MODELING OF BLOCK-ORIENTED SYSTEMS USING MODIFIED STATE VARIABLE APPROACH

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Abstract - The state variable method can be used for efficient simulation of block - oriented systems. The modified method of state variables is described in this contribution which has been developed as the analogy of well-known modified nodal analysis method. Described procedure solves following problem: Consider two dynamical systems with known state equations. Systems will be interconnected by defined way and excited by input sources. Our task is to compile state equations of resulting system.

INTRODUCTION

In case of block-oriented systems, the state variable method is frequently used. The state equations can be often obtained directly from system structure, considering outputs of first-order memory elements as state variables. Large systems consist of interconnected subsystems. It is important to know how to compile state equations of resulting system from state equations of given subsystems.

In this contribution, the problem of two subsystems is solved. From the mathematical point of view, the state equations of first system have to be modified, taking into account influence of connected second system.

The method is valid on the assumption that state variables of both subsystems are independent.

Described procedure will be demonstrated on the example of two cascaded dynamical systems with additional feedback. First system simulates generation of input signal for excitation of the second one.

PROCEDURE

Let us consider two systems with following state equations:

$$\frac{d}{dt}\mathbf{x}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1\mathbf{v}_1, \quad \mathbf{y}_1 = \mathbf{C}_1\mathbf{x}_1 + \mathbf{D}_1\mathbf{v}_1, \qquad (1)$$

$$\frac{d}{dt}\mathbf{x}_2 = \mathbf{A}_2\mathbf{x}_2 + \mathbf{B}_2\mathbf{v}_2, \quad \mathbf{y}_2 = \mathbf{C}_2\mathbf{x}_2 + \mathbf{D}_2\mathbf{v}_2, \qquad (2)$$

where $\mathbf{x}$, $\mathbf{y}$, $\mathbf{v}$ are the state, output and input vectors and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are well-known matrices of state equations.

Both systems are interconnected only using their inputs and outputs in accordance with Fig.1. These interconnections can be described as follows:

$$\mathbf{v}_1 = \alpha_1\mathbf{v} + \beta_{11}\mathbf{y}_1 + \beta_{12}\mathbf{y}_2, \qquad (3)$$

$$\mathbf{v}_2 = \alpha_2\mathbf{v} + \beta_{21}\mathbf{y}_1 + \beta_{22}\mathbf{y}_2, \qquad (4)$$

$$\mathbf{y} = \mathbf{F}_1\mathbf{y}_1 + \mathbf{F}_2\mathbf{y}_2 + \mathbf{F}\,\mathbf{v}, \qquad (5)$$

where symbols α, β and $\mathbf{F}$ represent weighted matrices of corresponding interconnections.

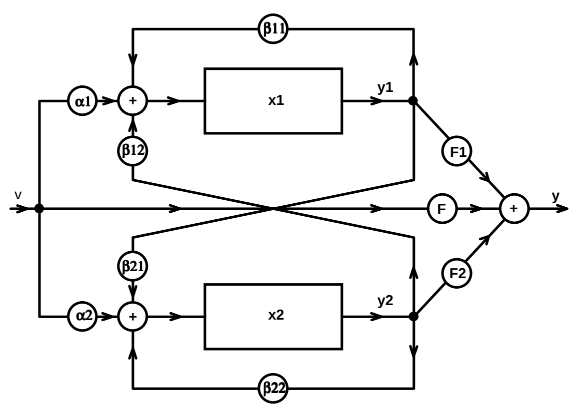

Fig.1. Considered interconnections between two systems.

Equations (1), (2), (3) and (4) can be arranged to the *modified state equation of dynamics*:

$$\frac{d}{dt}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ 0\\ 0\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}\mathbf{A}_1 & & \mathbf{B}_1 & & & & \\ & \mathbf{A}_2 & & \mathbf{B}_2 & & & \\ -\mathbf{C}_1 & & -\mathbf{D}_1 & & \mathbf{E} & & \\ & -\mathbf{C}_2 & & -\mathbf{D}_2 & & \mathbf{E} & \\ & & \mathbf{E} & & -\beta_{11} & -\beta_{12} & -\alpha_1\\ & & & \mathbf{E} & -\beta_{21} & -\beta_{22} & -\alpha_2\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \mathbf{v}_1\\ \mathbf{v}_2\\ \mathbf{y}_1\\ \mathbf{y}_2\\ \mathbf{v}\end{bmatrix} \qquad (6)$$

where $\mathbf{E}$ is the identity matrix.

Matrix equation (6) can be numerically reduced using partial pivoting to the form

$$\frac{d}{dt}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{B}_{11}\\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{B}_{21}\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \mathbf{v}\end{bmatrix} \qquad (7)$$

which represents resulting state equations of dynamics.

Procedure for computation of the output state equations is similar. Utilizing Eq. (1), (2), (3) and (4), set of equations (6) can be rewritten as *modified output equations*:

It should be noted that also the analytical derivation of resulting matrices in (7) and (9) can be performed from input matrices in (1)-(5) instead of numerical variable elimination with the aim to speed-up analysis. However, this procedure is without physical background.

