INITIAL CONDITIONS IN LINEAR SWITCHED NETWORKS

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ABSTRACT

The simple algorithm of initial condition computation immediately before and after switching is proposed in this paper. The algorithm avoids both limiting step $\Delta t \to 0$ [1] *and the time expensive numerical Laplace inversion* [2].

INTRODUCTION

Modeling of switched networks (switched-capacitor or switched-current networks, convertors *DC-DC*, switched modulators etc.) often leads to the discontinuity of networks variables at the time instances of switches state change. This phenomenon is called as *inconsistent initial conditions* [2]. The computer algorithm how to search some transfer matrices for recounting initial conditions after switching from ones before switching is described in [2]. This algorithm is based on so-called *two-step Laplace inversion* [2]. However, the computation can be time expensive due to searching of optimal step to minimize the error caused by incidental Dirac impulse contained in the calculated response. The necessity to use complex arithmetic can be further disadvantage.

The simple approach will be described in this contribution which consists in the solution of the set of linear algebraic equations. These equations are based on the two-graph modified nodal approach and they enable to compute initial conditions in arbitrary linear switched networks. Due to minimum number of independent equations, the algorithm is fast and stable.

The time instances immediately after/before switching are signed as $0^+/0^-$.

PROBLEM DEFINITION

Let us consider linear network comprising arbitrary resistive and inertia elements, controlled sources as amplifiers, and controlled analog switches with jump-varying switch resistances. In the time instant t = 0, some of switches change their state. Let regime immediately before/after switching be named as phase 1/phase2. In the phase 1/phase2, the network can be modeled as a linear stationary network with n_1/n_2 independent circuit variables, represented using vectors $\mathbf{v}^{(1)}/\mathbf{v}^{(2)}$. It is evident that circuit variables immediately after switching are linear dependent on the variables immediately before switching and on the input signal immediately after switching:

$$\mathbf{v}^{(2)}\left(0^+\right)=\mathbf{M}^{(12)}\mathbf{v}^{(1)}\left(0^-\right)+\mathbf{P}^{(2)}\mathbf{w}^{(2)}\left(0^+\right) \qquad (1)$$

Our task is to compute matrices $\mathbf{M}^{(12)}$ and $\mathbf{P}^{(2)}$ for transformation of one set of initial conditions to another one. The algorithm must work properly in case of both consistent and inconsistent initial conditions.

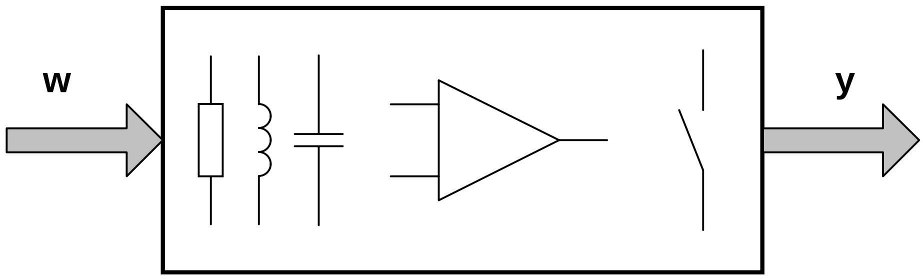
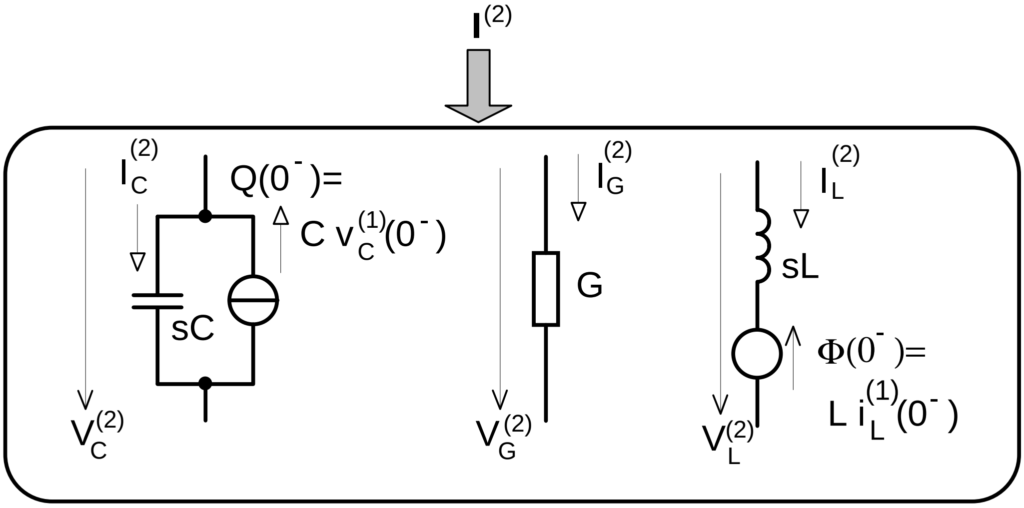


Fig.1. Block diagram of considered system.

