SEMI-SYMBOLIC STEADY STATE ANALYSIS OF DYNAMICAL SYSTEMS

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Abstract: *Novel method how to obtain results of steady-state analysis of linear network in semi-symbolic form is presented. On the assumption of periodical input signal, the steady-state time response during one period is obtained directly as a function of time. Model of investigated system can be considered in arbitrary known form as the set of differential-algebraic equations or s-domain transfer function.*

Introduction

In case of simulation of integrated circuits with large spread of time constants or circuits excited by narrow-band signals as AM, computer analysis is connected with serious problems. Time step of computation is very small in comparison with the period of low-frequency components in the response because it has to be adapted to the period of high-frequency spectral terms. Then the simulation is time- and memory consuming. In addition, the steady-state is usually reached during many periods of high-frequency components (10.000 and more). In the systems with large spread of time constants, the special procedures have to be performed to satisfy convergence which lead to the additional the additional deceleration of simulation. These factors mentioned above cause that the computer steady-state solution using classical methods is often impossible.

Proposed novel solution consists of three step. First, the forced response is find in semi-symbolic form as a function of time. Second, the vector of initial condition corresponded to the steady state is computed using simple algorithm which is based on the equality of initial conditions at time 0 and T where T is the period of input signal. Third, the complete response is compiled from the forced and natural responses where the natural response is response to the initial conditions obtained in second step.

As a result, the analytical function describing one period of steady state is obtained.

Algorithm

Algorithm is based on the state space model of investigated system:

$$\frac{d}{dt}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}v \tag{1}$$

$$y = \mathbf{C}\mathbf{x} + \mathbf{D}v \tag{2}$$

where $\mathbf{x}$ is the vector of state variables, y and v are output and input signals, $\mathbf{A}$ is the state matrix and $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are corresponding vectors of state description. In spite of different original model of investigated system, this model can be transferred to the state equations (1) and (2). Procedure how to perform it is described in [1]. In distinction from the transfer function, the state-space model enables to operate with initial conditions which is necessary assumption for the solution of steady state.

Equation (1) can be solved in the operator domain. Laplace transform of state vector is in the known form:

$$\mathbf{X}(s) = (s\mathbf{E} - \mathbf{A})^{-1}\mathbf{x}(0) + (s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B}V(s). \tag{3}$$

On the right side of (3), first term represents Laplace transform of natural response to the initial state $\mathbf{x}(0)$, second term is the Laplace transform of forced response to the input signal $v(t)$.

Equation (3) corresponds with the time-domain solution

$$\mathbf{x}(t) = \mathbf{g}^*(t)\mathbf{x}(0) + e^{\mathbf{A}t}\mathbf{B} * v(t) = \mathbf{x}_N(t) + \mathbf{x}_F(t) \tag{4}$$

where symbol * denotes convolution and $\mathbf{x}_N$ and $\mathbf{x}_F$ are natural and forced responses, respectively, and

$$\mathbf{g}^*(t) = e^{\mathbf{A}t}. \tag{5}$$

Let us consider that input signal is periodical with repeating period T. For periodical steady-state, following relation is fulfilled:

$$\mathbf{x}(T) = \mathbf{x}(0). \tag{6}$$

Equations (4) and (6) give result

$$[\mathbf{E} - \mathbf{g}^*(T)]\mathbf{x}(0) = \mathbf{x}_F(T) \tag{7}$$

which represents set of linear equations. After simple solution, vector of initial conditions $\mathbf{x}(0)$ is found on the assumption that we know vector of forced response and the matrix $\mathbf{g}^*(T)$.

Matrix g*(t) and vector $\mathbf{x}_F(t)$ will be obtained in semi-symbolical form using procedures described below. Then the matrix $\mathbf{g}^*(T)$ and vector $\mathbf{x}_F(T)$ in (7) will be obtained after setting $t = T$.

At the end, analytical function describing steady-state will be obtained after arrangement (2) and (4). Block diagram of whole algorithm is in Fig.1.

