VIRTUAL TRAJECTORIES OF DC CIRCUITS

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Abstract

Concept of virtual dynamic of the resistive DC circuits is presented. It is shown that so-called virtual eigenvalues can be attached to each linear model of resistive circuit linearized around its DC operating point. Corresponding virtual trajectories describe motion of operating point after its deflection from equilibrium state and help to decide if the operating point is stable.

Introduction

Investigated approach of virtual dynamic is based on some ideas published in [1] and [2]. In [1], concept of so-called potential stability of DC operating points is introduced including the sufficient condition of instability. In [2], the necessary condition of DC operating point instability is formulated with utilization of virtual eigenvalues of resistive circuits with controlled sources.

First, we distinguish resistance and conductance virtual eigenvalues. Second, we explain physical meaning of virtual trajectories. Third, we present illustration example how to investigate stability of operating points using virtual trajectories.

R- and G- virtual eigenvalues

Let us consider linear resistive circuit with *N* controlled sources. For simplicity, consider only voltage sources. However, this principle is valid for arbitrary type of controlled sources.

Deflection of the operating point from its equilibrium state is modeled using voltage sources of voltage variations (voltage virtual shiftings) δV_{0i} in series with the i-th output of controlled sources, where i = 1, .. , *N*. All voltage variations can be associated to the vector $\boldsymbol{\delta V}_0$. Resistive circuit responds to this deflection using current variations (current virtual shiftings) δI_{0i} where i = 1, .. , *N*. All current variations can be associated to the vector $\boldsymbol{\delta I}_0$.

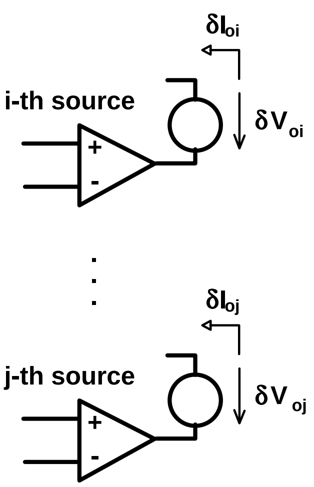

Fig.1. Voltage and current virtual shiftings.

Arbitrary linear resistive circuit with the controlled sources can be described with the resistance/conductance equations of virtual shiftings:

$$\delta \mathbf{V}_0 = \mathbf{R}_0 \delta \mathbf{I}_0 \text{ or } \delta \mathbf{I}_0 = \mathbf{G}_0 \delta \mathbf{V}_0 \qquad (1)$$

where $\mathbf{R}_0/\mathbf{G}_0$ are regular matrices of virtual resistances/conductances with the relations

$$\mathbf{R}_0 = \mathbf{G}_0^{-1}. \qquad (2)$$

Equations (1) define linear vector space, so called space of virtual shiftings or virtual space. This space is characterized by the eigenvalues of matrices $\mathbf{R}_0/\mathbf{G}_0$, so called resistance/conductance virtual eigenvalues. In [2], we formulated necessary condition of the DC operating point stability which can be extended as follows:

***DC* operating point of linear system compound of the resistors and ideal controlled sources is potentially stable if all eigenvalues of matrix $\mathbf{R_0}$ /$\mathbf{G_0}$ lie in the <u>right</u> complex half-plane.**

Proof of this theorem can be done on the basis of physical interpretation of virtual eigenvalues and for the case of conductance eigenvalues is described in following chapter.

Physical interpretation of virtual eigenvalues and virtual trajectories

Let us insert *N* real voltage sources with internal resistances *r* and internal voltages $\delta V_{0i,k}$, i = 1,..,*N* in series with the output of controlled sources as shown in Fig.2. Resistance *r* has to be chosen sufficiently small not to influence circuit parameters. Due to circuit reaction in the form of current variations $\delta I_{0i,k}$, i = 1,..,*N*, terminal voltages $\delta V_{0i,k+1}$, i = 1,..,*N* will be different immediately after virtual sources inserting then internal voltages. In vector form,

$$\delta\mathbf{V}_{0,k+1} = \delta\mathbf{V}_{0,k} - r\delta\mathbf{I}_{0,k} = (\mathbf{E} - r\mathbf{G}_0)\delta\mathbf{V}_{0,k}, r > 0, r \to 0 \qquad (3)$$

