Efficient Compilation of Circuit Equations of Switched Networks

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Abstract

The algorithm of finding the set of independent circuit variables of the networks with periodically operated switches is proposed. Our method is based on so-called voltage and current coefficients. As a result, number of circuit equations is considerably reduced in comparison to the classical modified nodal approach.

1 Basic Conception

For computer analysis of switched network, the number of circuit equations plays important role. Large number of equations leads to the time- and memory- expensive simulation. Classical modified nodal approach compiles additional equations for switches and transform cells and number of equations grows unsatisfactorily.

The two-graph modified nodal approach (MNA) [1,2] has been used as the basic method. To create minimum number of equations, our procedure is as follows:

- We find the set of independent nodal variables in each switching phase:

$$\mathbf{v}_1 = \left[v_1^1, v_1^2, .., v_1^{n_1}\right]^T \ldots \text{phase } 1, \qquad (1)$$
$$\mathbf{v}_2 = \left[v_2^1, v_2^2, .., v_2^{n_2}\right]^T \ldots \text{phase } 2,$$

where n_1 and n_2 are numbers of independent variables in phase 1 and phase 2, respectively. The algorithm how to obtain that sets will be described in following chapter.

- It is possible to create the set of differential equations of *MNA* in the operator domain for independent variables:

$$\underbrace{\left(\mathbf{G}_1^M + s\mathbf{C}_1^M\right)}_{\mathbf{Y}_1^M}\mathbf{V}_1 = \mathbf{D}_1 W_1 + \mathbf{C}_{12}^M \mathbf{v}_2(kT^-), \qquad (2)$$
$$\underbrace{\left(\mathbf{G}_2^M + s\mathbf{C}_2^M\right)}_{\mathbf{Y}_2^M}\mathbf{V}_2 = \mathbf{D}_2 W_2 + \mathbf{C}_{21}^M \mathbf{v}_1(kT + T_1^-),$$

where $\mathbf{G}_1^M$ and $\mathbf{G}_2^M$ are modified conductance matrices of dimensions ($n_1\, x\, n_1$) and ($n_2\, x\, n_2$),

$\mathbf{C}_1^M$ and $\mathbf{C}_2^M$ are modified reactance matrices of dimensions ($n_1\, x\, n_1$) and ($n_2\, x\, n_2$),

$\mathbf{C}_{12}^M$ and $\mathbf{C}_{21}^M$ are modified transitive matrices of dimensions ($n_1\, x\, n_2$) and ($n_2\, x\, n_1$),

$\mathbf{D}_1$ and $\mathbf{D}_2$ are certain incidence vectors of dimensions ($n_1\, x$ 1) and ($n_2\, x$ 1),

W_1 and W_2 are Laplace transformations of input signals in phases 1 and 2,

T_1 is length of phase 1.

Equations (2) have key significance for computer analysis. It is possible to compile them for arbitrary linear switched network no matter whether the consistent initial conditions occur in the time instances between two switching phases [3]. In practice, the simulator will produce correct results also in case of discontinuously changed nodal variables.

In following part, we describe the algorithm how to find independent circuitvariables.

2 Finding Of The Independent Circuit Variables

One way how to find independent circuit variables is given in [4] as a method of current and voltage graphs. Number of compiled equations will be equal to the number of independent circuit variables. To find the set of independent variables, we have to perform transformation and reduction of variables even before the compilation of circuit equations.

Two numbers - so called voltage and current coefficients - will be attached to each node in each switching phase [5]. Let us designate these coefficients as * and **. The current coefficient gives the number of row of the admittance matrix that corresponds to the equation of first KL for given node. The voltage coefficient gives the number of column of the admittance matrix that corresponds to the voltage of given node. The reference node has number 0 and corresponding coefficients 0*,0**.

In case of continuous-time circuit with the elements that have admittance matrices, both nodal coefficients will be equal and they will correspond to the order of nodes.

If two nodes are connected by switch with zero switch resistance in some switching phase, their nodal voltages and prospective outside currents will be equal. That is why their coefficients will be equal. Generally, *m* different nodes shorted by switches in given phase is described only by single equation of first KL instead of originally *m* equations.

