

Modeling of switched DC-DC converters by mixed s-z description

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Abstract

The paper explains basic ideas how to model dynamical properties of switched DC-DC regulators by means of so-called generalized transfer functions (GTFs). These functions were originally developed for modeling the real switched capacitor filters. However, they can be also used for modeling general linear systems with periodically varying parameters. Switched DC-DC converters like boost or buck types exhibit some properties of continuous- and discrete-time characters. These systems can be globally modeled by mixed s-z description, utilizing both the Laplace and z-transform operators. Then the resulting line-to-output and control-to-output transfer functions and the corresponding frequency responses can model some system behavior more truly than the classical s-domain zeros/poles location, including special effects above the Nyquist's frequency.

1 Introduction

Frequency-domain modeling of dynamic properties of switched DC-DC converters is based on so-called averaging approach [1]. Several methods are currently used, depending on which of the converter subcircuits will be included in the averaging process: if only the PWM switch (see Vorpérian model of PWM switch) [2], or the PWM switch together with the inductor (the so-called Switched Inductor Model SIM) [3], etc. Different approaches must be applied to the modeling of propagation the input disturbances to the output (the so-called line-to-output transfer function), and to the modeling of duty ratio perturbation to the output (the so-called control-to-output transfer function). The modeling also depends on the fact if converter operates in continuous current mode (CCM) or discontinuous current mode (DCM), and if the classical voltage mode control or the current mode control is applied. Especially the modeling of the last mentioned regulators is rather complicated. For more complex description of the dynamic features and for monitoring the stability, special approximate methods were developed [4] which model the influence of switching processes to the loop gain, especially in the frequency region in the vicinity of Nyquist's frequency $f_{switch}/2$.

Switched DC-DC converter can be considered as an analog linear time-varying system, where the externally controlled switches are modeled by time-varying resistances. For modeling of such systems, the so-called generalized transfer functions (GTFs) [5] can be applied. GTFs are functions of two well-known s and z operators, simultaneously describing both the continuous-time and discrete-time behavior of switched converter. Frequency responses are obtained after double substitution $s=j\omega$ and $z=\exp(j\omega T)$, where T is the switching period. The GTF-based modeling offers more precise description of converter dynamics than presently used approximate procedures, based only on s - or z -domain transfer functions.

This study contains demonstration of this new modeling approach for line-to-output transfer functions of switched converters operating in CCM.

2 GTF of switched DC-DC converters

Some basic types of well-known switched DC-DC converters are summarized in Figs. 1(a-c). All of them are two-phase switched circuits with a switching diagram in Fig. (d), when the active switch S causes by its ON/OFF operation inverse states of the passive switch D , i.e. OFF/ON. The relative duration of ON state of active switch with respect to the switching period is called duty ratio (D). This quantity is used to be controlled depending on the output voltage, which is then stabilized via pulse width modulation. After neglecting the nonlinear operation of switching devices, the converter can be described in each its switching phase by linear model with dominant couple of state variables – voltage across the capacitor and the inductor current.