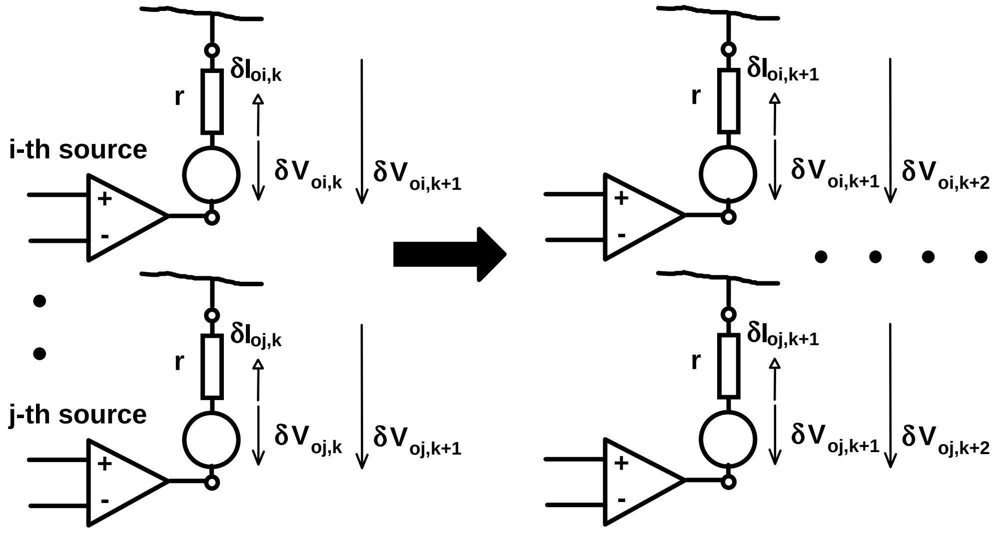

Fig.2. Generation of virtual trajectories.

In next step, we insert again to the circuit the set of real virtual sources but with the internal voltages $\delta V_{0i,k+1}$, i = 1,..,*N*. Due to system reaction, the terminal voltages $\delta V_{0i,k+2}$, i = 1,..,*N* appear. Repeating this procedure, we can observe evolution of voltage virtual shiftings in the virtual space. For the example of three-dimensional space, the principle of equation (3) is illustrated in Fig.3. If the virtual trajectory converges to the origin, then corresponding operating point is stable.

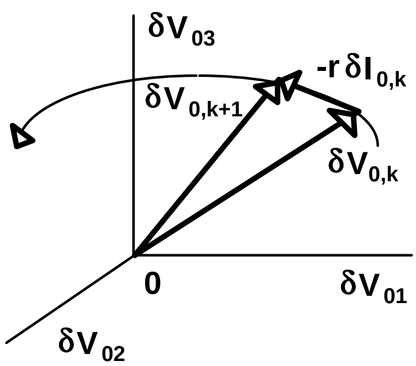

Fig.3. Virtual trajectory in three-dimensional virtual space.

Solution of equation (3) is in the form

$$\delta\mathbf{V}_{0,k} = (\mathbf{E} - r\mathbf{G}_0)^k \delta\mathbf{V}_{0,0} \qquad (4)$$

Convergence properties depend on the location of eigenvalues of the matrix **E** - *r* $\mathbf{G}_0$. It is evident that

λ'	=	$1 - r\lambda$
λ' is eigenvalue of		λ is eigenvalue of
$\mathbf{E} - r\,\mathbf{G}_0$		$\mathbf{G}_0$

Convergence condition (must be true for all eigenvalues):

$$|\lambda'| < 1 \Rightarrow 0 < r < 2\frac{\mathrm{Re}\{\lambda\}}{|\lambda|^2}, \mathrm{Re}\{\lambda\} > 0$$

If all eigenvalues of matrix G_0 lie in the right complex half-plane**,** the virtual trajectories are stable. Internal resistance *r* of virtual sources has to be chosen sufficiently small not to affect convergence properties.

For $r \to 0$, equation (4) tends to the result which confirms aforementioned conclusion:

$$\delta\mathbf{V}_{0,k} \to e^{-r\mathbf{G}_0 k}\delta\mathbf{V}_{0,0}.$$

Example

Consider DC resistive circuit in Fig.4 including virtual sources at outputs of controlled sources. We investigate necessary conditions of the operating point (in)stability.

Equation of virtual shiftings is in the form

$$\begin{bmatrix}\delta V_{01}\\ \delta V_{02}\end{bmatrix} = \begin{bmatrix} R_1(1-A_1)+R_2+R_3+R_4 & -R_1(1-A_1)-R_2 \\ -A_2R_4+(R_1+R_2)(1-A_2) & (R_1+R_2)(1-A_2)\end{bmatrix}\begin{bmatrix}\delta I_{01}\\ \delta I_{02}\end{bmatrix}$$

and corresponding characteristic equation

$$\lambda^2-(r_{11}+r_{22})\lambda+\det\mathbf{R}_0=0$$

where r_{11} and r_{22} are elements of the matrix $\mathbf{R}_0$ and det $\mathbf{R}_0$ is its determinant. Necessary and sufficient conditions of instability:

$$\mathrm{Re}\{\lambda_1\}=\mathrm{Re}\{\lambda_2\}<0 \Rightarrow A_2\left[A_1+\frac{R_3}{R_4}\left(1+\frac{R_2}{R_1}\right)\right] > \left(1+\frac{R_2}{R_1}\right)\left(1+\frac{R_3}{R_4}\right) \wedge A_1+(A_2-2)\left(1+\frac{R_2}{R_1}\right) > \frac{R_3+R_4}{R_1}$$