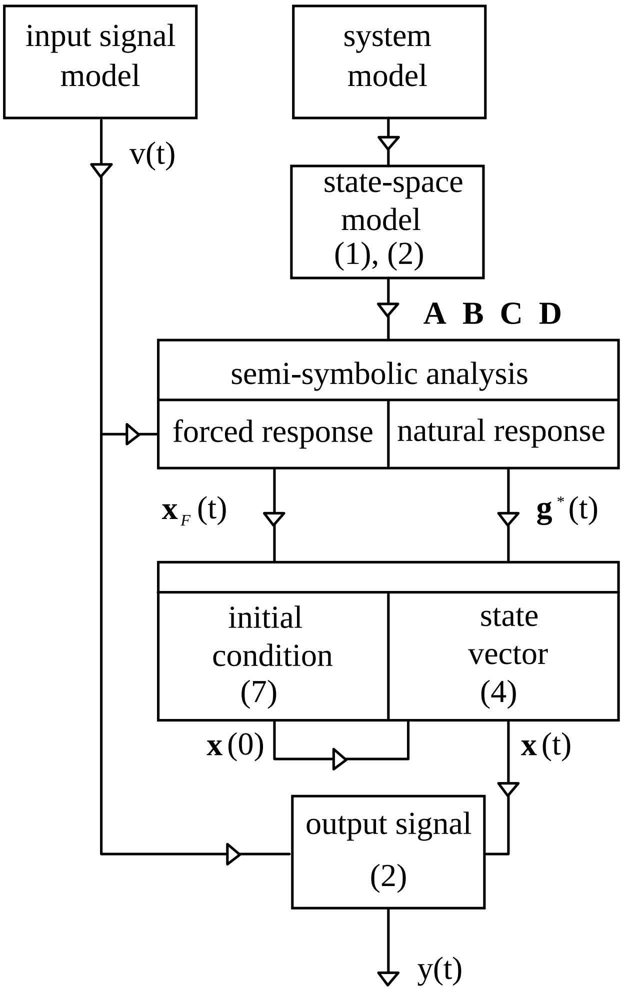

Fig.1.: Simplified diagram of the algorithm of semi-symbolic steady-state analysis.

Semi-symbolic analysis of natural and forced responses

Natural response is given by equation

$$\mathbf{x}_N(t) = \mathbf{g}^*(t)\mathbf{x}(0) \text{ where } \mathbf{g}^*(t) = e^{\mathbf{A}t} = L^{-1}[s\mathbf{E} - \mathbf{A}]^{-1}.$$

Symbol L^{-1} designates the Laplace inversion. We have to find each element of matrix $e^{\mathbf{A}t}$ in semi-symbolic form. It will be done in three steps:

1. Zeros and poles finding of all matrix elements in the *s*-domain.
2. Partial fraction expansion to obtain each element in basic form for Laplace inversion.
3. Laplace inversion of each element of matrix $\mathbf{g}^*(t)$.

First step is performed using multiple application of *QR* or *QZ* algorithm for solution of the eigenvalue problem. All matrix elements have the same poles - the eigenvalues of matrix **A**. Zeros of the element $\{a_{ij}\}$ are roots of the equation

$$\det(s\mathbf{E}_{i:j} - \mathbf{A}_{i:j}) = 0 \tag{8}$$

where the symbol det means determinant and subscript *i:j* denotes omitting of *i*th row and *j*th column. It should be noted that for $i \neq j$ the matrix $\mathbf{E}_{i:j}$ is not diagonal and then equation (9) has to be solved as the generalized eigenvalue problem with matrix deflation preceded *QZ* algorithm.

Second and third steps are realized using the algorithm described in [2]. This procedure of partial fraction expansion is also utilized in the program *LTP2* [3].

Laplace transform of forced response is given by equation

$$\mathbf{X}_F(s) = (s\mathbf{E} - \mathbf{A})^{-1}\mathbf{B}V(s). \tag{9}$$

As described above, zeros and poles of elements of the matrix $(s\mathbf{E} - \mathbf{A})^{-1}$ are obtained during the computation of natural response. In accordance with the equation (9), poles of $\mathbf{X}_F$ are eigenvalues of matrix **A** completed by the poles of input signal. Zeros of $\mathbf{X}_F$ are linear combinations of zeros of the matrix $(s\mathbf{E} - \mathbf{A})^{-1}$ completed by the zeros of input signal.

In many applications, the Laplace transform of input signal is not directly known. For example, signal is modeled using the set of measured samples. Semi-symbolic analysis of steady-state response can be performed also in these situations but detail description is beyond the scope of this contribution.

Conclusion

Semi-symbolic analysis offers solution of steady-state analysis in linear network as the analytical function of time. The numerical algorithm is based on the standard solution of the eigenvalue problem and on the algorithm of partial fraction expansion. Described algorithm is reliable also for the analysis of large networks excited by narrow-band periodical signals.

Acknowledgement

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References

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