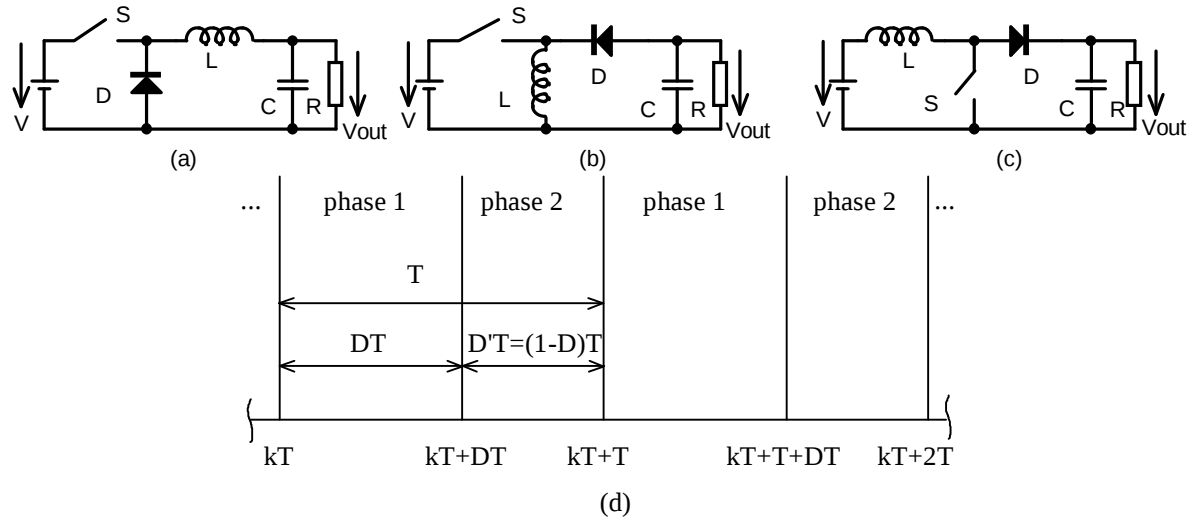


Fig. 1: Fundamental circuit diagrams of DC-DC converters, types: (a) buck, (b) boost, (c) buck-boost; the switching diagram (d).

Let us consider the following assumptions: Linear model in each switching phase, continuous conducting mode, constant duty ratio. Input voltage V will have a DC component V_0 and superposed AC component $\hat{v}(t)$. The aim of the analysis is to get frequency dependence of AC component of the output voltage and the other observed signals without limitation of the bandwidth of signal $\hat{v}(t)$.

Due to the switching processes, the circuit variables of switched converters are not smooth time domain functions. According to the theory of generalized transfer functions [5], these signals can be represented by smooth, so-called equivalent signals which fulfill two following conditions: (1) the original and equivalent signals have identical values at sampling instances, which we are interested in (mostly on the boundaries of switching phases), (2) bandwidth of the equivalent signals coincide with the bandwidth of input signal. The first condition stands that the equivalent signals form abstract envelopes of real signals. In case of negligible influence of switching processes, it is a good compliance between the real signals and their envelopes. The second condition enables to compare equivalent signals and input signal in the frequency domain by means of linear theory. In other words, it enables to define transfer functions.

Let us observe the converter state by vector $\mathbf{X}$, which consists – in the simplest case – of a couple of state variables V_C and I_L . All remaining circuit currents or voltages can be reached by linear combination of these state variables and input voltage. In the following, we will assume a continuity of state variables both in the frame of switching phases and at transition instances between them.

Let us introduce the following notation:

$\mathbf{G}_1, \mathbf{G}_2 \dots$ matrices of converter natural responses within phase 1 and phase 2, respectively, defining on the assumption of zero-input signal v :

$$\mathbf{X}(kT + DT) = \mathbf{G}_1 \mathbf{X}(kT), \quad \mathbf{X}(kT + T) = \mathbf{G}_2 \mathbf{X}(kT + DT) \quad (1)$$

$\mathbf{g}_1, \mathbf{g}_2 \dots$ matrices of converter impulse responses within phase 1 and phase 2, respectively, defining on the assumption of input signal v as Dirac impulse, operating at the beginning of switching phase.

$\mathbf{H}_1, \mathbf{H}_2 \dots$ vectors of converter forced responses to constant input signal V_0 within phase 1 and phase 2, respectively, considering zero initial state:

$$\mathbf{X}(kT + DT) = \int_0^{DT} \mathbf{g}_1(x) dx V_0 = \mathbf{H}_1 V_0, \quad \mathbf{X}(kT + T) = \int_0^{D'T} \mathbf{g}_2(x) dx V_0 = \mathbf{H}_2 V_0 \quad (2)$$

Accounting the above notations and assumptions, the switched converter can be described by the following equations:

End of the switching phase 1 – $t = kT + DT$:

$$\mathbf{X}(kT + DT) = \mathbf{G}_1 \mathbf{X}(kT) + \int_0^{DT} \mathbf{g}_1(DT - x) v(kT + x) dx. \quad (3)$$

