^{jk\omega_i t} =$$

$$= \sum_{k=-\infty}^{+\infty}\mathbf{A}^{(k)} e^{jk\omega_s t} \sum_{k=-\infty}^{+\infty}\left[\mathbf{x}^{(k,0)} + \mathbf{x}^{(k,1)} e^{j\omega_i t} + \mathbf{x}^{(k,-1)} e^{-j\omega_i t}\right] e^{jk\omega_s t} +$$

$$+ \sum_{k=-\infty}^{+\infty}\mathbf{B}^{(k)} e^{jk\omega_s t}.e^{j\omega_i t}. \qquad (8)$$

After a rearrangement we obtain

$$\sum_{k=-\infty}^{+\infty}[jk\omega_s \mathbf{x}^{(k,0)} + j(k\omega_s+\omega_i)\mathbf{x}^{(k,1)} e^{j\omega_i t} +$$

$$+ j(k\omega_s-\omega_i)\mathbf{x}^{(k,-1)} e^{-j\omega_i t}] e^{jk\omega_i t} =$$

$$= \sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\mathbf{A}^{(k-m)}\left[\mathbf{x}^{(m,0)} + \mathbf{x}^{(m,1)} e^{j\omega_i t} + \mathbf{x}^{(m,-1)} e^{-j\omega_i t}\right] e^{jk\omega_s t} +$$

$$+ \sum_{k=-\infty}^{+\infty}\mathbf{B}^{(k)} e^{j(k\omega_s+\omega_i)t}, \qquad (9)$$

representing an equality of the Fourier series of two signals. This means that individual spectral components with corresponding frequencies must be equal.

Let us exclude cases where the spectral terms cross due to aliasing from further analyses. With respect to the fact that the spectra of matrix **A** and vector **B** contain only terms at multiples of ω_s, the energetic balance of (9) is formed separately by spectral components with the same index ω_i. Therefore we can write a separate condition for components (•,+1)

$$\sum_{k=-\infty}^{+\infty}\left[j(k\omega_s+\omega_i)\mathbf{x}^{(k,1)}\right] e^{j(k\omega_s+\omega_i)t} =$$

$$\sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\mathbf{A}^{(k-m)}\mathbf{x}^{(m,1)} e^{j(k\omega_s+\omega_i)t} + \sum_{k=-\infty}^{+\infty}\mathbf{B}^{(k)} e^{j(k\omega_s+\omega_i)t}. \qquad (10)$$

Assuming $k = 0$ in (10) and reducing the factor $e^{j\omega i t}$ we obtain an equation that follows from the equality of spectral components at frequency ω_i:

$$j\omega_i \mathbf{x}^{(0,1)} = \sum_{m=-\infty}^{+\infty}\mathbf{A}^{(-m)}\mathbf{x}^{(m,1)} + \mathbf{B}^{(0)}. \qquad (11)$$

After a rearrangement we get

$$\left[\mathbf{A}^{(0)} - j\omega_i \mathbf{E}\right]\mathbf{x}^{(0,1)} + \sum_{\substack{m=-\infty \\ m\neq 0}}^{+\infty}\mathbf{A}^{(-m)}\mathbf{x}^{(m,1)} = -\mathbf{B}^{(0)}, \qquad (12)$$

where **E** is the unity matrix.

Without the second term on the left-hand side, (12) represents the well-known state equation of the averaged model. Thus the second term represents an error factor that determines the difference between the averaged model and the reality.

The impact of the error term on the accuracy of the averaged model depends on the spectrum of matrix **A** and the spectral components $\mathbf{x}^{(m,1)}$, which represent ripple of the state vector around its average value. The spectrum of matrix **A** is determined by state matrices in the individual quasistable phases of the converter and by the duty ratio d. The influence of spectral components $\mathbf{x}^{(m,1)}$ decreases with increasing ratio of the switching frequency to the input signal frequency.

Components of the error term in (12) can be computed numerically. Formula (12) is a part of the more general equation (10) for spectral terms of type $k\omega_s+\omega_i$, $k= \ldots-2, -1, 0, 1, 2, \ldots$. It represents an infinite-dimension system of linear equations (13).

