

# INITIAL CONDITIONS IN LINEAR SWITCHED NETWORKS

Dalibor Biolek\* and Jaafar Al Kheir\*\*

\* Department of Electrical Engineering, K301, VA Brno, PS13,  
612 00 Brno, Czech Republic

\*\* Department of Mechanical and Electrical Engineering, Teshreen  
University, P.Box 1531, Lattakia, Syria

## ABSTRACT

*The simple algorithm of initial condition computation immediately before and after switching is proposed in this paper. The algorithm avoids both limiting step  $\Delta t \rightarrow 0$  [1] and the time expensive numerical Laplace inversion [2].*

## INTRODUCTION

Modeling of switched networks (switched-capacitor or switched-current networks, convertors DC-DC, switched modulators etc.) often leads to the discontinuity of networks variables at the time instances of switches state change. This phenomenon is called as *inconsistent initial conditions* [2]. The computer algorithm how to search some transfer matrices for recounting initial conditions after switching from ones before switching is described in [2]. This algorithm is based on so-called *two-step Laplace inversion* [2]. However, the computation can be time expensive due to searching of optimal step to minimize the error caused by incidental Dirac impulse contained in the calculated response. The necessity to use complex arithmetic can be further disadvantage.

The simple approach will be described in this contribution which consists in the solution of the set of linear algebraic equations. The algorithm is explained on two examples: **1.** Ideal switched capacitor (SC) network, **2.** Arbitrary switched network consisting of both reactances and resistances. After generalization, the common algorithms for calculation of inconsistent initial conditions will be stated out.

The time instances immediately after/before switching are signed as  $0^+/0^-$ .

### Algorithm for idealized SC networks

Let us consider the simple SC network in Fig.1a. In the phase 1 (or 2), the network is described by the set of independent nodal voltages  $\mathbf{v}^1: [v_2^1 = v_3^1, v_4^1]$  or  $\mathbf{v}^2: [v_1^2 = v_2^2, v_3^2 = v_4^2]$ , respectively. Knowing vector  $\mathbf{v}^1(0^-)$  immediately before switching, it is necessary to determine vector  $\mathbf{v}^2(0^+)$  immediately after switching.

The corresponding operator diagram in phase 2 is in Fig.1b, considering initial charges  $Q_1$ ,  $Q_2$ ,  $Q_3$  and  $Q_4$ . Fig.1b leads to the *two-graph modified nodal approach* (2-graph MNA) equations [2]

$$\begin{array}{c} \boxed{1} \\ \boxed{0} \end{array} \mathbf{W}^2 = \begin{array}{c|c} 1 & 0 \\ \hline -sC_4 & s(C_2 + C_3 + C_4) \end{array} \begin{array}{c} V_1^2 \\ V_3^2 \end{array} - \begin{array}{c} 0 \\ Q_2 + Q_3 - Q_4 \end{array}. \quad (1)$$

$$\mathbf{D}^2 \quad \mathbf{Y}_M^{22} = \mathbf{G}_M^{22} + s\mathbf{C}_M^{22} \quad \mathbf{V}^2 \quad \mathbf{Q}_M^2$$

