

Frequency domain analysis of switched networks by generalized transfer functions

Dalibor Biolek and Karel Zaplatilek

Department of Electrical Engineering, VA Brno, Kounicova 65, PS 13, 61200 Brno, Czech republic

This paper describes modelling of switched networks using the generalized transfer function approach. Both analog and discrete parts of network response are described simultaneously by the single function of two variables s and z . Focus is concentrated on computer implementation.

1. INTRODUCTION

The brief introduction to the theory of generalized transfer functions (GTFs) of networks with periodically varying parameters is described in [1]. This approach has been used for the frequency domain simulation of real switched networks. As a result of periodically varying parameters, the system response contains both analog and discrete parts. Both aspects can be expressed simultaneously by the generalized transfer function $K(s,z)$ of the well known Laplace and z operators. In the sense of the equivalent signal theory by Tsividis [2], the frequency response of the system is obtained after double substitution $s=j\omega$, $z=\exp(j\omega T)$, where T is the period of system parameters alternation.

This "hybrid" description includes well-known transfer functions of continuous-time and discrete-time systems as boundary cases. Computer programs, based on this approach, provide good simulation of mixed continuous- and discrete-time systems and switched systems with consistent and inconsistent initial conditions, respectively. It is also possible to compute frequency responses over the Nyquist's frequency.

In this contribution, the computer algorithm of the GTFs evaluation is introduced, placing emphasis on the networks with externally controlled switches. If no switches are included in the network, the algorithm provides the same results as an analog linear simulator.

Let us consider the linear N -phased switched network with switching diagram in Fig. 1. The main idea of the method is as follows:

- i) Description of switched network by two-graph modified nodal analysis (2-graph MNA) [3] to obtain a minimum set of equations in each switching phase.
- ii) Circuit equation pre-processing to derive equations for equivalent signals.
- iii) Applying the Laplace transform to

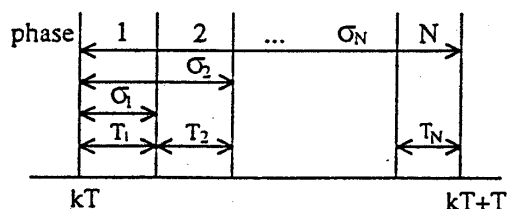


Figure 1. The switching diagram

equivalent signals yields equations for GTFs.

In the paper, the superscripts $-/+$ are used to denote circuit conditions immediately before/after switching.

2. DESCRIPTION OF SWITCHED NETWORK BY TWO-GRAPH MNA

The switched network is described in each switching phase separately, taking into account the initial conditions due to switching from the previous phase. As mentioned in [4], the Laplace transform correctly handles both consistent and inconsistent initial conditions. Thus

$$Y_1(s)V_1(s) = C_1 v_N(kT^-) + W_1(s), Y_j(s)V_j(s) = C_j v_{j-1}(kT + \sigma_{j-1}^-) + W_j(s), j = 2, 3, \dots, N, \quad (1)$$

where V_j is the vector of nodal voltages and incidental branch currents, W_j is a vector of input sources, v_{j-1} is the vector of initial conditions at the end of the previous phase. Y_j is a modified admittance matrix. The matrix C_j contains the parameters of reactance elements which mediate the phase-to-phase energy transfer.

Two examples how to determine elements of the Y_j and C_j matrices are summarised in Table 1. The symbols $\Delta, \square$ denote the voltage and current nodes in conformity with Vlach and Singhal [3]. The sign ! denotes the ban of further corresponding row filling.