Equation (12) is highlighted in (13). The magnitudes of the components of the generalized vector $\mathbf{x}^{(m,1)}$ quickly decrease with increasing m. The same holds for the spectral components of matrix **A** and vector **B**. It is therefore possible to use a finite number of equations and to determine, with a certain error, the components of $\mathbf{x}^{(m,1)}$ by solving (13). It requires the knowledge of matrices $\mathbf{A}^{(k)}$ and $\mathbf{B}^{(k)}$, and

$$\cdots\begin{pmatrix} & & \cdots & & \\ \mathbf{A}^{(0)}\text{-}j(\omega_i\text{-}2\omega_s)\mathbf{E} & \mathbf{A}^{(-1)} & \mathbf{A}^{(-2)} & \mathbf{A}^{(-3)} & \mathbf{A}^{(-4)} \\ \mathbf{A}^{(1)} & \mathbf{A}^{(0)}\text{-}j(\omega_i\text{-}\omega_s)\mathbf{E} & \mathbf{A}^{(-1)} & \mathbf{A}^{(-2)} & \mathbf{A}^{(-3)} \\ \mathbf{A}^{(2)} & \mathbf{A}^{(1)} & \mathbf{A}^{(0)}\text{-}j\omega_i\mathbf{E} & \mathbf{A}^{(-1)} & \mathbf{A}^{(-2)} \\ \mathbf{A}^{(3)} & \mathbf{A}^{(2)} & \mathbf{A}^{(1)} & \mathbf{A}^{(0)}\text{-}j(\omega_i\text{+}\omega_s)\mathbf{E} & \mathbf{A}^{(-1)} \\ \mathbf{A}^{(4)} & \mathbf{A}^{(3)} & \mathbf{A}^{(2)} & \mathbf{A}^{(1)} & \mathbf{A}^{(0)}\text{-}j(\omega_i\text{+}2\omega_s)\mathbf{E} \\ & & \cdots & & \end{pmatrix}\begin{pmatrix} \cdots \\ \boldsymbol{x}^{(-2,1)} \\ \boldsymbol{x}^{(-1,1)} \\ \boldsymbol{x}^{(0,1)} \\ \boldsymbol{x}^{(1,1)} \\ \boldsymbol{x}^{(2,1)} \\ \cdots \end{pmatrix} = -\begin{pmatrix} \cdots \\ \mathbf{B}^{(-2)} \\ \mathbf{B}^{(-1)} \\ \mathbf{B}^{(0)} \\ \mathbf{B}^{(1)} \\ \mathbf{B}^{(2)} \\ \cdots \end{pmatrix} \qquad (13)$$

frequencies ω_i and ω_s.

Let us assume a limited number of harmonic components with indexes $k = -K, ..., 0, ..., K$ for the converter with M accumulating elements (the state vector has M components). Equation (13) represents a set of $(2K+1)M$ scalar linear equations. Numerical experiments show that starting from a certain value of K, a further increase in the number of harmonic components does not influence the component (0,1). The number of harmonics is a trade-off between accuracy and the size of the resulting system of equation.

Equation (13) confirms the validity of the averaged model in the case of negligible ripple of the state variables. Considering $\omega_s \to \infty$, the amplitudes of all spectral components of the state vector, which represent the ripple, vanish and the only nonzero variables in (13) are the components $x_i^{(0,+1)}$. Equation (13) transforms into (12) with the zero error term on the left-hand side, which is the classical averaged model of switched converter.

3 Numerical experiments

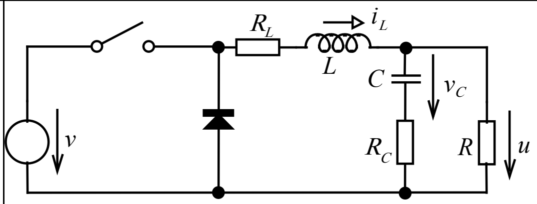
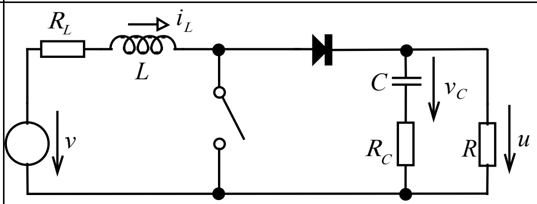
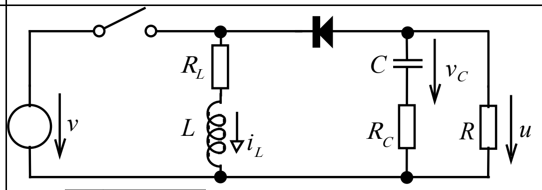
In order to be able to compute small-signal transfer functions in a general case, it is necessary to complete (2) with the output equation to the full form of state description

$$u = \mathbf{C}(t)\mathbf{x} + D(t)v\,, \qquad (14)$$

where u is the output voltage, $\mathbf{C}$ is a vector of dimension $(1, M)$, and D is a scalar.