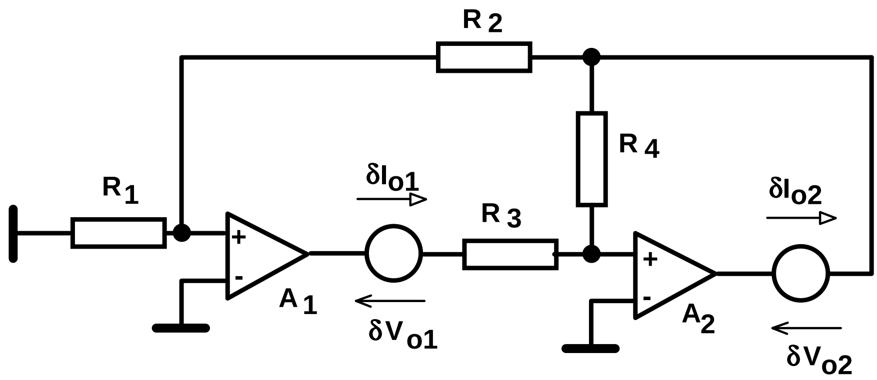

Fig.4. Example of investigated circuit with the zero-state operating point.

Using Green's algorithm [1], only first condition is obtained. Both conditions are sufficient for operating point instability but together create necessary one.

In Fig.5, the dependence of virtual eigenvalues on the gain constants A_1 and A_2 is depictured. Shaded area is the area of operating point instability. In Fig. 5, virtual trajectories are associated to certain eigenvalues. We can observe various classical types of trajectories as stable focus ($\lambda_1=\lambda_2=200$), unstable focus ($\lambda_1=\lambda_2=-200$), unstable saddle ($\lambda_1=141$, $\lambda_2=-141$) and limit cycle ($\lambda_1=j200$, $\lambda_2=-j200$).

Conclusion

For the purpose of stability testing of DC operating points in the resistive circuits, it is advantageous to use theory of virtual motion. This approach enables to compile equations of virtual shiftings and matrices of virtual resistances/conductances. Eigenvalues of these matrices have to lie in the right-half complex plane in case of operating point stability. Stability of given operating point can be observed also using so called virtual trajectories which illustrate motion of resistive circuit in the space of virtual shiftings after its deflection from the equilibrium state.

In distinction from Green's algorithm, our approach gives necessary and sufficient condition of operating point instability. Its computer algorithmization is without problems.

Acknowledgement: This work is supported by the Grant Agency of Czech Republic under Grant No. 102/94/1080.

