OPTIMIZATION OF FREQUENCY FILTERS VIA VERTEX GRAPHS

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ABSTRACT

A method of systematic investigation of the influence of element parameters on the desired properties of a linear dynamical system –frequency filter in particular – is described. The so-called coupling (or vertex) graphs are utilized. One of the results is the partition of a set of parameters into disjoint sets. Within these sets, it is possible to vary the parameters independently to obtain the required circuit behavior.

1. INTRODUCTION

Parameters of filter elements participate in filter properties by means of variously linked relations, which result from the filter circuit functions. The most frequently followed circuit functions are voltage/current gains and immittances.

We will focus on some questions that have been inspired by the following optimization tasks [1-5]:

- Some of the element parameters are changed (e.g. by replacement the element with a different type). The problem is to find a minimum number of other parameters, whose variation will fully compensate the effect of the original change on the monitored filter properties (full compensation), or to find other parameters, whose variation will compensate the effect of the original change in a defined way (partial compensation).

Except the apparent utilization of full or partial compensation, this principle is often applied to the optimization of element ratio of analogue filters [1]. As an example, consider a dual $R_1C_1R_2C_2$ cell of the lowpass type. We increase R_1 *a*-times and then we look for ways of compensating this change to keep the voltage transfer unchanged. The universal solution consists in impedance scaling: we increase/decrease/decrease $R_2/C_1/C_2$ *a*-times. However, if the partial R_1C_1 and R_2C_2 cells are designed or realized such that the second cell does not affect the first one, then we need only decrease C_1.

- It is necessary to find a minimum number of parameters whose proportional variation (see later) results in the required control of filter parameters.

A typical application is optimization of dynamic range of active filters [1-4]. As an example, let us have a multiple *OpAmp* active filter, where we have to change the voltage level at a certain output while preserving the voltages in remaining nodes.

In the paper, the couplings among the element parameters are examined by means of so-called vertex graph with the aim of algorithmic circuit optimization. The paper is organized as follows. In Section 2 we discuss an example of simple filter where the problem of coupling conditions among element parameters is demonstrated, including the description of these conditions by using coupling (vertex) graphs. In Sections 3 and 4 we then define these graphs and their utilization for algorithmic filter optimization. Finally we demonstrate the features of our approach on an example of *KHN* (Kerwin, Huelsman, and Newcomb) filter [1].

2. OPENING EXAMPLE

Consider the passive *RC* bandpass filter in Fig. 1. The transfer function is in the form

$$\frac{V_{out}}{V_{in}} = \frac{a_1 s}{1 + b_1 s + b_2 s^2} ,$$

where

$$\begin{aligned} a_1 &= R_2C_2 = 16\mu s , \\ b_1 &= R_1C_1 + R_2C_2 + R_1C_2 = 32.16\mu s , \\ b_2 &= R_1R_2C_1C_2 = 256 ps^2 . \end{aligned} \qquad (1)$$

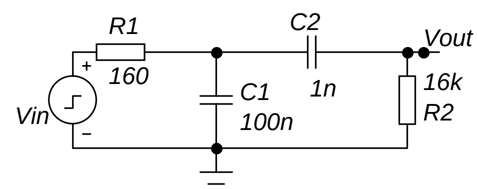

Figure 1. Example of a passive RC bandpass filter with f_0=10kHz.

Equations (1) represent coupling conditions among four circuit parameters R_1, R_2, C_1, and C_2. Varying these parameters to keep the coupling conditions, all the coefficients of the transfer function will remain unchanged. The coupling conditions can be diagrammatized by so-called coupling (vertex) graph [5] according to Fig. 2(a). The procedure of graph simplification is shown in the left-side part of Fig. 2 (subfigures a-d). Taking into account the numerical values of parameters and neglecting the R_1C_2 term in the expression of b_1 coefficient (an error of less than 0.5%) leads to the reduced vertex graph in Fig. 2(e). The right-side part of the figure (subfigures e-h) then depicts its successive simplification.