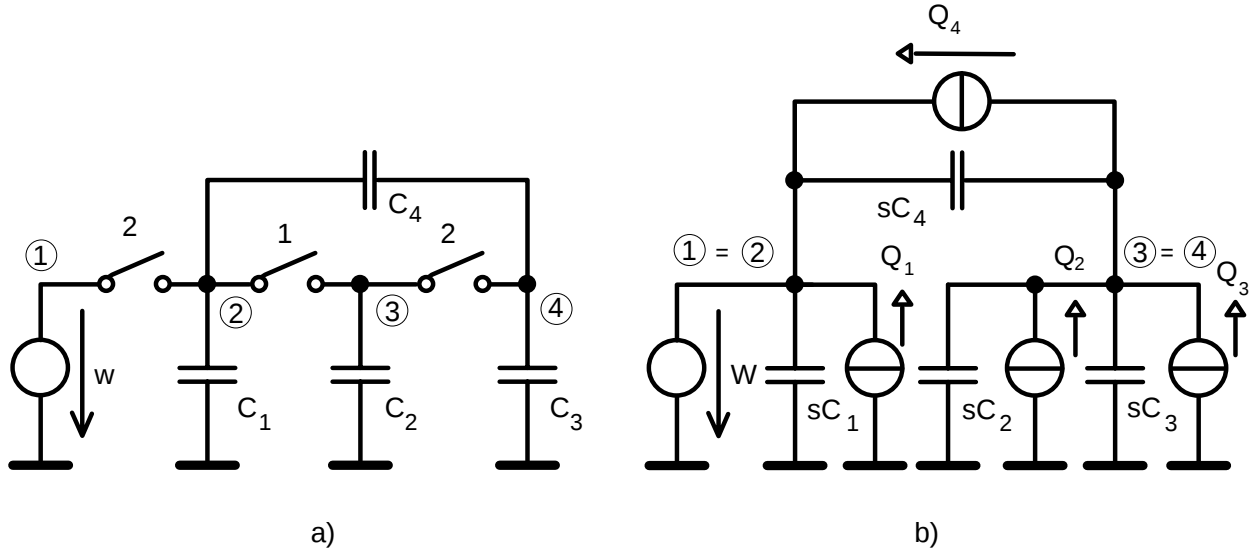


Fig.1. a) example of idealized two-phase SC network, b) operator diagram for the phase 2.

The original well-known admittance matrix  $\mathbf{Y}^{22}$  is modified to the matrix  $\mathbf{Y}_M^{22}$  due to influence of voltage source in the node 1.

For our network, the *charge conservation law* can be described as follows:

$$\begin{aligned} Q_1 &= C_1 v_1^2(0^+) = C_1 v_2^1(0^-) \\ Q_2 &= C_2 v_3^2(0^+) = C_2 v_2^1(0^-) \\ Q_3 &= C_3 v_3^2(0^+) = C_3 v_4^1(0^-) \\ Q_4 &= C_4 [v_1^2(0^+) - v_3^2(0^+)] = C_4 [v_2^1(0^-) - v_4^1(0^-)] \end{aligned} \quad (2)$$

There are two ways how to determine vector  $\mathbf{Q}_M^2$  in (1):

$$\begin{bmatrix} 0 \\ Q_2 + Q_3 - Q_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -C_4 & C_2 + C_3 + C_4 \end{bmatrix} \begin{bmatrix} v_1^2(0^+) \\ v_3^2(0^+) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ C_2 - C_4 & C_3 + C_4 \end{bmatrix} \begin{bmatrix} v_2^1(0^-) \\ v_4^1(0^-) \end{bmatrix} \quad (3)$$

$\mathbf{Q}_M^2 \quad \mathbf{C}_M^{22} \quad \mathbf{v}^2(0^+) \quad \mathbf{C}_M^{12} \quad \mathbf{v}^1(0^-)$

Matrices  $\mathbf{C}_M^{22}$  and  $\mathbf{C}_M^{12}$  arose by simple modification of original matrices  $\mathbf{C}^{22}$  and  $\mathbf{C}^{12}$  after resetting the row corresponding the input voltage source. Algorithm how to compile these matrices is described in [4]. In the case of RC networks, the principle is indicated in Tab.1.

| phase                       | element                     | $\mathbf{Y}^{22} = \mathbf{G}^{22} + s\mathbf{C}^{22}$                                                                                                                                                                                                                                     | $\mathbf{C}^{12}$          |                             |                             |                            |                                                                                                                                                                                                                                                                                                                  |              |               |               |              |
|-----------------------------|-----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|-----------------------------|-----------------------------|----------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------|---------------|--------------|
| 2:<br>1:                    |                             | <table><tr><td><math>\mathbf{G} + s\mathbf{C}</math></td><td><math>-\mathbf{G} - s\mathbf{C}</math></td></tr><tr><td><math>-\mathbf{G} - s\mathbf{C}</math></td><td><math>\mathbf{G} + s\mathbf{C}</math></td></tr></table> <p style="text-align: center;"><math>a^* \qquad b^*</math></p> | $\mathbf{G} + s\mathbf{C}$ | $-\mathbf{G} - s\mathbf{C}$ | $-\mathbf{G} - s\mathbf{C}$ | $\mathbf{G} + s\mathbf{C}$ | <table><tr><td><math>\mathbf{C}</math></td><td><math>-\mathbf{C}</math></td></tr><tr><td><math>-\mathbf{C}</math></td><td><math>\mathbf{C}</math></td></tr></table> <p style="text-align: center;"><math>c^* \qquad d^*</math></p> <p style="text-align: right;"><math>e^{**}</math><br/><math>f^{**}</math></p> | $\mathbf{C}$ | $-\mathbf{C}$ | $-\mathbf{C}$ | $\mathbf{C}$ |
| $\mathbf{G} + s\mathbf{C}$  | $-\mathbf{G} - s\mathbf{C}$ |                                                                                                                                                                                                                                                                                            |                            |                             |                             |                            |                                                                                                                                                                                                                                                                                                                  |              |               |               |              |
| $-\mathbf{G} - s\mathbf{C}$ | $\mathbf{G} + s\mathbf{C}$  |                                                                                                                                                                                                                                                                                            |                            |                             |                             |                            |                                                                                                                                                                                                                                                                                                                  |              |               |               |              |
| $\mathbf{C}$                | $-\mathbf{C}$               |                                                                                                                                                                                                                                                                                            |                            |                             |                             |                            |                                                                                                                                                                                                                                                                                                                  |              |               |               |              |
| $-\mathbf{C}$               | $\mathbf{C}$                |                                                                                                                                                                                                                                                                                            |                            |                             |                             |                            |                                                                                                                                                                                                                                                                                                                  |              |               |               |              |