References

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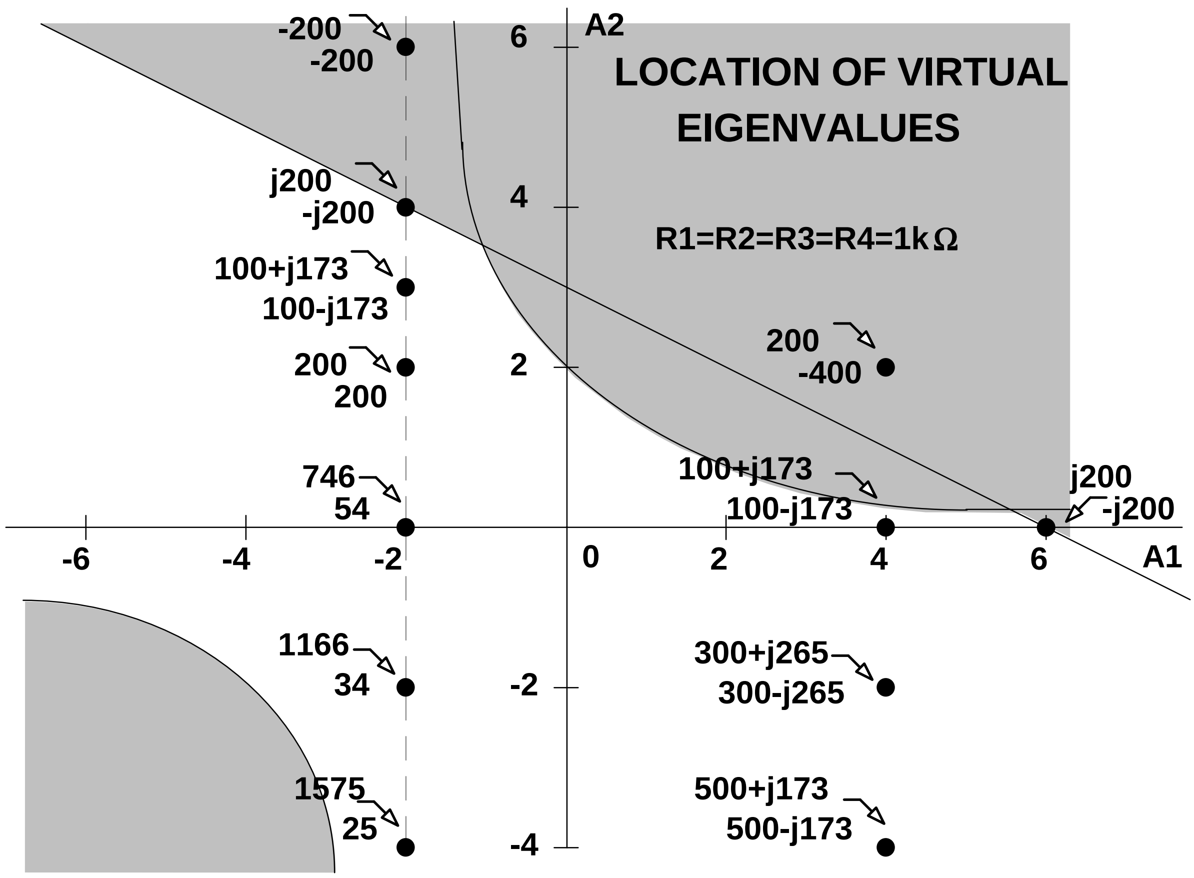

Fig.5. Location of resistive eigenvalues for the circuit in Fig.4. Shaded area is the area of instability.

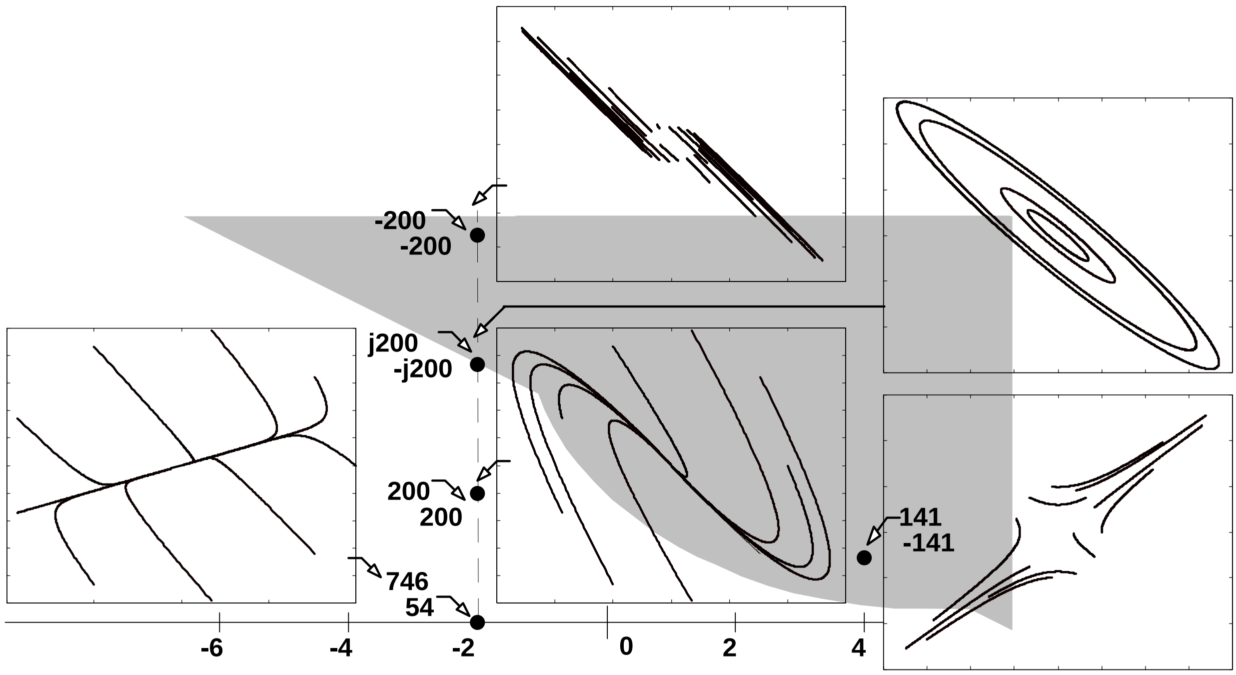

Fig.6. Virtual trajectories for given eigenvalues.