Identification of Unstable dc Operating Points

Dalibor Biolek

Dept. of EE, VA Brno, Kounicova 65, PS 13, 612 00 Brno, Czech Republic
tel: (+420) 5 4118 2487, fax: (+420) 5 4118 2888, email: biolek@cs.vabo.cz

Abstract *– The Green's "Γ-test" for identifying of some unstable DC operating points of nonlinear resistive networks is analyzed. We show that some of unstable operating points cannot be identified using this procedure. Described method is equivalent to the sign testing of certain determinant of the DC circuit linearized around its operating point. At the end, we indicate more general method based on the necessary condition of the DC operating point (in)stability.*

I. INTRODUCTION

In [1], the definition of so-called potential stability of DC operating point is introduced. On the basis of this definition, the algorithm of identifying of the DC operating point instability is proposed. We summarize main steps of this algorithm, avoiding some details which are not necessary for next considerations:

1. The nonlinear circuit is linearized around investigated operating point and the linear resistive circuit with controlled sources is obtained.
2. Two vectors are defined:
   $\mathbf{x}_d$ voltages across the output branches of current sources and across the input branches of voltage-controlled sources;
   currents through the voltage sources and through the input branches of current-controlled sources;
   $\mathbf{y}_d$ currents through the current sources and to the input branches of voltage-controlled sources;
   voltages across the voltage sources and across the input branches of current-controlled sources.
3. Then two equations can be compiled:

$$\mathbf{Q}_A\mathbf{y}_d + \mathbf{x}_d = 0 \tag{1}$$

$$\mathbf{y}_d = \mathbf{A}_A\mathbf{x}_d \tag{2}$$

(simplification of Eq. (4) and (5) in [1] in case of resistive circuit).

The matrix $\mathbf{Q}_A$ can consist only of the resistances/conductances in the circuit but not of the parameters of controlled sources. Its compilation is not easy in some cases.

The matrix $\mathbf{A}_A$ consists only of the parameters of controlled sources and its compilation is trivial.

4. The constant Γ is computed:

$$\Gamma = \det\left(\mathbf{Q}_A\mathbf{A}_A + \mathbf{I}\right) \tag{3}$$

where $\mathbf{I}$ is the identity matrix of the same order as the order of matrix $\mathbf{Q}_A$ $\mathbf{A}_A$. If $\Gamma < 0$, the corresponding operating point is unstable. If $\Gamma > 0$, we are not able to decide about the stability using this method.

We show that the constant Γ corresponds with the determinant of normalized set of circuit matrix and therefore it can be computed more efficiently using the modified nodal approach (MNA) or other well-known methods. Then we remind of some examples when unstable operating points cannot be identified using the testing of constant Γ. Finally, the necessary and sufficient condition of operating point instability will be indicated.

II. CONSTANT Γ AND DETERMINANTS OF CIRCUIT GRAPHS AND MATRICES

Equations (1) and (2) can be expressed by following Mason's signal flow graph (SFG) on Fig.1.

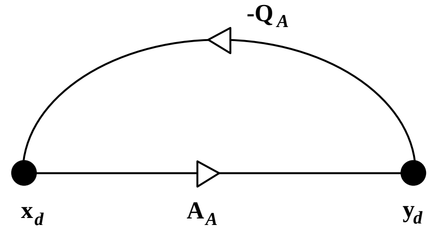

Fig.1. Signal flow graph corresponded to the equations (1) and (2).

Applying Mason's rule for computing of the graph determinant we obtain equation (3) for the constant Γ.

For the modeling of concrete resistive circuits with controlled sources, the MNA is frequently used or corresponding modified Mason-Coates SFGs. Different types of circuit equations lead to the different graph and matrix determinants which differ only by some multiplying constant. We will see on following example that this constant is the gain product of self-loops in the Mason-Coates graph or the product of the diagonal elements of the circuit matrix.

<u>Example 1:</u> Consider linearized circuit in Fig.2 with voltage-controlled voltage source. We investigate conditions of the instability of corresponding operating point using Green's method.