Tab.1. The principle of circuit matrices compilation in case of RC circuit elements. The symbols \* and \*\* denote the *voltage* and *current nodal coefficients* in conformity with the 2-graph MNA [3].

Due to aforementioned modification, the matrix equation (3) consists of two scalar equations. The first one is trivial:  $0 = 0$ . This equation can be replaced by the equality  $v_1^2(0^+) = w^2(0^+)$ :

$$\begin{array}{|c|c|c|} \hline 1 & 0 & v_1^2(0^+) \\ \hline -C_4 & C_2 + C_3 + C_4 & v_3^2(0^+) \\ \hline \end{array} \underset{\mathbf{M}^{22}}{=} \begin{array}{|c|c|} \hline 0 & 0 \\ \hline C_2 - C_4 & C_3 + C_4 \\ \hline \end{array} \underset{\mathbf{C}_M^{12}}{=} \begin{array}{|c|c|} \hline v_2^1(0^-) & v_4^1(0^-) \\ \hline \end{array} \underset{\mathbf{v}^1(0^-)}{=} \begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array} \underset{\mathbf{D}^2}{=} w^2(0^+). \quad (4)$$

The matrix  $\mathbf{M}^{22}$  is regular as a consequence of  $\mathbf{v}^2$  is the vector of independent nodal voltages. Solving (4) yields required transform equation

$$\mathbf{v}^2(0^+) = \mathbf{M}^{12} \mathbf{v}^1(0^-) + \mathbf{P}^2 w^2(0^+). \quad (5)$$

This equation stated that the network state at beginning phase 2 is determined by the state at the end of phase 1 and at the input signal value at beginning phase 2. The transform matrix  $\mathbf{M}^{12}$  and vector  $\mathbf{P}^2$  can be formally calculated as follows:

$$\mathbf{M}^{12} = (\mathbf{M}^{22})^{-1} \mathbf{C}_M^{12}, \mathbf{P}^2 = (\mathbf{M}^{22})^{-1} \mathbf{D}^2. \quad (6)$$

For our network is

$$\mathbf{M}^{12} = \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \frac{C_2 - C_4}{C_2 + C_3 + C_4} & \frac{C_3 + C_4}{C_2 + C_3 + C_4} \\ \hline \end{array}, \quad \mathbf{P}^2 = \begin{array}{|c|} \hline 1 \\ \hline 0 \\ \hline \end{array}.$$

After generalization of results mentioned above, we can state the algorithm of transform matrices creation in case of **ideal SC networks**:

1. We compile matrices  $\mathbf{C}^{22}$  and  $\mathbf{C}^{12}$  according to Tab.1 or [5].
2. If the input voltage/current source is active in phase 2, i.e. if the voltage/current coefficient  $a^*/a^{**}$  of input node is nonzero, we write "1" to the corresponding row  $a^{**}$  of vector  $\mathbf{D}^2$ . In case of voltage source, we fill "0" the row  $a^{**}$  of matrices  $\mathbf{C}^{22}$  and  $\mathbf{C}^{12}$ . Then we write "1" to the position: row  $a^{**}$ , column  $a^*$  of  $\mathbf{C}^{22}$ .  
After these operations, the creation of matrices  $\mathbf{C}_M^{12}$  and  $\mathbf{M}^{22}$  is finished.  
If the input sources are not active in phase 2, then  $\mathbf{C}_M^{12} = \mathbf{C}^{12}$  and  $\mathbf{M}^{22} = \mathbf{C}^{22}$ .
3. We compute transform matrix  $\mathbf{M}^{12}$  and vector  $\mathbf{P}^2$  using (6).