phase	element	$Y_j(s)$	C_j										
j: j-1:		<table> <tr> <td>$G+sC$</td> <td>$-G-sC$</td> </tr> <tr> <td>$-G-sC$</td> <td>$G+sC$</td> </tr> </table> $\triangle a$ $\triangle b$	$G+sC$	$-G-sC$	$-G-sC$	$G+sC$	<table> <tr> <td>C</td> <td>$-C$</td> <td>$\square e$</td> </tr> <tr> <td>$-C$</td> <td>C</td> <td>$\square f$</td> </tr> </table> $\triangle c$ $\triangle d$	C	$-C$	$\square e$	$-C$	C	$\square f$
$G+sC$	$-G-sC$												
$-G-sC$	$G+sC$												
C	$-C$	$\square e$											
$-C$	C	$\square f$											
j: j-1:	 $A(s) = 1/(1/A_0 + s/\omega_T)$	<table> <tr> <td>-1</td> <td>1</td> <td>$\frac{1}{A_0} + \frac{s}{\omega_T}$</td> </tr> </table> $\triangle a$ $\triangle b$ $\triangle c$	-1	1	$\frac{1}{A_0} + \frac{s}{\omega_T}$	<table> <tr> <td>$\frac{1}{\omega_T}$</td> <td>$\square !$</td> </tr> </table> $\triangle f$	$\frac{1}{\omega_T}$	$\square !$					
-1	1	$\frac{1}{A_0} + \frac{s}{\omega_T}$											
$\frac{1}{\omega_T}$	$\square !$												

Table 1 Two examples of matrices Y_j and C_j creation

3. CIRCUIT EQUATIONS PRE-PROCESSING

Solving equations (1) in the time domain, we obtain values $v(kT + \sigma_{j-1}^-)$ at the end of each phase:

$$v_1(kT + \sigma_1^-) = \int_0^{T_1} g_1(\xi) W_1(kT + \sigma_1 - \xi) d\xi + g_1^* v_N(kT^-),$$

$$v_j(kT + \sigma_j^-) = \int_0^{T_j} g_j(\xi) W_j(kT + \sigma_j - \xi) d\xi + g_j^* v_{j-1}(kT + \sigma_{j-1}^-), j = 2, 3, \dots, N, \quad (2)$$

$$g_j(t) = L^{-1}\left\{\left(Y_j(s)\right)^{-1}\right\}, \quad g_j^* = g_j(T_j)C_j. \quad (3)$$

Equations (2) can be solved by means of an equivalent signal and GTF theory [1],[5],[6].

Let us suppose the following property of input signals $W_j(t)$, $j=1,2,\dots,N$.

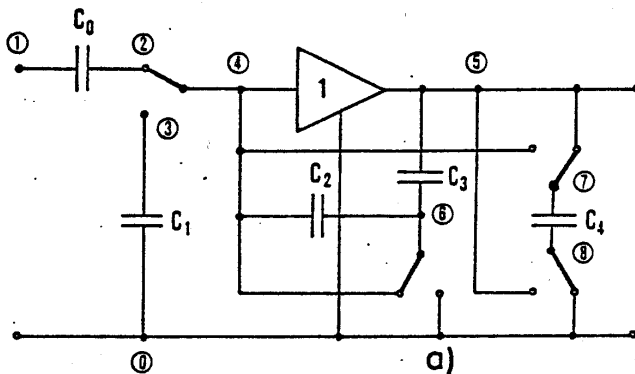
- i) The sampling frequency $f_s = 1/T$ is given. After ideal sampling of $W_j(t)$, the new spectral components appear. The overlapping effect of nonzero components has not arisen. Thus the input signal belongs to the class of "easily reconstructed" signals, or
- ii) The input signals $W_j(t)$, $j=1,2,\dots,N$ have a sampled-held character so that they are constant during the j -th phase: $W_{j+1}(t) = W_{j+1}(kT + \sigma_j^-)$, $kT + \sigma_j \leq t < kT + \sigma_{j+1}$. These signals arose by Sample-Hold process from original signals fulfilling condition i).

