STABILITY OF FILTERS WITH SINGLE-OpAmp FDNRs

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Abstract

Analysis of a single-OpAmp FDNR is performed in this paper with the aim to derive conditions of its possible unstable behavior.

1 Introduction

An active 5th-order lowpass filter shown in Fig. 1 has a cutoff frequency of 10 Hz, a passband ripple of 2 dB, and an attenuation of at least 50 dB from a frequency of 15 Hz. This filter, published for the first time in [1], has been designed from a passive RCL ladder cell by using the Bruton transformation, where two grounded FDNRs are realized by the circuit.
Under some circumstances, it is possible to observe a predisposition of these circuits to oscillate. As follows from a detailed analysis, the circuit can become unstable due to an unstable DC operating point.

2 Method of Stability Testing

First of all, we test the stability by investigating the pole location. However, the result can strongly depend on the complexity of models of individual circuit elements, especially of the OpAmp [2]. To avoid these equivocations, we additionally test the stability of DC operating point [2]. If we find such conditions that lead to DC instability, then this will necessarily involve unstable behavior of all the dynamical circuits that are adjoined to this DC operating point, independently of the complexity of dynamical models of circuit elements. It is interesting that this instability is not generally discovered by the analysis of pole location.
However, the negative result of DC instability testing does not imply stable behavior of an adjoint dynamical circuit. Possible dynamical instability can be discovered by “pole-location” testing.

3 Recapitulation of Circuit Properties

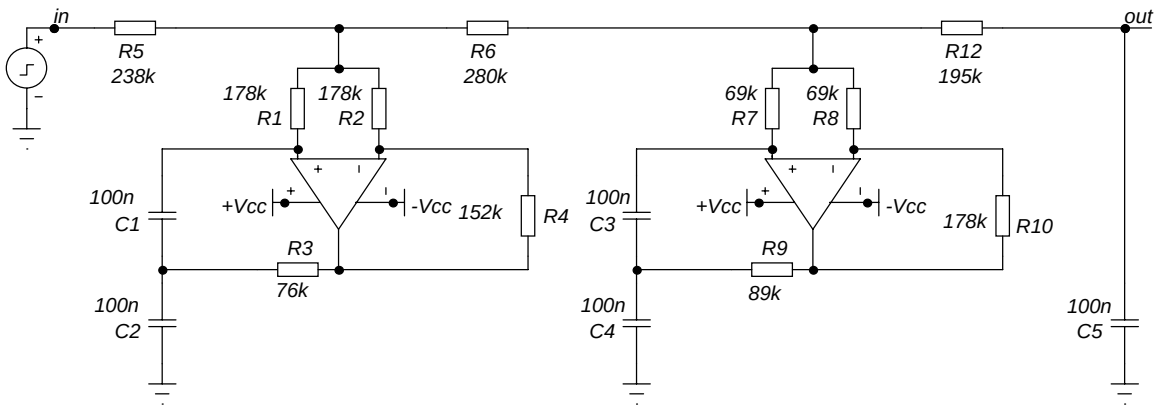

Fig.1. Lowpass filter with two FDNRs, 10 Hz, Cauer type.

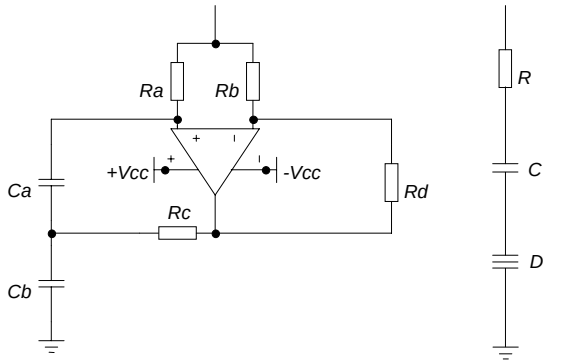

Fig. 2. Equivalent circuit.

In general, the resistance, capacitance, and FDNR in series are modeled by the given single-OpAmp circuit (see Fig. 2).
The following equations are valid for the parameters of equivalent elements:

$$R = \frac{R_a R_b}{R_a + R_b} \tag{1}$$

$$C = \frac{(R_a + R_b)R_c}{R_b R_c C_a + R_b R_c C_b - R_a R_d C_a} C_a C_b \tag{2}$$

$$D = \left(1 + \frac{R_a}{R_b}\right) R_c C_a C_b . \tag{3}$$

For the ladder simulation according to Fig. 1, it is necessary to exclude the capacitor in the equivalent circuit, i.e. to satisfy the condition C→∞. We reach this by choosing (see eq. 2)

$$\frac{R_a}{R_b}\frac{R_d}{R_c} = 1 + \frac{C_b}{C_a} . \tag{4}$$

It is advisable to choose capacitances $C_a = C_b$ to be identical. Condition (4) can then be respected e.g. by choosing $R_a = R_b$, $R_d = 2R_c$, as can be seen in Fig. 1.
In fact, condition (4) will be fulfilled with only some accuracy. If the denominator in (2) is negative, the capacitance will be also negative, which is a potential source of oscillation. For a proper functioning, it is necessary to keep the left side of equation (4) less than the right side. For example, increasing R_a above the designed value leads to instability.
In the next part, these propositions will be made more exact by the pole-location analysis. We will confine ourselves to the single-pole model of operational amplifier. The two-pole model does not bring essential improvements.