For instance, the product paths connecting the parameters R_2-C_2 with vertex a_1 represent a graph of the first equation in (1). In

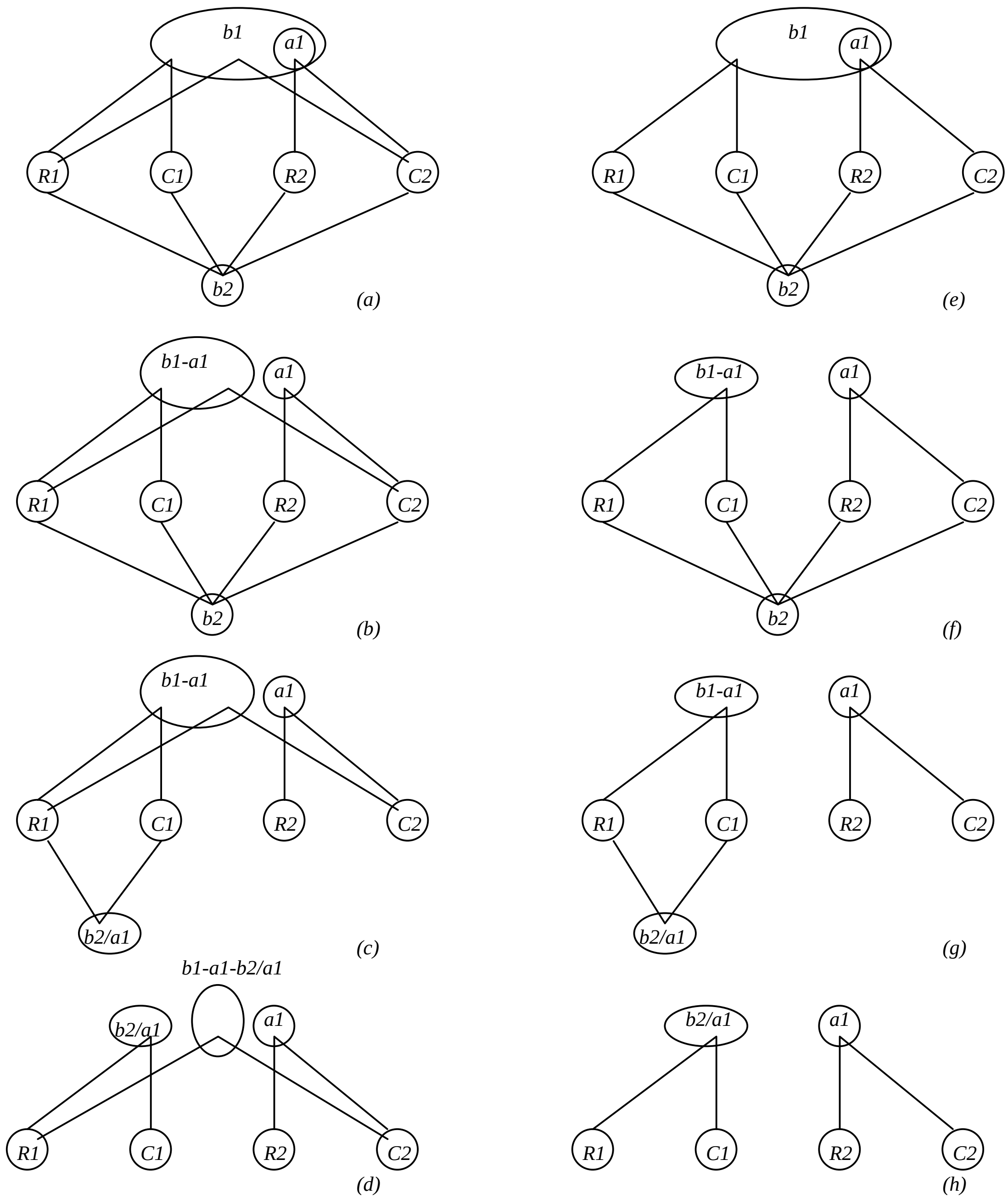

Figure 2. Coupling graph of the circuit in Fig. 1 and its successive simplification: (a)-(d) full, (e)-(h) reduced after neglecting „weak“ couplings. It holds: b1-a1=R1C1+R1C2 (full graph), or R1C1 (reduced); b2/a1=R1C1; b1-a1-b2/a1=R1C2 (full graph), or 0 (reduced).

Fig. 2(a), three vertex graphs are constructed which correspond to three equations in (1). Substituting the first equation into the second leads to a modification, where the summing vertex b_1 is partitioned (see Fig. (b)). Substituting the first equation into the third leads to the reduction of vertex b_2 to b_2/a_1 (see Fig. (c)). Thus the product R_1C_1 is defined, enabling splitting the summing vertex into two simple ones (Fig. (d)). The resulting graph does not contain any loops and all the parameters are connected by a continuous path. It means that if we change an arbitrary parameter, we have to change all the remaining parameters from the set [R_1 C_1 R_2 C_2] in order to preserve the transfer function. For example, the change described by the relation $R_1 \to a\ R_1$, where a is an arbitrary positive real number, has to be accompanied by the changes $C_1 \to C_1/a$, $R_2 \to a\ R_2$, $C_2 \to C_2/a$ (the so-called proportional variation). It is the well-known impedance scaling. The parameter variation is characterized by one degree of freedom.