### Algorithm for arbitrary switched networks

Let us consider switched network in Fig.2a. In the phase 1, the network is described by the set of three independent voltages  $\mathbf{v}^1: [v_1^1 = v_2^1, v_3^1, v_4^1]$ . As a result of disconnecting input source in the phase 2, the set of variables is then reduced to two independent voltages  $\mathbf{v}^2: [v_2^2, v_3^2]$ .

The operator diagram for phase 2 in Fig. 2b) yields

$$\begin{array}{|c|} \hline 0 \\ \hline 0 \\ \hline \end{array} \underset{\mathbf{D}^2}{=} \begin{array}{|c|c|} \hline G_1 + G_2 & -G_2 \\ \hline -G_2 & G_2 + G_3 + s(C_1 + C_3) \\ \hline \end{array} \underset{\mathbf{Y}^{22} = \mathbf{G}^{22} + s\mathbf{C}^{22}}{=} \begin{array}{|c|c|} \hline V_2^2 & V_3^2 \\ \hline \end{array} \underset{\mathbf{V}^2}{=} \begin{array}{|c|} \hline 0 \\ \hline Q_1 + Q_3 \\ \hline \end{array} \underset{\mathbf{Q}^2}{=} \quad (7)$$

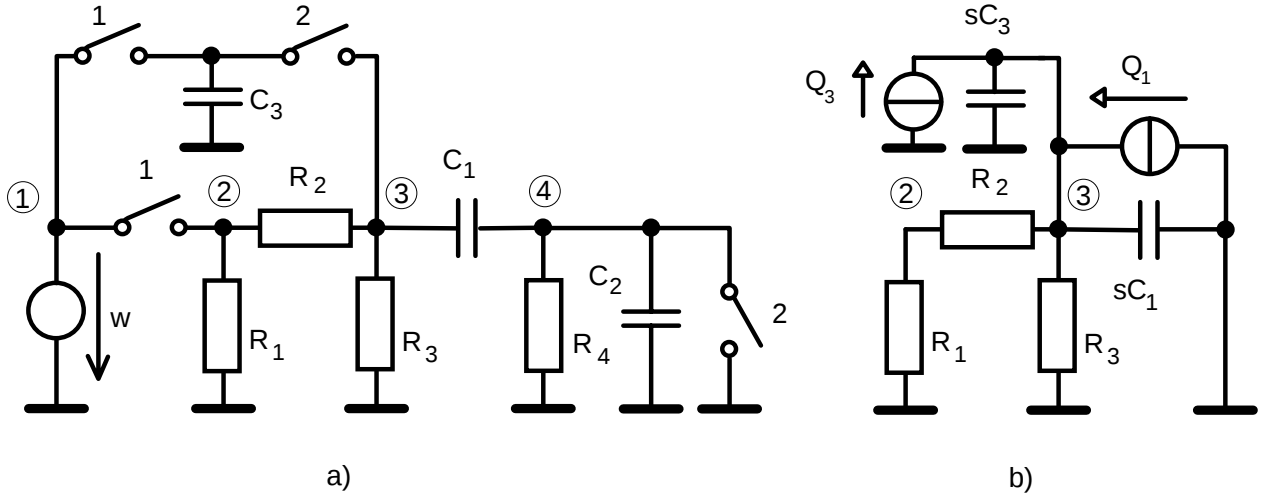


Fig.2. a) example of network with external switching, b) its operator diagram for phase 2.

The *charge conservation law* leads to the equations

$$Q_1 = C_1 v_3^2(0^+) = C_1 [v_3^1(0^-) - v_4^1(0^-)]$$

$$Q_3 = C_3 v_3^2(0^+) = C_3 v_1^1(0^-),$$

or

$$\begin{array}{c|c} 0 & \\ \hline Q_1 + Q_3 & \end{array} = \begin{array}{c|c} 0 & 0 \\ \hline 0 & C_1 + C_3 \end{array} \begin{array}{c} v_2^2(0^+) \\ v_3^2(0^+) \\ \hline v^2(0^+) \end{array} = \begin{array}{c|c|c} 0 & 0 & 0 \\ \hline C_3 & C_1 & -C_1 \end{array} \begin{array}{c} v_1^1(0^-) \\ v_3^1(0^-) \\ v_4^1(0^-) \\ \hline v^1(0^-) \end{array} \quad (8)$$

$$\mathbf{Q}^2 = \mathbf{Q}_M^2 \quad \mathbf{C}^{22} = \mathbf{C}_M^{22} \quad \mathbf{C}^{12} = \mathbf{C}_M^{12}$$