We choose x and y variables in accordance with step 2 (see Fig.2 (b)) and compile equations (1) and (2):

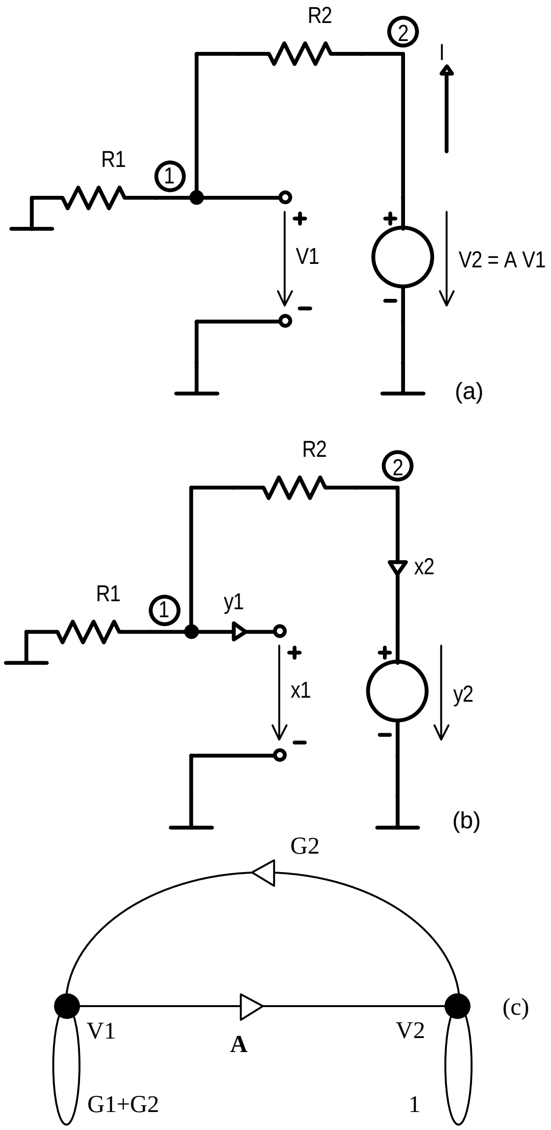

Fig.2. (a) Simple investigated circuit, (b) definition of circuit variables for the Green's test, (c) Mason-Coates SFG.

$$\underbrace{\begin{bmatrix} \dfrac{R_1R_2}{R_1+R_2} & -\dfrac{R_1}{R_1+R_2} \\ \dfrac{R_1}{R_1+R_2} & \dfrac{1}{R_1+R_2} \end{bmatrix}}_{\mathbf{Q}_A} \underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathbf{y}_d} + \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}_d} = \mathbf{0}$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\mathbf{y}_d} = \underbrace{\begin{bmatrix} & \\ A & \end{bmatrix}}_{\mathbf{A}_A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}_d}$$

Then

$$\Gamma = \det(\mathbf{Q}_A\mathbf{A}_A + \mathbf{I}) = 1 - A\frac{R_1}{R_1+R_2} \qquad (4)$$

Operating point is unstable if

$$\Gamma < 0 \Rightarrow A > 1 + \frac{R_2}{R_1} \qquad (5)$$

For $\Gamma > 0 \Rightarrow A < 1 + R_2/R_1$ and we cannot decide about the stability using Green's method.

In Fig.2 (c), the Mason-Coates SFG of analyzed circuit is drawn. The graph determinant Δ is

$$\Delta = G_1 + G_2 - AG_2 = (G_1+G_2)\left(1 - A\frac{R_1}{R_1+R_2}\right) = (G_1+G_2)\Gamma$$

We can see that the determinant is divided to two parts - the gain product of self-loops and the constant Γ. As results from the connections between the graph and matrix description of circuits, the self-loop gains correspond with the diagonal elements of the circuit matrix. For our example, proper matrix description corresponding to the graph is as follows:

$$\underbrace{\begin{bmatrix} G_1+G_2 & -G_2 \\ -A & 1 \end{bmatrix}}_{\mathbf{G}^M} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{0}$$

where V_1 and V_2 are nodal voltages of nodes 1 and 2, respectively, and $\mathbf{G}^M$ is the modified conductance matrix.