End of the switching phase 2 – $t = kT + T$:

$$\mathbf{X}(kT + T) = \mathbf{G}_2 \mathbf{X}(kT + DT) + \int_0^{D'T} \mathbf{g}_2(D'T - x) v(kT + DT + x) dx. \quad (4)$$

Now consider the input voltage as a sum of DC and AC components:

$$v = V_0 + \hat{V}e^{st}, \quad s = j\omega. \quad (5)$$

As a consequence of circuit linearity, the steady-state vector $\mathbf{X}$ will be also compound of DC and AC terms:

$$\mathbf{X} = \mathbf{X}_0 + \hat{\mathbf{X}}e^{st}. \quad (6)$$

Taking into account equations (5) and (6), the following results can be derived from (3) and (4):

$$\mathbf{X}_0 = (\mathbf{I} - \mathbf{G}_1 \mathbf{G}_2)^{-1} [\mathbf{G}_1 \mathbf{H}_2 + \mathbf{H}_1] V_0 = (\mathbf{I} - \mathbf{G}_2 \mathbf{G}_1)^{-1} [\mathbf{G}_2 \mathbf{H}_1 + \mathbf{H}_2] V_0 \quad (7)$$

$$\hat{\mathbf{X}}_1 = (\mathbf{I} - \mathbf{G}_1 \mathbf{G}_2 z^{-1})^{-1} [\mathbf{G}_1 \hat{\mathbf{H}}_2(s) z^{-D} + \hat{\mathbf{H}}_1(s)] \hat{V} \quad (8)$$

$$\hat{\mathbf{X}}_2 = (\mathbf{I} - \mathbf{G}_2 \mathbf{G}_1 z^{-1})^{-1} [\mathbf{G}_2 \hat{\mathbf{H}}_1(s) z^{-D'} + \hat{\mathbf{H}}_2(s)] \hat{V} \quad (9)$$

Equations (7) describe two equivalent ways of computing the state vector in DC steady state, whereas equations (8) and (9) represent state vectors of equivalent state signals of the converter, which correspond to the state variables after their sampling at the end of phases 1 ($\hat{\mathbf{X}}_1$) and phases 2 ($\hat{\mathbf{X}}_2$), respectively. The following quantities figure in the equations:

$z = e^{sT}$.. operator of the z-transform,

$$\hat{\mathbf{H}}_1(s) = \int_0^{DT} \mathbf{g}_1(t) e^{-st} dt, \quad \hat{\mathbf{H}}_2(s) = \int_0^{D'T} \mathbf{g}_2(t) e^{-st} dt \quad (10)$$

...running Laplace transforms of converter impulse responses within phases 1 and 2, respectively. These functions generate s-domain poles of converter frequency responses.

Note that equations (7) for DC steady state directly result from equations (8), (9), (10) and (2) for $s = 0$.

Equations (8) a (9) describe generalized transfer functions of switched converter. Applying the double substitution $s = j\omega$, $z = e^{j\omega T}$ yields line-to-output frequency responses for arbitrary bandwidth, i.e. without limitation to one half of switching frequency, as usual for so far published procedures.

3 Possibilities of computer simulation

The above modeling can be utilized for converter computer simulation, either by programming the equations into MATLAB-type software packages, or by direct implementation into SPICE-family simulation programs. In the first case, and also partly in the second one, it is advantageous to do analytical equation preprocessing for concrete converter topology.

Matrices $\mathbf{G}_1$, $\mathbf{G}_2$ and vectors $\mathbf{H}_1$ and $\mathbf{H}_2$ can be determined by numerical analyses of natural and forced responses of the converter separately during each quasistationary switching phase. For example, we can run multiple simulation from single PSPICE circuit file for this purpose. Numerical computation of vectors (10) is more complicated. The details are described in [5]. Considering two dominant state variables in the converter, it is possible to have a go at analytical preprocessing of results and their subsequent programming. It must be also considered than in some converter topologies, when the state variables are not directly affected by input voltage within some switching phase, the corresponding vectors $\mathbf{H}$ and $\hat{\mathbf{H}}$ are simply zero. For instance, vectors $\mathbf{H}_2$ a $\hat{\mathbf{H}}_2$ will be zero for the buck and boost converters.