It follows from the reduced graph and its successive simplification that the circuit could (under some conditions) be optimized in such a way as if it consisted of two *RC* cells that do not affect each other: It suffices to hold both time constants R_1C_1 and R_2C_2. The resulting graph is decomposed into two disjoint subgraphs, corresponding to two independent coupling conditions. Parameters can now be varied with two degrees of freedom.

3. COUPLING (VERTEX) GRAPH

Let circuit functions depend on *m* parameters $P=[p_1, p_2,.., p_m]$. These parameters are coupled by *n* coupling conditions of the type of

$$F^{j}(P)=f^{j},\ j=1, 2, .., n, \tag{2}$$

where f^{j} are constants (for example, transfer function coefficients).

Function *F* has a special form when analyzing linear circuits symbolically: it is an algebraic sum of *N* products Π_k (k=1,2,..,*N*) of *M* parameters, where *N* and *M* are natural numbers. Some products can enter the sum with negative sign, depending on the type of circuit solved:

$$F^{j}=\sum_{k=1}^{N}\left(\pm\Pi_k^{j}\right)=f^{j},\ j=1, 2, .., n. \tag{3}$$

The coupling graph in Fig. 3 corresponds to Equation (3).

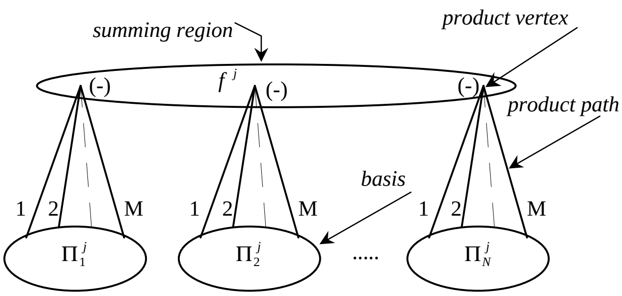

Figure 3. Definition of the coupling (vertex) graph.

The so-called *product paths* start from the individual M parameters and they converge at a point which we call *product vertex*, or simply *vertex*. *M* parameters form so-called *basis*. From the mathematical point of view, the vertex represents the product of parameters in the basis. In practice, the individual bases can contain some shared parameters.

In a simple case, coupling condition (3) is formed only by one product. Then we speak of the graph with *single vertex* (or the *single graph*). If summing more products, we have a graph with *multiple vertex* (or *multiple graph*), which is sunk into the so-called *summing region*. From the mathematical point of view, the summing region is represented by formula (3). If some products enter the sum with negative sign, then it is labeled near the corresponding vertex (see Fig. 3).

The total coupling graph is a synthesis of individual graphs of all the coupling conditions. Evaluating the graph, we can obtain important information for optimization, for instance, which parameters can be modified independently while preserving the target functions.

The fundamental contrast between the vertex graphs and both the signal-flow-graph technique first introduced by Mason [6] and Coates' flow graphs [7] should be noted. The last-mentioned graphs are graphical methods of solving linear algebraic equations, whereas the vertex graphs have been designed for solving nonlinear equations of type (3).

4. ALGORITHMIC UTILIZATION OF VERTEX GRAPHS

The opening example illustrated one of the possibilities of utilizing vertex graphs for subsequent circuit optimization. An algorithmic utilization of these graphs for computer optimization can be performed in the following steps:

- Algorithmic generation of required circuit functions in the symbolic form.
- Definition of coupling conditions according to the optimization objective.
- Interpretation of the vertex graph in the data structure.
- Algorithmic simplification of the vertex graph to so-called minimum form.
- Utilization of the minimum graph for final optimization.

A detailed description of individual stages, especially No's. 3 and 4, is beyond the scope of this paper.

Next we will show the utilization in a nutshell on the example of a 2nd-order active filter.

5. EXAMPLE OF UTILIZATION

For the OPA1/OPA2/OPA3 outputs, the circuit in Fig. 4 behaves as high/band/low-pass filter. The symbolic analysis using the *SNAP* program [8-10] offers the following results:

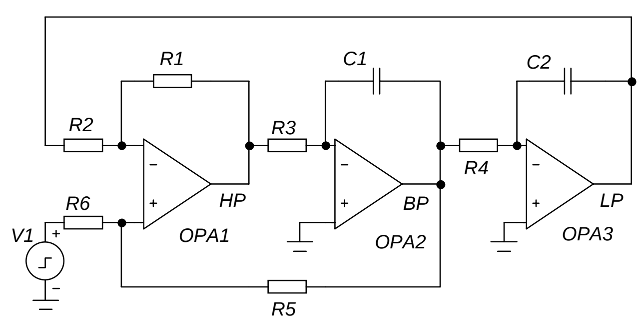

Figure 4. Active KHN filter [1].