It should be noted that if all diagonal elements of the circuit matrix are positive, then instead of testing of the sign of Γ we can test sign of the matrix determinant:

$$\Gamma < 0 \Leftrightarrow \det(\mathbf{G}^M) = G_1 + G_2 - AG_2 < 0 \qquad (6)$$

This result corresponds with (5).

The MNA often leads to the situation when some of diagonal elements are negative or zero. Then the simple linear operations (multiplication of proper row by -1, interchange of two rows or two columns) can be performed to reach demanded sign and value of these elements. Let us name such arranged matrix as **ordered** matrix. For example, classical MNA leads to the matrix equation of circuit in Fig.2 (a)

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} G_1+G_2 & -G_2 & \\ -G_2 & G_2 & 1 \\ -A & 1 & \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I \end{bmatrix} = \mathbf{0}$$

where I is the current flowing to the node 2 from the output of controlled source (see Fig. 2(a)). The modified conductance matrix is not ordered because of zero element g_{33}. The ordered forms are for example

$$\begin{array}{c} 1 \\ 3 \\ 2 \end{array} \begin{bmatrix} G_1+G_2 & -G_2 & \\ -A & 1 & \\ -G_2 & G_2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I \end{bmatrix} = \mathbf{0} \quad \text{or}$$

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} G_1+G_2 & & -G_2 \\ -G_2 & 1 & G_2 \\ -A & & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ I \\ V_2 \end{bmatrix} = \mathbf{0}$$

In both cases, the testing of the sign of the determinant gives the same results as in (6).

Aforementioned conclusions indicate the possibility to test sign of the determinant of the easy compiled ordered MNA matrix instead of whole Green's procedure.

However, the condition $\Gamma < 0$ or $\det\left(\mathbf{G}^M\right) < 0$ is only sufficient but not necessary condition of operating point instability. This known fact will be illustrated on the examples of cascade and multiple feedback circuits.

III. Sufficient or Necessary Condition of Operating Point Instability?

First consider linear resistive circuit with single feedback loop. Let us sign the loop gain as K_L. It is known that this single-feedback circuit is stable if $K_L < 1$ and unstable if $K_L > 1$. However, it is evident from previous text that

$$\Gamma = 1 - K_L \tag{7}$$

Therefore,

the single-feedback circuit is stable if $\Gamma > 0$ and unstable if $\Gamma < 0$. Thus, $\Gamma < 0$ is both sufficient and necessary condition of operating point instability.

For this reason, (5) is both sufficient and necessary condition of instability of the operating point corresponded to the circuit in Fig.2 (a) because it has only one feedback loop.

However, the situation is more complicated in case of manifold feedback. According to the Mason's rule, the constant Γ is derived using the loop gains of Mason's graph:

$$\Gamma = 1 - \sum_m P_m^1 + \sum_m P_m^2 - \sum_m P_m^3 + \cdots \tag{8}$$

where P_m^r is the gain product of the mth possible combination of r nontouching loops.

It is evident that $\Gamma < 0$ means the instability due to total unstable positive feedback. However, $\Gamma > 0$ need not necessary mean stability because some of internal feedback loops can cause unstable behavior in spite of positive „global“ factor Γ. We illustrate it on the example of cascade circuit.

Example 2: Let us do cascade connection of two simple circuits from Example 1. In reality, the operating point corresponding to the resulting circuit is potentially stable in case of stability of both blocks. With utilization of (5), the necessary and sufficient instability conditions are

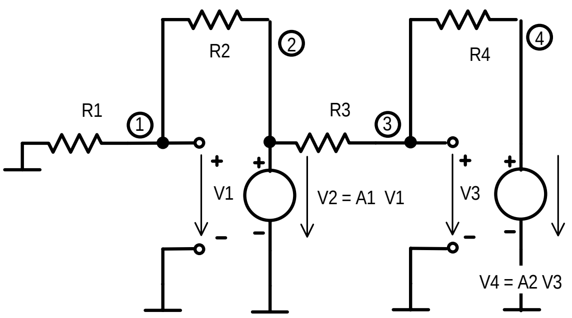

Fig.3. Cascade connection of two similar circuits.