4 Demonstration of computer simulation of buck converter

An example of SPICE simulation of line-to-output frequency response of buck converter is given in [1]. The circuit parameters are as follows (see also Fig. 1 (a)):

$$L=50\mu\text{H}, C=500\mu\text{F}, R=3\Omega.$$

The input voltage 28V is converted to the output voltage 12.8V, and the corresponding duty ratio is $D = 0.543$. The switching frequency is 100kHz. Converter operates in CCM. The modeling in [1] does not include ESRs of inductor and capacitor. Switches were considered as ideal.

In Fig. 2 are three pairs of magnitude and phase frequency responses of this converter as results of MATLAB simulations for the following models: model of buck converter containing Vorpérian average model of PWM switch, model based on GTF when integrals (10) we approximated by DC vectors of forced responses $\mathbf{H}$ according to (2), and full model based on GTF. In addition, the following real properties has been included into the models: $R_{on} = 0.01\Omega$ of the active switch, series resistance 0.05Ω of the inductor, 0.1Ω capacitor ESR. The frequency response • with Vorpérian model corresponds to the results of SPICE simulation in [1]. It does not reflect switching processes and its validity is limited to the frequency region below one half of switching frequency.

The second response , is repeated periodically with the sampling frequency. It describes phenomena, which are associated with signal sampling. However, this modeling does not reflect inertial processes during the active switch in ON states.

The third frequency response $\mathbf{f}$ is a full representative of converter generalized transfer functions. It models in complex the crosstalk of wideband input signal to the output, without the $f_{switch}/2$ limitation. For example, it can be deducted from this response how the individual spectral terms of switching signal will leak into the converter output through poorly shielded input terminals.

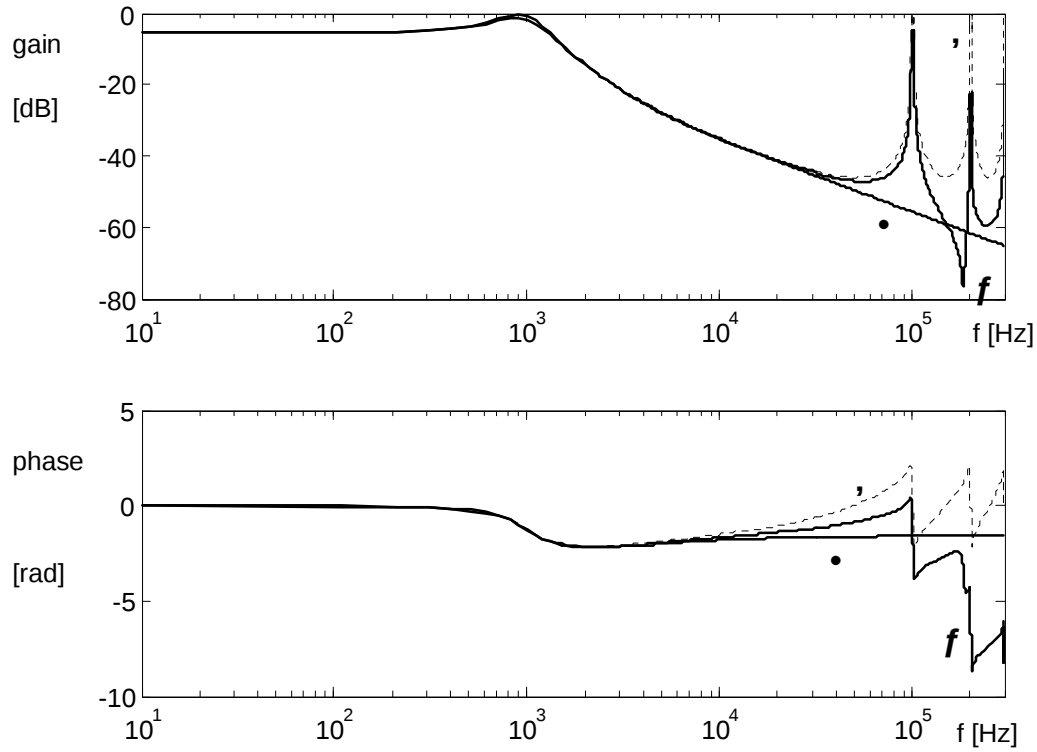


Fig. 2: Frequency responses of buck converter: (1) with Vorpérian model of PWM switch, (2) with GTF-based z-domain model, (3) with full GTF-based s-z-domain model. ESR=0.1 Ω .