In most cases, the system interconnection is simple without feedback paths. Typical case is the cascade connection. Then the compilation of resulting state equations can be trivial. However, in general situation, utilization of our algorithm can lead to the efficient solution.

$$\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\\ 0\\ 0\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}\mathbf{C}_1 & & & & \mathbf{D}_1 & & \\ & \mathbf{C}_2 & & & & \mathbf{D}_2 & \\ -\mathbf{A}_1 & & \mathbf{E} & & -\mathbf{B}_1 & & \\ & -\mathbf{A}_2 & & \mathbf{E} & & -\mathbf{B}_2 & \\ -\beta_{11}\mathbf{C}_1 & -\beta_{12}\mathbf{C}_2 & & & \mathbf{E}-\beta_{11}\mathbf{D}_1 & -\beta_{12}\mathbf{D}_2 & -\alpha_1\\ -\beta_{21}\mathbf{C}_1 & -\beta_{22}\mathbf{C}_2 & & & -\beta_{21}\mathbf{D}_1 & \mathbf{E}-\beta_{22}\mathbf{D}_2 & -\alpha_2\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \mathbf{x}_1'\\ \mathbf{x}_2'\\ \mathbf{v}_1\\ \mathbf{v}_2\\ \mathbf{v}\end{bmatrix} \qquad (8)$$

where the superscript ‘ denotes time derivation.

Note that if $\mathbf{D}_1 = \mathbf{0}$ and $\mathbf{D}_2 = \mathbf{0}$, the output equations of separated systems together with equation (5) are output equations of resulting system after interconnection. Otherwise, matrix equation (8) can be reduced like in previous case to the form

$$\begin{bmatrix}\mathbf{y}_1\\ \mathbf{y}_2\end{bmatrix} = \begin{bmatrix}\mathbf{C}_{11} & \mathbf{C}_{12} & \mathbf{D}_{11}\\ \mathbf{C}_{21} & \mathbf{C}_{22} & \mathbf{D}_{21}\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \mathbf{v}\end{bmatrix}. \qquad (9)$$

Resulting output state equations are then obtained using (5).

EXAMPLE

Consider connection of systems according to Fig.2. First system is harmonic generator with the signal frequency ω_0 =10 rads/s and adjustable initial conditions (blocks *a* and *b*). Second system is excited by first one so that its input is directly connected with the output of the generator. Output of second system serves as input signal for first block. In this way, resulting system is autonomous without external inputs.

Let us compile state equations of resulting system. State equations of system 1 and 2 are as follows:

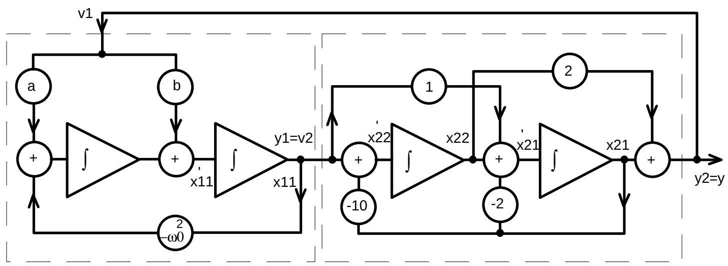

Fig.2. Illustration example

System 1:

$$\frac{d}{dt}\begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} & 1 \\ -100 & \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} b \\ a \end{bmatrix} v_1$$

$$y_1 = \begin{bmatrix} 1 & \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}.$$

System 2:

$$\frac{d}{dt}\begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -10 & \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_2 .$$

$$y_2 = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}$$

Equations of interconnection (3)-(5) are

$$v_1 = y_2$$
$$v_2 = y_1$$
$$y = y_2$$

Modified state equations of dynamics:

$$\frac{d}{dt}\begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} & 1 & & & b & & & & \\ -100 & & & & a & & & & \\ & & -2 & 1 & & 1 & & & \\ & & -10 & & & 1 & & & \\ -1 & & & & & & 1 & & \\ & & -1 & -2 & & & & 1 & \\ & & & & 1 & & & -1 & \\ & & & & & 1 & -1 & & \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \\ v_1 \\ v_2 \\ y_1 \\ y_2 \\ v \end{bmatrix}$$

The variable need not be considered because of empty last column (external input source v does not exist). After elimination of redundant variables we obtain resulting state equations of dynamics:

$$\frac{d}{dt}\begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \end{bmatrix} = \begin{bmatrix} & 1 & b & 2b \\ -100 & & a & 2a \\ 1 & & -2 & 1 \\ 1 & & -10 & \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{21} \\ x_{22} \end{bmatrix}.$$

The eigenvalues of state matrix depend on the constants a and b. It should be noted that the resulting system can be unstable due to the feedback path from the output of second system to the input of first one.

Output equations are in the simple form

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}.$$

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