Both continuous-time and discrete-time signals occur in Eq (2). That fact complicates frequency analysis. To overcome this problem, each discrete-time signal $v_j(kT + \sigma_j^-)$ can be replaced by the equivalent continuous-time signal $v_j^*(t)$ with properties mentioned in [1]:

- 1) Both signals are identical in the instances $t = kT + \sigma_j^-$:

$$v_j(kT + \sigma_j^-) = v_j^*(t) \Big|_{t = kT + \sigma_j^-}, \quad k = \dots, -1, 0, 1, 2, \dots \quad (4)$$

- 2) All nonzero spectral components of equivalent signal fall into the spectral area of the input signal.



To illustrate how to replace the real signal by the equivalent signal, let us examine the time response of the two-phase second order SC filter in Fig.2. Due to switching, the output signal contains high-frequency spectral components. Computation of the complete output spectrum is expensive and time consuming. Two continuous-time equivalent

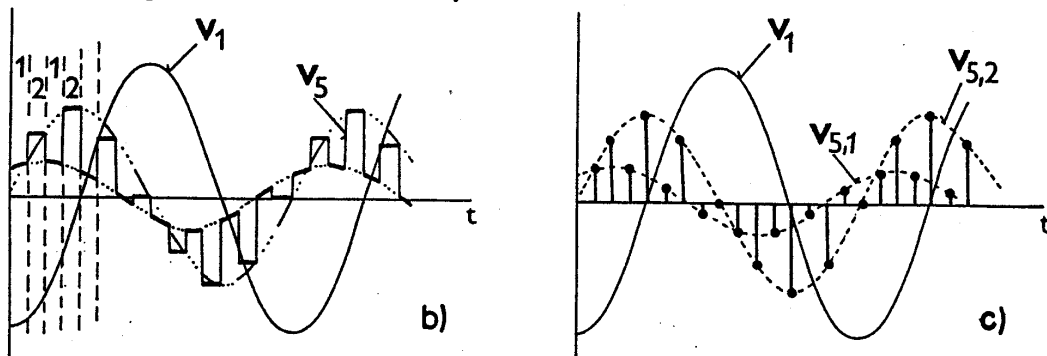


Figure 2. a) SC filter: $C_0 = C_1 = 15.6 \text{ nF}$, $C_2 = 200 \text{ pF}$, $C_3 = C_4 = 2 \text{ nF}$, $f_s = 100 \text{ kHz}$
b,c) real and equivalent time responses.

signals $v_{5,1}$ and $v_{5,2}$ create a certain envelopes of impulse signals. Comparing equivalent signals with input signal $v_1 = v_1^*$ for various frequencies, much information on transient properties in both switching phases can be obtained.

Replacing discrete time by continuous time $kT + \sigma_j = t$, $j=1,2,\dots,N$, the equations (2) can be re-written for continuous-time equivalent signals. The Laplace transform leads to the large set of equations:

1	0	0	...	0	$-g_1^* e^{-sT_1}$	V_1^*
$-g_2^* e^{-sT_2}$	1	0	...	0	0	V_2^*
0	$-g_3^* e^{-sT_3}$	1	...	0	0	V_3^*
.	.	.	...	.	.	.
0	0	0	...	$-g_N^* e^{-sT_N}$	1	V_N^*

$$= \begin{matrix} G_1(s)W_1(s) \\ G_2(s)W_2(s) \\ G_3(s)W_3(s) \\ . \\ . \\ G_N(s)W_N(s) \end{matrix} \quad (5)$$

where

$$G_j(s) = \int_0^{T_j} g_j(\xi) e^{-s\xi} d\xi \quad \text{or} \quad G_j(s) = e^{-sT_j} \int_0^{T_j} g_j(\xi) d\xi = e^{-sT_j} h_j(T_j) \quad (6) \quad (7)$$

for condition *i*) or *ii*), respectively. The symbol $h_j(T_j)$ denotes the step response at the end of the j -th phase.

The equations (5) are in well-known form but with a different physical interpretation than in equations published so far. Solving unknown vectors V_j^* , $j=1,2,\dots,N$, the set of generalized transfer functions will be obtained.