COMPILATION OF CIRCUIT EQUATIONS

In each switching phase, the network is described using the two-graph modified nodal approach [1]. The operator scheme of some elements in the phase 2 is in Fig.2. Inertia elements enter their energy to the phase 2 stored from phase 1. This fact is modeled using initial charge $Q(0^-)$ and magnetic flux $\Phi(0^-)$. $\mathbf{I}^{(2)}$ is vector of currents flowing into the circuit in the phase 2. Applying Kirchhoff's laws yields



$$\mathbf{I}^{(2)} = \mathbf{I}_C^{(2)} + \mathbf{I}_G^{(2)} + \mathbf{I}_L^{(2)} - \mathbf{Q}\left(0^-\right) = s\mathbf{C}^{(2)}\mathbf{V}^{(2)} + \mathbf{G}^{(2)}\mathbf{V}^{(2)} + \mathbf{A}^{(2)}\mathbf{I}_L^{(2)} - \mathbf{C}^{(12)}\mathbf{v}^{(1)}\left(0^-\right),$$

$$0 = \text{linear combination of } \left\{\mathbf{V}^{(2)}, \mathbf{I}_L^{(2)}, \Phi\left(0^-\right)\right\} = \mathbf{B}^{(2)}\mathbf{V}^{(2)} + s\mathbf{L}^{(2)}\mathbf{I}_L^{(2)} - \mathbf{L}^{(12)}\mathbf{i}_L^{(1)}\left(0^-\right)$$

or in the matrix form

$$\begin{array}{|c|}\hline \mathbf{I}^{(2)} \\ \hline \\ \hline\end{array} = \begin{array}{|c|c|}\hline s\mathbf{C}^{(2)} + \mathbf{G}^{(2)} & \mathbf{A}^{(2)} \\ \hline \mathbf{B}^{(2)} & s\mathbf{L}^{(2)} \\ \hline\end{array} \begin{array}{|c|}\hline \mathbf{V}^{(2)} \\ \hline \mathbf{I}_L^{(2)} \\ \hline\end{array} - \begin{array}{|c|c|}\hline \mathbf{C}^{(12)} & \\ \hline & \mathbf{L}^{(12)} \\ \hline\end{array} \begin{array}{|c|}\hline \mathbf{v}^{(1)}\left(0^-\right) \\ \hline \mathbf{i}_L^{(1)}\left(0^-\right) \\ \hline\end{array}$$

$$\mathbf{W}^{(2)} = \mathbf{Y}_G^{(22)}\,\mathbf{V}_G^{(2)} - \mathbf{C}_G^{(12)}\,\mathbf{V}_G^{(1)}\left(0^-\right) \qquad (2)$$

$$= s\mathbf{C}_G^{(2)} + \mathbf{G}_G^{(2)}$$

where $\mathbf{W}^{(2)}, \mathbf{V}_G^{(2)}, \mathbf{V}_G^{(1)}\left(0^-\right), \mathbf{Y}_G^{(22)}, \mathbf{C}_G^{(12)}$ are vector of input signals in the phase 2, vector of generalized network coordinates in the phase 2, vector of generalized „final"conditions from the phase 1, generalized addmitance matrix in the phase 2 and generalized transitive matrix between phases 1 and 2. The last stated form is generally valid independently on the type of linear network elements. In [2], we have described the algorithm how to compile generalized matrices from equation (2).

ALGORITHM

$$\begin{tikzcd} \mathbf{C}_G^{(12)} \arrow[d] & \mathbf{C}_G^{(22)} \arrow[d] \\ \mathbf{C}_M^{(12)} & \mathbf{M}^{(22)} \end{tikzcd}$$

1) Compilation of matrices $\mathbf{G}_G^{(22)}, \mathbf{C}_G^{(22)}$ and $\mathbf{C}_G^{(12)}$

2) Searching of empty rows of $\mathbf{C}_G^{(22)}$ (except of „input“ node)

    If (empty row) then

        row of $\mathbf{G}_G^{(22)} \rightarrow$ empty row

3) If (excitation in phase 2) then

    begin

        modification of $\mathbf{C}_G^{(22)}$ and $\mathbf{C}_G^{(12)}$;

        $\mathbf{D}^{(2)} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$

        $\uparrow$"input" row

    end

        else $\mathbf{D}^{(2)} = \mathbf{0}$

4) Calculation of $\mathbf{v}^{(2)}\left(0^+\right)$

$$\mathbf{M}^{(22)}\mathbf{v}^{(2)}\left(0^+\right) = \mathbf{C}_M^{(12)}\mathbf{v}^1\left(0^-\right) + \mathbf{D}^{(2)}\mathbf{w}^{(2)}\left(0^+\right) \Rightarrow \mathbf{v}^{(2)}\left(0^+\right)$$

After generalization of aforementioned conclusions we can state the algorithm for the compilation of transform matrices in case of **arbitrary switched networks:**

1. We compile matrices $\mathbf{G}^{22}$, $\mathbf{C}^{22}$ and $\mathbf{C}^{12}$ according to Tab.1 or [5].

2. We scan all rows of matrix $\mathbf{C}^{22}$ exeption to the row corresponding to the input node. In case of empty rows they will be replaced by corresponding rows of matrix $\mathbf{G}^{22}$.
This step is unnecessary in case of idealized SC networks.

3. If the input voltage/current source is active in phase 2, i.e. if the voltage/current coefficient a*/a** of input node is nonzero, we write "1" to the corresponding row a** of vector $\mathbf{D}^2$. In case of voltage source, we fill "0" the row a** of matrices $\mathbf{C}^{22}$ and $\mathbf{C}^{12}$. Then we write "1" to the position: row a**, column a* of $\mathbf{C}^{22}$.
After these operations and prospective modification in accordance with the item **2**, the creation of matrices $\mathbf{C}_M^{12}$ and $\mathbf{M}^{22}$ is finished.

4. We compute transform matrix $\mathbf{M}^{12}$ and vector $\mathbf{P}^2$ using (6).

CONCLUSION

The presented algorithm of inconsistent initial condition computation in the switched networks is based on the *two-graph modified nodal approach*. Using this method, the network is described by the set of independent circuit variables in each switching phase. As a result, the regular minimum set of equations is obtained. Novel algorithm utilizes this regularity and compiles transform matrices between the vectors of nodal variables immediately before and after switching. The algorithm is implemented to the general program for the fast analysis of real switched networks [4], [5].

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