$$A_1 > 1 + \frac{R_2}{R_1} \text{ or } A_2 > 1 + \frac{R_4}{R_3} \tag{9}$$

However, Green's approach leads to the different result:

$$\Gamma = \left(1 - A_1 \frac{R_1}{R_1 + R_2}\right)\left(1 - A_2 \frac{R_3}{R_3 + R_4}\right)$$

Sufficient instability conditions:

$$\Gamma < 0 \Rightarrow \quad A_1 > 1 + \frac{R_2}{R_1} \quad \text{and} \quad A_2 < 1 + \frac{R_4}{R_3}$$

or

$$A_1 < 1 + \frac{R_2}{R_1} \quad \text{and} \quad A_2 > 1 + \frac{R_4}{R_3}$$

Conditions mentioned above mean that the unstable operating point is identified only if just one and only one block in the cascade is unstable.

The case $\Gamma > 0$ means that we cannot decide about the stability. This case includes two possible conditions:

$$\Gamma > 0 \Rightarrow \quad A_1 < 1 + \frac{R_2}{R_1} \quad \text{and} \quad A_2 < 1 + \frac{R_4}{R_3}$$

or

$$A_1 > 1 + \frac{R_2}{R_1} \quad \text{and} \quad A_2 > 1 + \frac{R_4}{R_3}$$

In other words, we cannot decide about stability using Green's method if both blocks have either stable or unstable behavior.

It is evident that the testing of the sign of Γ is insufficient in the circuits with multiple feedback. For cascade circuit from the Example 2, it is necessary to test each feedback loop separately. However, in many circuits there exist influences among more feedback loops and it is not evident how to evaluate these influences. This fact is illustrated on following example.

Example 3: Circuit on Fig.4 has two feedback loops. There is question how to choose amplifications A_1 and A_2 to ensure stable operating point.

Computation of the constant Γ leads to the sufficient condition of instability:

$$A_2\left(A_1 + 2\right) > 4 \tag{10}$$

This result is in conformity with the physical experience, because both A_1 and A_2 must be sufficiently large to perform instability due to positive feedback. However, operating point is also unstable for other combinations of A_1 and A_2 which do not fulfill condition (10), for example $A_1 = -3$, $A_2 = 5$. In reality, condition (10) has to be completed by the second one

$$A_1 + 2A_2 > 6 \tag{11}$$

which cannot be obtained using Green's method. Then (10) and (11) are the necessary and sufficient conditions of the operating point instability.

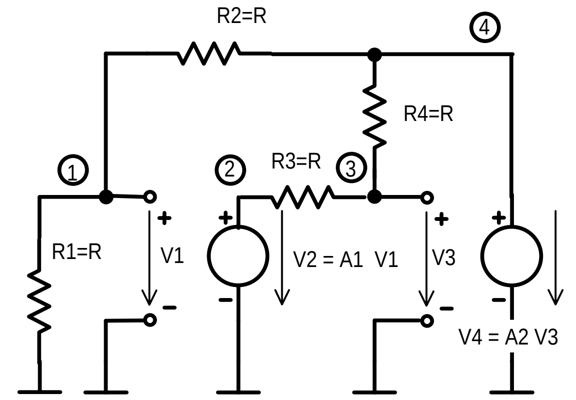

Fig.4. Multiple-feedback structure.

Relation (11) can be compiled using our method based on the so-called virtual dynamics of DC circuits. This method has been described in [2]. In following chapter, we only describe resulting algorithm how to find necessary and sufficient conditions of the operating point (in)stability.