Before the numerical solution, some pre-processing of (5) can produce [4]. For simplicity, this pre-processing will be performed for two-phase networks. Rearranging Eq.(5) for $N=2$ yields

$$\begin{aligned} V_1^* &= K_{11}(s,z)W_1 + K_{21}(s,z)W_2 & V_2^* &= K_{12}(s,z)W_1 + K_{22}(s,z)W_2 \\ K_{11}(s,z) &= F_1(z)G_1(s), & K_{21}(s,z) &= F_1(z)g_1^*G_2(s)e^{-sT_1}, \\ K_{12}(s,z) &= F_2(z)g_2^*G_1(s)e^{-sT_2}, & K_{22}(s,z) &= F_2(z)G_2(s) \\ F_1(z) &= (E - z^{-1}g_1^*g_2^*)^{-1}, & F_2(z) &= (E - z^{-1}g_2^*g_1^*)^{-1}, & z &= e^{sT}. \end{aligned} \quad (8)$$

The transmission properties of two-phase switched network can be derived by $2^2=4$ generalized transfer functions. The network behaviour depends on the s - and z - pole location. It can be proved that

α) The system stability depends only on the z - pole location in the known sense ($|z_{\text{pole}}| < 1$).

As results from Eq.(8), the z -domain poles are eigenvalues of matrices $g_1^*g_2^*$ (or $g_2^*g_1^*$; both products have the same eigenvalues).

β) The s -domain poles determine the character of transient phenomena during each switching phase.

The s -domain poles are generated by $G_j(s)$. These poles are eigenvalues of the admittance matrices $Y_j(s)$. In case of SH inputs, the s -domain poles are not present (see Eq. 7).

4. NUMERICAL ASPECTS

Applying the precision algorithm of numerical Laplace inversion to Eq.(1) for $t=T_j$, matrices g_j^* , $j=1,2,\dots,N$ can be obtained [4]. To calculate matrices $F_j(z)$ in semi-symbolic form and z -pole location, the QZ algorithm or other standard procedures can be applied.

In case of SH inputs, the problem of $G_j(s)$ calculation is reduced to the step response evaluation using Laplace inversion. Otherwise, the matrix G_j depends on variable $s=j\omega$. There are two suitable ways how to compute $G_j(s)$, $s=j\omega$:

a) Using numerical Laplace inversion of Eq.(1), taking into account rules about Laplace transform of signal multiplied by $\exp(-st)$ and the integral of the signal:

$$\int_0^{T_j} g_j(\xi) e^{-s\xi} d\xi \approx -\sum_i K_i Y_j \left(\frac{z_i}{T_j} + s \right) / z_i, \quad (9)$$

where K_i, z_i are precalculated coefficients of Laplace inversion [4].

b) To compute $G_j(s)$ in semi-symbolic form with utilisation of the QZ algorithm, Eq.(8) and the interpolation procedure using FFT. However, this method is beyond the scope of this contribution and will be published separately.

5. SIMULATION RESULTS

The numerical algorithms have been optimized and adjusted during the development of computer program SPIN. By this program, the linear switched networks with both consistent and inconsistent initial conditions may be analysed. The s - and z - domain parts of generalized transfer functions are computed in semi-symbolic form. On the assumption of a sampled-held input signal, the algorithm is simplified essentially. However, this simplification can lead to simulation errors if the impulse response $g_j(t)$ contains derivatives of Dirac impulses.