The denominator of the corresponding transfer functions is in the form of

$$b_2s^2+b_1s+b_0,$$

where

$$b_2 = R_2R_3R_4C_1C_2\left(R_5 + R_6\right),$$
$$b_1 = R_4R_6C_2\left(R_1 + R_2\right), \qquad (4)$$
$$b_0 = R_1\left(R_5 + R_6\right).$$

The numerators for HP/BP/LP are

a_0 , $-a_1 s$, $a_2 s^2$,

where

$$a_0 = R_5\left(R_1 + R_2\right),$$
$$a_1 = R_4R_5C_2\left(R_1 + R_2\right), \qquad (5)$$
$$a_2 = R_3R_4R_5C_1C_2\left(R_1 + R_2\right).$$

The objective is to find disjoint sets of element parameters whose proportional variations will have no influence on the above transfer functions.

The vertex graph constructed from equations (4) and (5) is rather complicated and not very clear. Applying simplification rules (which cannot be described here) yields a transformation into the graph in Fig. 5(a). It corresponds to the modified equations (4) and (5):

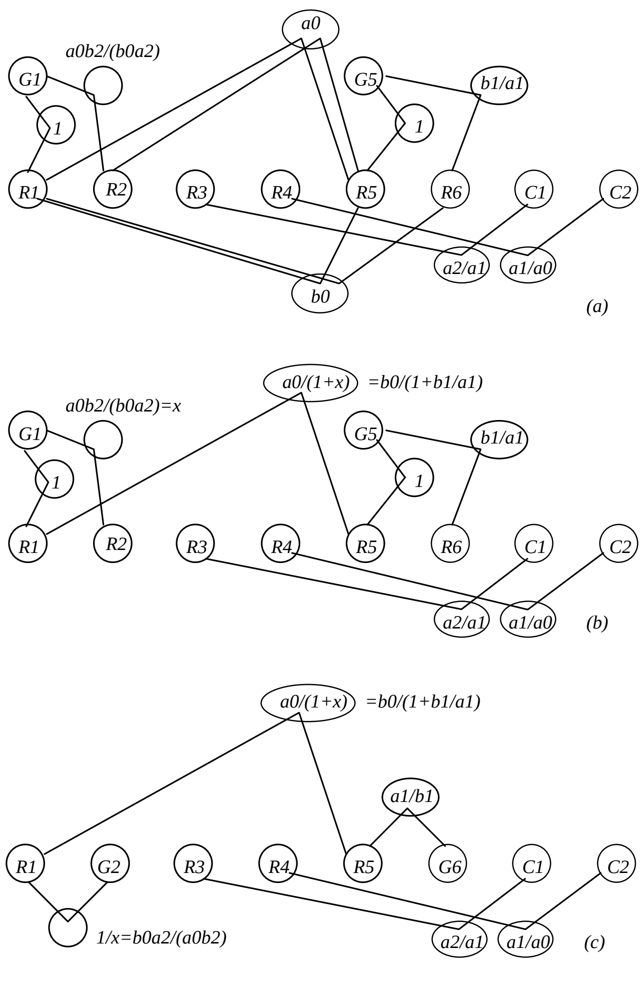

Figure 5. Vertex graph of the active filter in Fig. 4, (a) initial, (b) modified, (c) minimum.

$$a_0 = R_5\left(R_1 + R_2\right),\ a_1 / a_0 = R_4C_2\,, a_2 / a_1 = R_3C_1\,,$$
$$b_0 = R_1\left(R_5 + R_6\right),\ b_1 / a_1 = R_6G_5\,, \qquad (6)$$
$$a_0b_2 /(a_2b_0) = R_2G_1$$

Parameters G_1 and G_5 appear in these equations. They can be incorporated in the graph by using the equalities R_1G_1=1 and R_5G_5=1. Another graph modification will reduce double vertices a_0 and b_0 (see the graph in Fig. 5(b)). After the final transformation of graph basis, the minimum graph in Fig. 5(c) consists of three disjoint parts. We have found three desired subsets of parameters: [R_1 R_2 R_5 R_6], [R_3 C_1], [R_4 C_2].

6. SUMMARY

The method described enables the design of algorithms of finding mutual couplings between the element parameters of a complicated linear dynamic system. These couplings are defined by the features of required circuit functions. Typical examples of application are the optimization of element ratios and of the dynamic range of filters, and the compensation of the influence of undesirable changes in element parameters on the total circuit performance.

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