If some node is disconnected in some phase from the rest of the circuit, we set the coefficients 0*,0**. In this way we eliminate corresponding nodal voltage and current from subsequent computation. Otherwise, compiled circuit matrix would be singular and the solution routines would fail.

Nodal voltages of the positive and negative inputs of ideal operational amplifier (OpAmp) are equal and voltage coefficients of these nodes will be the same. However, the current coefficients will be different because both inputs can be excited independently by different current sources. Excitation to the output of OpAmp (to the ideal voltage source) does not make sense and corresponding current coefficient has to be chosen zero. Obviously, the ideal OpAmp decreases number of independent voltages and currents in each switching phase by 1. Classical *MNA* leads to the increasing of this number.

Influence of the unity gain buffer in the circuit has similar consequences. Nodal voltages of the input and output are identical and both nodes have the same voltage coefficients. The current coefficient of the output node is again zero.

In following chapter we describe the compilation of circuit matrices in case of nonideal OpAmp with the consideration of final DC gain and the first and second poles and the output resistance. Then the order of the circuit matrices will increase likewise for the circuits with the single and mutual inductances.

Nevertheless, it follows from the above mentioned that the total number of different voltage and current coefficients is equal in given switching phase. After their consecutive numbering, the highest coefficient gives the order of admittance matrix in the corresponding phase.

Described principles will be demonstrated on the following example of *SC* filter.

3 Example

Consider *SC* filter with the economical FDNR in Fig. 1 [6]. Let us suppose ideal unity gain buffer and switches with the zero switch-on and infinity switch-off resistances.

Tab. 1 demonstrates the algorithm how to generate voltage and current coefficients. First of all, the transformation due to switches is performed: the nodes connected by the switch are labeled by the identical coefficients. The reference node and the isolated nodes have coefficients 0*,0**. In this step, the voltage and current coefficients are identical.

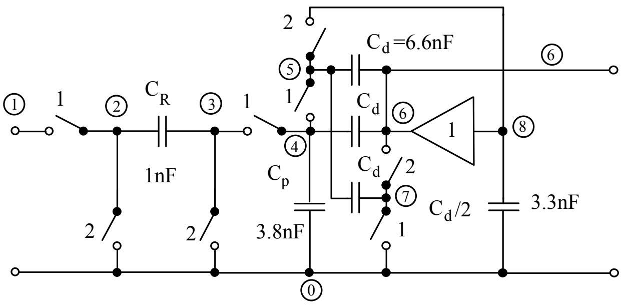

Figure 1. Economical switched FDNR

In following step, the transformation due to nonregular elements (buffer) is performed. After arrangement of the coefficients we can see that the orders of the admittance matrices will be 3x3 in the phase 1 and 2x2 in the phase 2. The independent circuit variables are for example:

phase 1: v_1, v_3, v_6, i_1,i_3,i_8; $n_1 = 3$;

phase 2: v_4, v_5, i_4,i_5; $n_2 = 2$.

phase	node	consecutive numbering		transformation due to				arrangement	
				switches		buffers			
		*	**	*	**	*	**	*	**
1	1	1	1	1	1	1	1	1	1
	2	2	2	1	1	1	1	1	1
	3	3	3	3	3	3	3	2	2
	4	4	4	3	3	3	3	2	2
	5	5	5	3	3	3	3	2	2
	6	6	6	6	6	6	0	3	0
	7	7	7	0	0	0	0	0	0
	8	8	8	8	8	6	8	3	3
2	1	1	1	0	0	0	0	0	0
	2	2	2	0	0	0	0	0	0
	3	3	3	0	0	0	0	0	0
	4	4	4	4	4	4	4	1	1
	5	5	5	5	5	5	5	2	2
	6	6	6	6	6	5	0	2	0
	7	7	7	6	6	5	0	2	0
	8	8	8	5	5	5	5	2	2

Tab.1. Finding of the set of independent variables of the filter in Fig.1.