IV. Our Results

Our algorithm of stability testing is as follows (for more details, see [2]):

1. Each dependent voltage (current) source is completed by so-called virtual voltage (current) source δV_A (δI_A) in series (in parallel) with this source. The circuit reaction is in the form of virtual current δI_R through the virtual voltage source (virtual voltage δV_R across the virtual current source). The source orientation of these reactions is considered.
2. The virtual voltages (currents) create the vector of virtual voltages $\delta\mathbf{V} = [\delta\mathbf{V}_A \;\; \delta\mathbf{V}_R]^T$ (vector of virtual currents $\delta\mathbf{I} = [\delta\mathbf{I}_A \;\; \delta\mathbf{I}_R]^T$). So-called *equation of virtual shiftings* is compiled using the MNA:

$$\delta\mathbf{V} = \mathbf{R}\delta\mathbf{I} \text{ or } \delta\mathbf{I} = \mathbf{G}\delta\mathbf{V} \qquad (12)$$

   where **R** (**G**) is the regular matrix of virtual resistances (conductances).
3. Eigenvalues λ_R (or λ_G) of the matrix **R** (or **G**) computation. Decision about the stability in accordance with following theorem:

Theorem (Necessary and sufficient condition of potential stability of the operating point):
All virtual eigenvalues λ_R (or λ_G) have to lie on the right-half complex plane.

Let us use our method to the circuit on Fig. 4 from Example 3. Fig. 5 illustrates insertion of virtual voltage sources δV_{A1} and δV_{A2} and the circuit reactions δI_{R1} and δI_{R2}. Equations of virtual shiftings are then

$$\begin{bmatrix}\delta V_{A1}\\ \delta V_{A2}\end{bmatrix} = \underbrace{\begin{bmatrix}(4-A_1)R & (2-A_1)R\\ (2-3A_2)R & 2(1-A_2)R\end{bmatrix}}_{\mathbf{R}}\begin{bmatrix}\delta I_{R1}\\ \delta I_{R2}\end{bmatrix}$$

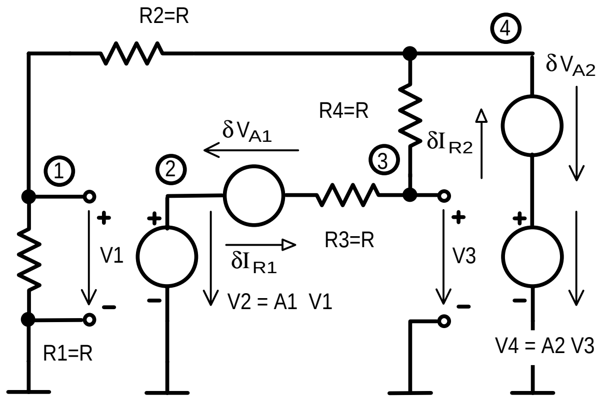

Fig.5. Virtual sources inserted to the circuit in Fig. 4.

Equation for virtual eigenvalues (type λ_R):

$$\lambda_R^2 + (A_1 + 2A_2 - 6)R\lambda_R + (4 - 2A_2 - A_1A_2)R^2 = 0$$

The operating point will be unstable if and only if at least one eigenvalue will lie on the left-side complex plain. Thus, the necessary and sufficient condition of the operating point instability is

$$A_1 + 2A_2 - 6 > 0 \text{ and } 4 - 2A_2 - A_1A_2 < 0$$

After simple arrangement we obtain conditions (10) and (11). Recall that condition (11) cannot be found using the Green's algorithm.

V. Conclusion

1. Green's method of identifying of the stability of the DC operating point is based only on the sufficient but not necessary condition of instability. As a result, some of unstable operating points cannot be identified by this procedure.
2. Stability test using the constant Γ is equivalent to the test using the determinant of so-called ordered circuit matrix. This enables to compute constant Γ in more efficient way using the MNA approach.
3. Identifying of all unstable operating points is possible using the method of virtual dynamics of the DC circuits. The stability test is performed using the testing of the location of so-called virtual eigenvalues of DC circuit that are computed from the equations of virtual dynamics.

Acknowledgement: This work is supported by the Grant Agency of Czech Republic under project No. 102/97/0765.

References

[1] M.M.Green and A.N.Wilson, „How to Identify Unstable DC Operating Point," *IEEE Trans. Circuit Syst.*, Vol. CAS-39, pp. 820-832, October 1992.

[2] D.Biolek and Z.Biolek, "Stability of DC Operating Points and Corresponding Dynamical Systems", MS'97 Melbourne, pp. 236-240.