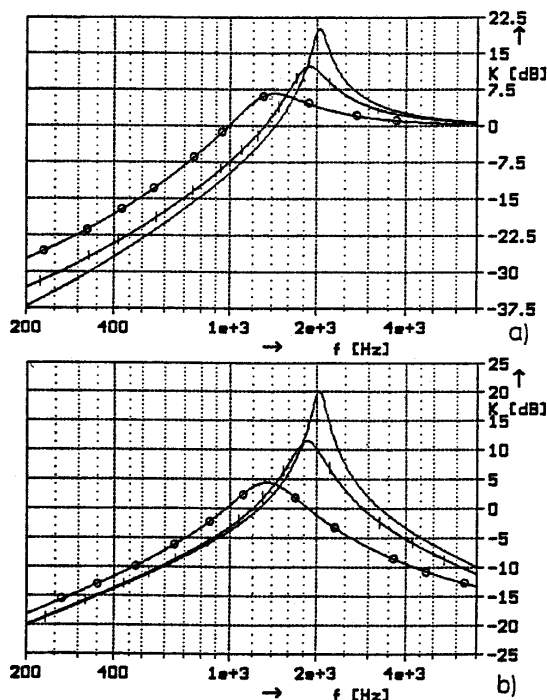


Figure 3. Frequency responses of filter in Fig. 2a. Output in phase 1 (a), 2(b). R_{ON} : - 0Ω ' 500Ω $\circ$ 1000Ω

It has been confirmed that results of s - z semi-symbolic analysis of continuous-time systems do not depend on the way to divide time axis to phase lengths T_j . In this case, only the s -domain transfer function is obtained. On the other hand, analysis of ideal switched capacitor circuits with inconsistent initial conditions provides well known z -domain system functions and periodically frequency responses.

As an example, some simulation results of the aforementioned second order SC filter are presented. In Fig. 3, the frequency responses for zero- and nonzero- switch resistance R_{ON} are shown. In Table 2, results of semi-symbolic analysis are summarized. It should be noted that the z -domain system order can depend on switch resistance.

z-domain poles	s-domain poles
$R_{ON}=0$	
0.98555637455373+j0.126569690366855	0
0.98555637455373-j0.126569690366855	0
$R_{ON}=200\Omega$	
1.54717712405126E-8	0
7.42385839547281E-8	0
0.985347368838751+j0.126295650868273	-1.25E6
0.985347368838751-j0.126295650868273	-2.27272727272727E6
$R_{ON}=500\Omega$	
9.69651500209532E-4+j3.76632889349822E-4	0
9.69651500209532E-4-j3.76632889349822E-4	0
0.979073972323854+j0.113655688962891	-5E5
0.979073972323854-j0.113655688962891	-9.09090909090909E5
$R_{ON}=1000\Omega$	
3.14676331976010E-2+j8.86688444706181E-3	0
3.14676331976010E-2-j8.86688444706181E-3	0
0.976164317525296+j0.0798327064310718	-2.5E5
0.976164317525296-j0.0798327064310718	-4.54545454545454E5

Table 2 The s- and z- domain dominant poles location. Analysis results, filter from Fig.2.

6. CONCLUSIONS

In this paper, we have proposed a novel switched networks simulation technique based on the generalized transfer function theory. A key advantage is that this approach is suitable both for discrete-time, continuous-time and mixed networks. To verify the aforementioned theory, computer program SPIN for linear switched networks has been constructed. Due to replacing real impulse signals by equivalent "smooth" signals, the frequency analysis is very fast.

REFERENCES

1. D.Biolek, ISSSE'92, URSI, Paris, Book 1, pp.458-461.
2. Y.Tsividis, Proceedings of the IEEE, 71, 1983, No.1, pp.181-183.
3. J.Vlach and K.Singhal, Computer Methods for Circuit Analysis and Design, Van Nostrand Reinhold, N.Y., 1983.
4. A.Opal and J.Vlach, IEEE Trans. on Circuits and systems, 37, 1990, No.3, pp.364-372.
5. C.K.Sewell and J.I.Pun, Proceedings of ISCAS'85, pp.1165-1168.
6. D.Biolek and Z.Biolek, Electronic Horizon, 48, 1987, No.9, pp.410-417 (Czechoslovakia).