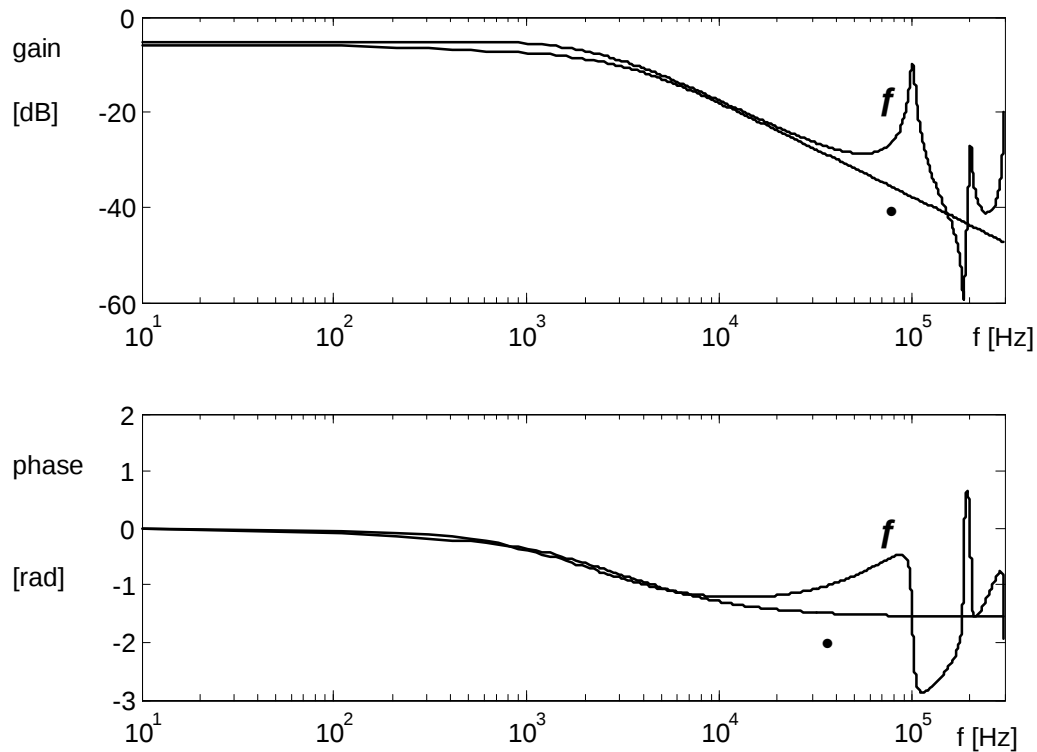


Fig. 3: Frequency responses for ESR = 1 Ω .

Fig. 3 shows the frequency responses of types $\bullet$ and f of the converter for capacitor ESR of 1 Ω . The differences between the classical and generalized modeling are now apparent also in

the frequency range below $f_{switch}/2$. Interesting phenomena can be observed when changing the duty ratio. For example, when D is given by the fraction $m:n$, m and n integers, then the input-to-output gain is suppressed on integer multiples of the frequency $n.f_{switch}$. Such results cannot be acquire from the average models.

5 Conclusions

A novel method of AC analysis of switched DC-DC converters is described. This method utilizes so-called generalized transfer functions. In comparison with the classical methods based on average modeling, the advantages consist in more credible modeling of converter behavior, especially in the frequency range around $f_{switch}/2$, as well as in the ability to model correctly the transfers above this border frequency. A drawback consists in more complicated mathematical models and thus in their more difficult implementation in current simulation programs.

Modeling of the line-to-output transfer functions is described in the paper. The above approach can be also extended to a similar modeling of control-to-output frequency responses.

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