4 Algorithm

It is evident that the algorithm of finding the independent circuit variables will have following parts:

(a) transformation due to switches,

(b) skipping the isolated nodes,

(c) transformation due to nonregular elements,

(d) rising numbering of the coefficients.

First of all, two matrices $\mathbf{U}_1$ and $\mathbf{U}_2$ $[0..N,0..1]$ are declared, where N is the total number of nodes except for the reference node. The voltage/current coefficients of the k-th node will be recorded to the first/second column and k-th row of matrix $\mathbf{U}_1$ for the phase 1 and $\mathbf{U}_2$ for the phase 2. The result of the algorithm will be as follows:

voltage coefficient of the k-th node: $k^*=\mathbf{U}_1(k,0)$ for the phase 1, $\mathbf{U}_2(k,0)$ for the phase 2;

current coefficient of the k-th node: $k^{**}=\mathbf{U}_1(k,1)$ for the phase 1, $\mathbf{U}_2(k,1)$ for the phase 2.

Before starting of the algorithm, both matrices are filled so that both coefficients in the k-th row are k (see Tab. 1, the column "consecutive numbering"). It corresponds to the initial state without transformations.

(a) Transformation Due To Switches

In this stage of the algorithm, the voltage and current coefficients of node are equal. If the connection of nodes a and b is indicated in given phase, it is necessary to perform following operations:

1. To read values of coefficients $S1=a^*=a^{**}$ and $S2=b^*=b^{**}$ from the corresponding matrix U. Comparing both coefficients and their prospective interchange so that $S1 \leq S2$. Then to replace higher coefficient by the lower on the given position.

2. To rewrite all coefficients $S2 \rightarrow S1$ in all following rows of the matrix U.

In this way, the algorithm will work reliably even by multiple transformation.

(b) Skipping The Isolated Nodes

The algorithm marks by the coefficients 0*,0** each node which becomes isolated in certain phase due to switching.

If the tested node has not zero coefficients in advance, its connection with the passive and active elements is tested. In case of finding first connection, we pass immediately to the test of next node. We can speed up this process considerably if the test is performed using the already compiled coefficients. The algorithm can be described by following steps:

- we read node coefficients step by step from the first row of matrix **U**, omitting the already read coefficients;
- we compare each coefficient with the coefficients of nodes to which the passive and active elements are connected.

In this way we test all connected nodes at the same time.

(c) Transformation Due To Nonregular Elements

Let us consider two types of ideal nonregular elements: the ideal differential OpAmp and the unity gain buffer.

Ideal differential OpAmp

Let the input nodes of the OpAmp be a_1, a_2 and the output node be a_3. Then the algorithm of the transformation of voltage coefficients a_1*, a_2* and a_3* is the same as for the transformation due to switches.

The current coefficient of the output node has to be rewritten to zero on all positions of the matrices $\mathbf{U}_1$ and $\mathbf{U}_2$.

Buffer

Let the input/output node of the buffer be a_1/a_2. Then the algorithm of the transformation of voltage coefficients a_1*and a_2* is the same as in the preceding case.

The current coefficient of the output node has to be rewritten to zero on all positions of the matrices $\mathbf{U}_1$ and $\mathbf{U}_2$.

(d) Rising Numbering Of The Coefficients

With the utilization of general properties of the voltage and current coefficients, following numbering algorithm has been developed and successfully verified:

1. We browse the matrix $\mathbf{U}_1/\mathbf{U}_2$ row by row. The voltage and current coefficients are classified separately. In each step, by now the highest coefficient S is stored. We set $S = 0$ before the first step.
2. If the coefficient in the row is greater then S, we increase the variable S by 1 and we set the value of this coefficient as S. In case this original coefficient appears also in the other matrix rows, we replace also its value by S. If the coefficient in the actual row is less or equal to S, the values of S and of the coefficient remain unchanged. Then the algorithm passes to the next row.

5 CONCLUSION

Novel algorithm how to find the set of independent circuit variables in the switched circuits is presented. Computer simulator which utilizes this algorithm compiles only the minimum set of equations. The analysis is then fast and not memory consuming.

Proposed algorithm has been incorporated to the program SPIN for the analysis of general networks with the external switching [7].

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