Similar to the derivation of (10), we obtain the frequency domain form of (14)

Table 1. Analyzed switched converters.

type	schematic, output description	state description
Buck	 $\mathbf{C}_1 = \begin{bmatrix} R_C \parallel R & \frac{R}{R_C+R} \end{bmatrix}, D_1 = 0$ $\mathbf{C}_2 = \begin{bmatrix} R_C \parallel R & \frac{R}{R_C+R} \end{bmatrix}, D_2 = 0$	$\mathbf{A}_1 = \begin{bmatrix} -\frac{R_L + R_C \parallel R}{L} & -\frac{R}{L(R_C+R)} \\ \frac{R}{C(R_C+R)} & -\frac{1}{C(R_C+R)} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$ $\mathbf{A}_2 = \begin{bmatrix} -\frac{R_L + R_C \parallel R}{L} & -\frac{R}{L(R_C+R)} \\ \frac{R}{C(R_C+R)} & -\frac{1}{C(R_C+R)} \end{bmatrix}, \mathbf{B}_2 = \mathbf{0}$
Boost	 $\mathbf{C}_1 = \begin{bmatrix} 0 & \frac{R}{R_C+R} \end{bmatrix}, D_1 = 0,$ $\mathbf{C}_2 = \begin{bmatrix} R_C \parallel R & \frac{R}{R_C+R} \end{bmatrix}, D_2 = 0$	$\mathbf{A}_1 = \begin{bmatrix} -\frac{R_L}{L} & 0 \\ 0 & -\frac{1}{C(R_C+R)} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$ $\mathbf{A}_2 = \begin{bmatrix} -\frac{R_L + R_C \parallel R}{L} & -\frac{R}{L(R_C+R)} \\ \frac{R}{C(R_C+R)} & -\frac{1}{C(R_C+R)} \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$
Buck-Boost	 $\mathbf{C}_1 = \begin{bmatrix} 0 & \frac{R}{R_C+R} \end{bmatrix}, D_1 = 0$ $\mathbf{C}_2 = \begin{bmatrix} -R_C \parallel R & \frac{R}{R_C+R} \end{bmatrix}, D_2 = 0$	$\mathbf{A}_1 = \begin{bmatrix} -\frac{R_L}{L} & 0 \\ 0 & -\frac{1}{C(R_C+R)} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$ $\mathbf{A}_2 = \begin{bmatrix} -\frac{R_L + R_C \parallel R}{L} & \frac{R}{L(R_C+R)} \\ -\frac{R}{C(R_C+R)} & -\frac{1}{C(R_C+R)} \end{bmatrix}, \mathbf{B}_2 = \mathbf{0}$

$$\sum_{k=-\infty}^{+\infty} u^{(k,1)}\, e^{j(k\omega_s+\omega_i)t} = \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \mathbf{C}^{(k-m)} \mathbf{x}^{(m,1)}\, e^{j(k\omega_s+\omega_i)t} + $$
$$+ \sum_{k=-\infty}^{+\infty} D^{(k)} e^{j(k\omega_s+\omega_i)t}\ . \qquad (15)$$

For the component (k, 1) and a finite number of harmonic components we obtain finally

$$u^{(k,1)} = \sum_{m=-K}^{+K} \mathbf{C}^{(k-m)} \mathbf{x}^{(m,1)} + D^{(k)}\ . \qquad (16)$$

The spectral components $\mathbf{C}^{(k-m)}$ and $D^{(k)}$ are given by formulae similar to (4).

The analysis has been implemented in Matlab for an arbitrary K and for an arbitrary number of switching phases. Boost, Buck, and Buck-Boost converters from Table 1 were analyzed using the following parameters that assure operation in the continuous-current mode:
d = 0.25, f_s = 10 kHz, R = 60 Ω, C = 1 mF, L = 6 mH, R_C = 1 Ω, R_L = 3 Ω.

The table shows all matrices and vectors of the complete state description. Fig. 2 shows the results of AC analyses obtained by the numerical solution of (13) for K = 10.

The comparison of spectral component (0,1) with the LTO transfer functions generated by the SSA method shows a good agreement for the Buck and Boost converters in a wide frequency band, i.e. also above the Nyquist frequency $f_s/2$ = 5 kHz. But for the Buck-Boost converter, a substantial difference occurs above the Nyquist frequency.

In the case of Buck converter, the agreement between the results of the SSA method and the frequency response (0,1) can be explained based on the form of state matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ in Table 1. The matrices are the same. According to (4a) this means that all spectral components except for the DC component are zero: $\mathbf{A}^{(k)} = \mathbf{0}$, $k \neq 0$. It means that the error term on the left- hand side of (12) is zero, irrespective of nonzero higher spectral components of the state vector, which represent the ripple of circuit quantities. The equality of the two state matrices means that the SSA method gives correct results and provides an accurate AC analysis for all frequencies, i.e. also above the Nyquist frequency.

The spectrum of matrices $\mathbf{A}$ and $\mathbf{C}$ is rich for the Boost converter as the matrices are time-varying. Thus the frequency-conversion effects occur and there is energetic interaction between the spectral terms. As vector $\mathbf{B}$ is constant, the higher harmonics are not directly excited. Their nonzero magnitude results from the conversion of the (0,1) component.