4 Testing of Dynamical (In)Stability

A symbolic analysis of the circuit in Fig. 2 using the SNAP program [3] yields a characteristic function on the assumption of a single-pole OpAmp model with the following parameters:

DC open-loop gain A0→∞,
finite GBW,
non-zero output resistance Ro.

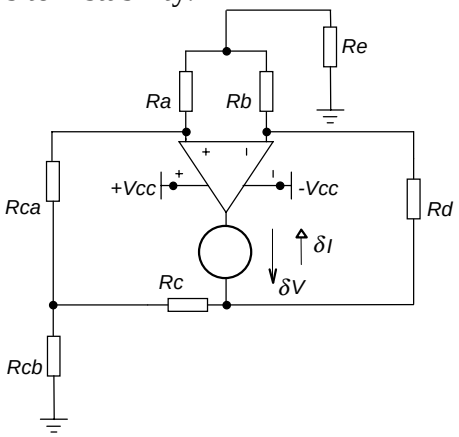

Fig. 3. Simplified model of circuit from Fig. 2 for DC stability testing.

An analysis of the coefficients yields the stability condition

$$\frac{R_a}{R_b}\frac{R_d}{R_c} \le 1 + \frac{C_b}{C_a} + a, \tag{5}$$

$$a = \frac{R_b + R_d + R_0}{\omega_t R_b R_c C_a} . \tag{6}$$

Here the statement from the foregoing chapter about equation (4) is refined: Due to OpAmp real properties, the instability boundary is shifted by a constant *a*, which depends on the OpAmp GBW and output resistance. For ideal amplifier, this shifting is zero.
Violating condition (5) makes for the oscillation.

5 Testing of DC (In)Stability

In ideal case, if the signal source does not affect the input, the filter is in DC steady state, when all nodal voltages are zero. In this equilibrium state, all resistive elements play an important role, including leak resistances and input/output resistances of the OpAmp. The question is if such an equilibrium can be unstable and thus unobservable. Then an arbitrary deflection from the equilibrium point, which may be caused by accidental fluctuation, leads to the system moving away from this point.

Resistance R_d in Fig. 2 introduces a negative feedback and thus the equilibrium tends to be stable. Even the input OpAmp resistances cannot be the primary causes of instability because they do not work in the positive-feedback loop. However, the leak resistances R_{ca} and R_{cb} of capacitors C_a and C_b can play a negative role, as shown in Fig. 3.

The resulting resistance between the crossing point of resistors R_a a R_b and the ground is modeled by the resistance R_e.

The voltage fluctuation for the stability testing of DC equilibrium state is modeled by the source δV of voltage variation at the OpAmp output. The circuit reaction is in the form of current fluctuation δI. The equilibrium state is unstable if the corresponding resistance R_f, which is the loading resistance of the source of fluctuation, is negative [2]:

$$R_f = \frac{\delta V}{\delta I} < 0 . \tag{7}$$

The following symbolic computations have again been performed by the SNAP program. The equation of R_f is a fraction whose denominator consists of only positive elements. Considering OpAmp DC open-loop gain A → ∞, then the instability condition is as follows:

$$\left(R_{ca} + R_{cb} + R_a + R_e\right)R_b R_c + R_a R_c R_e + R_{cb}\left(R_{ca} R_b - R_a R_d\right) < 0 . \tag{8}$$

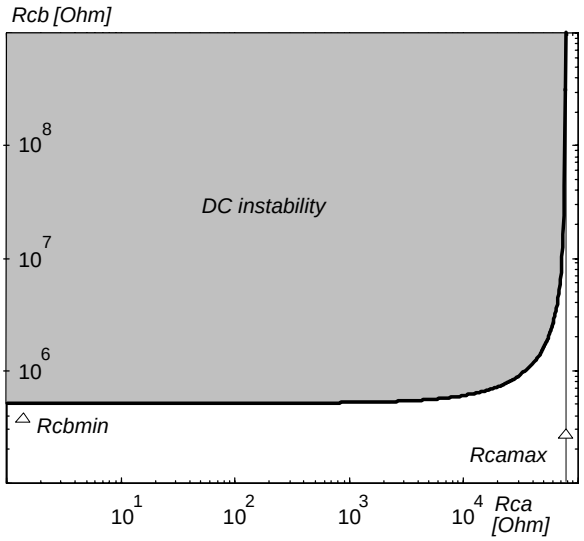

Fig. 4. DC instability region for the left FDNR branch in the filter from Fig. 1.