This is the set of two equations. The first one is again trivial  $0 = 0$ . However, the reason consists now in the absence of accumulation elements connected to node 2. In this way, the first equation in the set (7) is algebraical (in the operator form without the operator  $s$ ). That means **this equation is also true for instantaneous values and for the limits on the right:**

$$0 = (G_1 + G_2) v_2^2(0^+) - G_2 v_3^2(0^+).$$

We add this equation to the set (8):

$$\begin{array}{c|c} G_1 + G_2 & -G_2 \\ \hline 0 & C_1 + C_3 \end{array} \begin{array}{c} v_2^2(0^+) \\ v_3^2(0^+) \\ \hline v^2(0^+) \end{array} = \begin{array}{c|c|c} 0 & 0 & 0 \\ \hline C_3 & C_1 & -C_1 \end{array} \begin{array}{c} v_1^1(0^-) \\ v_3^1(0^-) \\ v_4^1(0^-) \\ \hline v^1(0^-) \end{array} \quad (9)$$

$$\mathbf{M}^{22} \quad \mathbf{C}^{12} = \mathbf{C}_M^{12}$$

Comparing (9) and (4) yields following conclusions:

$\mathbf{D}^2 = \mathbf{0}$  ( the network is not exited in the phase 2).

$\mathbf{C}_{12}^M$  is not the square matrix (the numbers of independent nodal variables in phases 1 and 2 are different).

$\mathbf{M}^{22}$  contains the parameters of both capacitive and resistive elements.

The desired solution can be obtained using inversion of matrix  $\mathbf{M}^{22}$ :

$$\begin{pmatrix} v_2^2(0^+) \\ v_3^2(0^+) \\ \mathbf{v}^2(0^+) \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{G_2}{G_1+G_2} \frac{C_3}{C_1+C_3} & \frac{G_2}{G_1+G_2} \frac{C_1}{C_1+C_3} & -\frac{G_2}{G_1+G_2} \frac{C_1}{C_1+C_3} \\ \frac{C_3}{C_1+C_3} & \frac{C_1}{C_1+C_3} & -\frac{C_1}{C_1+C_3} \end{pmatrix}}_{\mathbf{M}^{12}} \begin{pmatrix} v_1^1(0^-) \\ v_3^1(0^-) \\ v_4^1(0^-) \\ \mathbf{v}^1(0^-) \end{pmatrix}$$

After generalization of aforementioned conclusions we can state the algorithm for the compilation of transform matrices in case of **arbitrary switched networks**:

1. We compile matrices  $\mathbf{G}^{22}$ ,  $\mathbf{C}^{22}$  and  $\mathbf{C}^{12}$  according to Tab.1 or [5].
2. We scan all rows of matrix  $\mathbf{C}^{22}$  exception to the row corresponding to the input node. In case of empty rows they will be replaced by corresponding rows of matrix  $\mathbf{G}^{22}$ .  
This step is unnecessary in case of idealized SC networks.
3. If the input voltage/current source is active in phase 2, i.e. if the voltage/current coefficient  $a^*/a^{**}$  of input node is nonzero, we write "1" to the corresponding row  $a^{**}$  of vector  $\mathbf{D}^2$ . In case of voltage source, we fill "0" the row  $a^{**}$  of matrices  $\mathbf{C}^{22}$  and  $\mathbf{C}^{12}$ . Then we write "1" to the position: row  $a^{**}$ , column  $a^*$  of  $\mathbf{C}^{22}$ .  
After these operations and prospective modification in accordance with the item 2, the creation of matrices  $\mathbf{C}_M^{12}$  and  $\mathbf{M}^{22}$  is finished.
4. We compute transform matrix  $\mathbf{M}^{12}$  and vector  $\mathbf{P}^2$  using (6).

## CONCLUSION

The presented algorithm of inconsistent initial condition computation in the switched networks is based on the *two-graph modified nodal approach*. Using this method, the network is described by the set of independent circuit variables in each switching phase. As a result, the regular minimum set of equations is obtained. Novel algorithm utilizes this regularity and compiles transform matrices between the vectors of nodal variables immediately before and after switching. The algorithm is implemented to the general program for the fast analysis of real switched networks [4], [5].

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