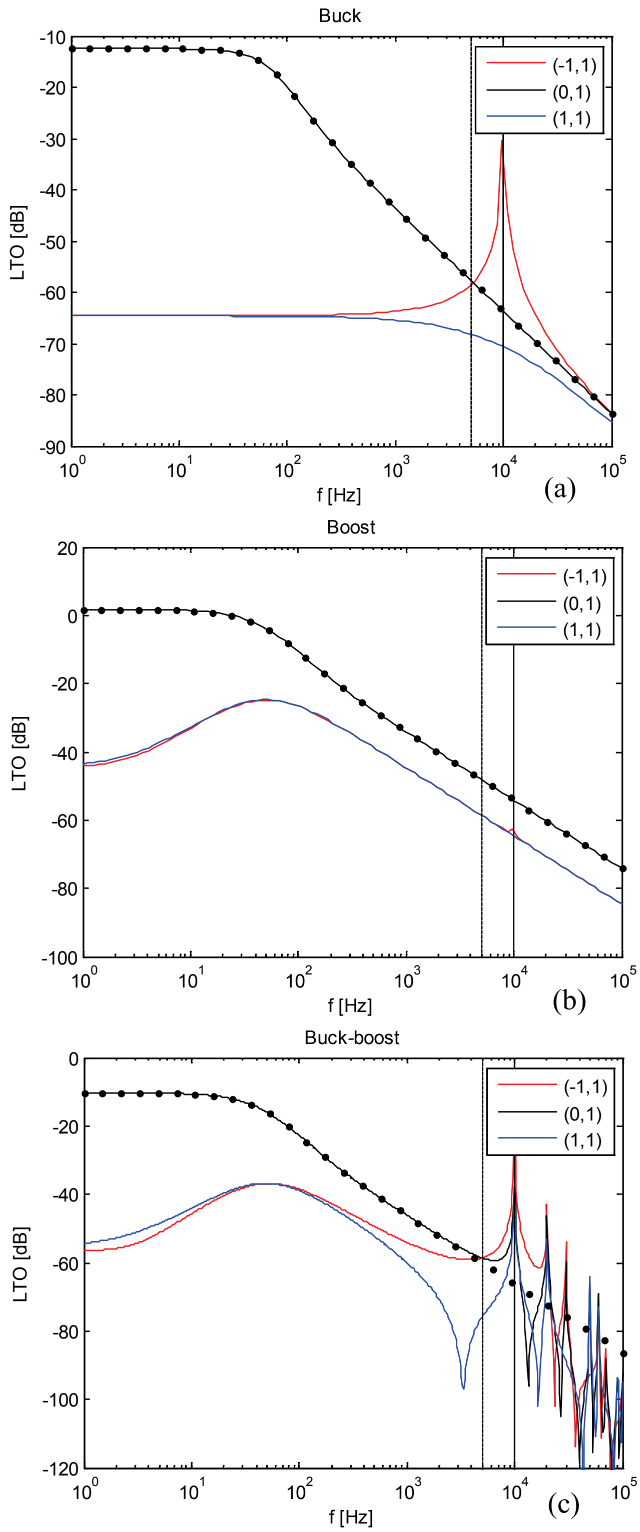


Fig. 2. Comparison of LTO frequency response obtained from the averaged model (dots) with spectral components (-1,1), (0,1), and (1,1) for (a) Buck, (b) Boost, (c) Buck-Boost converters. The vertical dashed line represents the 5 kHz Nyquists frequency and the full line represents the 10 kHz switching frequency.

The higher harmonics cause the nonzero error term in (12), i.e. the difference between component (0,1) and LTO of the SSA model. However, their influence is negligible.

State equations of the Buck-Boost converter are similar to those of the Boost converter. As vector **B** is time-varying, the higher harmonics are directly excited in (13) and their magnitude is comparable to the (0,1) component. The difference between (0,1) and LTO occurs even below the Nuquist frequency as shown in Fig. 2c.

4 Conclusions

The paper shows that:

(a) There is a simple numerical method for determining an arbitrary number of spectral components of the response of a switched converter excited by a single-tone input signal. It allows performing a generalized AC analysis of the converter.

(b) The classical SSA equations can be seen as a simplified case of the general equation of switching converter that determines the response to the harmonic input signal in the frequency domain.

(c) The difference between SSA model and actual converter behavior is given by the error term that depends on state matrices in quasistable switching phases and on the higher spectral components of state variables.

(d) The SSA method provides the most accurate results for the Buck converter, whose state matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are the same in both switching phases. The results of AC analysis are valid in a broad frequency band, i.e. also above the Nuquist frequency. On the other hand, inaccuracies occur even below the Nyquist frequency for the Buck-Boost converter.

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