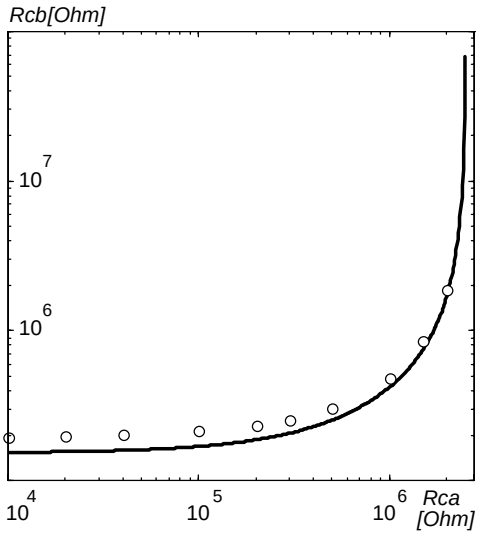

Fig. 5. Borderline of DC stability of the FDNR circuit with modified parameters.

With respect to variables R_{ca} and R_{cb}, formula (8) is the equation of a hyperbola which is the borderline of stability in the R_{ca} - R_{cb} space. For the first FDNR section of the filter in Fig. 1, the corresponding diagram is in Fig. 4. The boundary values R_{camax} and R_{cbmin} can be derived from condition (8):

$$R_{ca} < R_{ca\max} = \frac{R_a R_d}{R_b} - R_c , \; R_{cb} > R_{cb\min} = R_c \frac{R_b\left(R_a + R_e\right) + R_a R_e}{R_a R_d - R_b R_c} . \tag{9}$$

The first filter section has the values R_{camax} = 76kΩ, R_{cbmin}= 506kΩ.

This result means that, for instance, if leak resistance R_{ca} is greater than 76kΩ, the circuit will

be sure-fire DC stable, independent of the value of R_{cb}.
If the synthesis takes into account condition (4) (only R and D elements are in series), then conditions (9) are simplified to:

$$R_{ca} < R_{ca\max} = R_c \frac{C_b}{C_a},\ R_{cb} > R_{cb\min} = \frac{C_a}{C_b}\left(R_a + R_e + \frac{R_a R_e}{R_b} \right). \tag{10}$$

As is obvious from (10), the circuit could be unstable for a large resistance R_c and for a large ratio of capacitances. While minimizing the capacitance in the OpAmp feedback, an improper procedure can lead, for instance, to the following design of the first FDNR branch:

$$R_a = 534k\Omega,\ R_b = 107k\Omega,\ R_c = 253k\Omega,\ R_d = 557k\Omega,\ C_a = 10nF,\ C_b = 100nF\ .$$

Then the boundary values are as follows:

$$R_{ca\max} = 2{,}53M\Omega,\ R_{cb\min} = 152k\Omega\ .$$

For example, the filter will be DC unstable for the same leak resistances 2MΩ.
Instability condition (8) is not quite precise because it does not take into consideration the influence of additional parasitic resistances and the OpAmp output resistance. Though the influence of these resistances partly moves the borderline of stability, the following rule for designing elements from equations (1), (3) and (4) does not change:

It is useful to choose capacitances C_a and C_b to be identical and resistance R_c to be smaller than the expected leak resistance of capacitor C_a. Then the circuit will be DC stable.

Note: the leak resistances have to be sufficiently high to avoid undesirable warping of frequency response.

6 Experimental Verification

In the first step, the filter from Fig. 1 was compiled and tested. The OpAmp LF156 was used by reason of the relatively large working resistances on the OpAmp inputs. Then the first FDNR-branch was modified to the aforementioned values:

$$R_a = 534k\Omega,\ R_b = 107k\Omega,\ R_c = 253k\Omega,\ R_d = 557k\Omega,\ C_a = 10nF,\ C_b = 100nF$$

The shunting resistors were connected to capacitors C_a and C_b and their influence on stability was observed. Some of the measurement results are summarized in Fig. 5. Circles mark the measured points. The agreement with theoretical shape is good.

7 Conclusion

The analysis demonstrates that the DC instability of filters with the FDNR circuits discussed can easily be excluded on the assumption of proper design procedure. The dynamical instability can be more significant. This instability may consist in the wrong compensation of capacitance C according to equations (4) and (5).

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References

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