S-Z SEMI-SYMBOLIC SIMULATION OF SWITCHED NETWORKS

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Abstract - The algorithm of all *z*- and *s*- domain poles/zeros finding of real switched network with external switching is described in this contribution. This mixed *s-z* analysis enables realistic simulation of network behavior both in the classical and non-standard regimes as oversampling.

INTRODUCTION

Computer simulation of switched networks is not satisfactorily solved so far. For example, well-known *SPICE*-family network simulators do not provide frequency analysis of these systems.

Many special programs for both ideal and real switched capacitor networks have been developed [1], [2], [3], [4], [5], [8]. However, these programs often have some disagreeable limitations. In most cases, the frequency analysis provides periodical frequency responses regardless of the physical reality. Both the simple consideration and laboratory experiments lead to the conclusion that frequency responses of some real switched networks cannot be periodical on principle. The reason is in the inertia of the input branch which causes that the time-aliasing effect cannot exert its influence in full degree.

To describe correctly frequency-domain behavior of switched network over the Nyquist's frequency, the network simulator has to utilize special definition of transfer functions. The basis idea is concentrated to the so-called **generalized transfer function** (*GTF*) which is the function of both classical operators *s* and *z* [10-16]. The *GTF* represents some synthesis of well-known *s*- and *z*-domain transfer functions of continuous-time and discrete-time systems. This function describes transmission properties of switched network in a more complex way than the simple system function $K(z)$ which is restricted to the description of only discrete-time part of behavior. Main ideas concerning the *GTF* have been described in [13].

There is outstanding for computer simulation that the *GTF* can be obtained in semi-symbolic form. Frequency response can be computed after double substitution $s = j\omega$ and $z = exp(j\omega T)$ where T is the switching period. However, the algorithm of this mixed semi-symbolic analysis is not simple numerical task.

PLAN OF SOLUTION

All following considerations will be concerned to the switched networks with the switching diagram in Fig.1. Computer simulation of these networks is performed in following steps:

- *Compilation of circuit equations.*
- *Time-frequency preprocessing of circuit equations.*
- *Semi-symbolic computation of generalized transfer functions.*
- *Frequency analysis.*

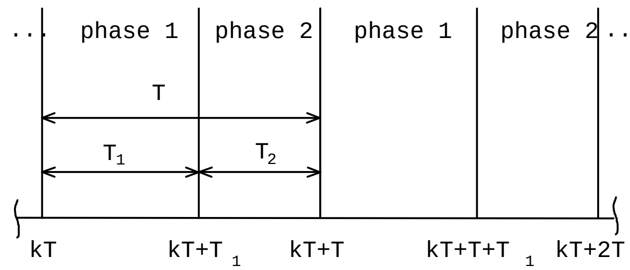

Fig.1. The switching diagram.

Aforementioned steps have been briefly described in [13] and [16]. It is necessary to summarize basic ideas.

The *two-graph modified nodal approach* (*MNA*) [7] has been used as the basic method. To create minimum number of equations, we find the set of independent nodal variables in each switching phase:

$$\mathbf{v}_1 = \left[v_1^1, v_1^2, .., v_1^{n_1}\right]^T \text{... phase 1,} \qquad (1)$$
$$\mathbf{v}_2 = \left[v_2^1, v_2^2, .., v_2^{n_2}\right]^T \text{... phase 2,}$$

where n_1 and n_2 are numbers of independent variables in phase 1 and phase 2, respectively. The algorithm how to obtain these sets is described in [9].

The set of differential equations of *MNA* in the operator domain for independent variables is compiled:

$$\underbrace{\left(\mathbf{G}_1^M + s\mathbf{C}_1^M\right)}_{\mathbf{Y}_1^M}\mathbf{V}_1 = \mathbf{D}_1 W_1 + \mathbf{C}_{12}^M \mathbf{v}_2(kT^-), \qquad (2)$$
$$\underbrace{\left(\mathbf{G}_2^M + s\mathbf{C}_2^M\right)}_{\mathbf{Y}_2^M}\mathbf{V}_2 = \mathbf{D}_2 W_2 + \mathbf{C}_{21}^M \mathbf{v}_1(kT + T_1^-),$$

where W_1 and W_2 are Laplace transformations of input signals in phases **1** and **2**,
T_1 is length of phase **1**,
matrices $\mathbf{Y}_1^M$, $\mathbf{Y}_2^M$, $\mathbf{C}_{12}^M$ and $\mathbf{C}_{21}^M$ and vectors $\mathbf{D}_1$ and $\mathbf{D}_2$ can be compiled using the algorithm described in [11] and [16].

Circuit equations (2) must be pre-processed with the aim to derive equations for *GTF*s in closed form. The general solution is as follows [16]

$$\mathbf{V}_1^e(s,z) = \mathbf{K}_{11}(s,z)W_1^e(s) + \mathbf{K}_{12}(s,z)W_2^e(s), \qquad (3)$$
$$\mathbf{V}_2^e(s,z) = \mathbf{K}_{21}(s,z)W_1^e(s) + \mathbf{K}_{22}(s,z)W_2^e(s).$$

The superscript ^{e} denotes so-called equivalent signal [6], [13]. The generalized transfer functions $\mathbf{K}_{11}$, $\mathbf{K}_{12}$, $\mathbf{K}_{21}$ and $\mathbf{K}_{22}$ depend on the input signal character (*Sample-Hold* response, arbitrary response...). For more details, see [13].

The frequency responses can be easily obtained by double substitution $s = j\omega$ and $z = exp(j\omega T)$.

SEMI-SYMBOLIC ANALYSIS OF GTFS

Semi-symbolic analysis is based on following equations for *GTF*s [16]

$$\begin{aligned}
\mathbf{K}_{11}(s,z) &= \left[\mathbf{E} - \mathbf{F}_1 z^{-1}\right]^{-1}\mathbf{G}_1(s,z),\\
\mathbf{K}_{12}(s,z) &= \left[\mathbf{E} - \mathbf{F}_1 z^{-1}\right]^{-1}\mathbf{g}_1^*(T_1)\mathbf{G}_2(s,z)z^{-\frac{T_1}{T}},\\
\mathbf{K}_{21}(s,z) &= \left[\mathbf{E} - \mathbf{F}_2 z^{-1}\right]^{-1}\mathbf{g}_2^*(T_2)\mathbf{G}_1(s,z)z^{-\frac{T_2}{T}},\\
\mathbf{K}_{22}(s,z) &= \left[\mathbf{E} - \mathbf{F}_2 z^{-1}\right]^{-1}\mathbf{G}_2(s,z),
\end{aligned} \tag{4}$$

where

$$\mathbf{F}_1 = \mathbf{g}_1^*(T_1)\mathbf{g}_2^*(T_2), \mathbf{F}_2 = \mathbf{g}_2^*(T_2)\mathbf{g}_1^*(T_1), \tag{5}$$

$$\mathbf{G}_i(s,z) = \int_0^{T_i}\mathbf{g}_i(\xi)e^{-s\xi}d\xi, i = 1,2, \tag{6}$$

$$z = e^{sT}. \tag{7}$$

The unity matrix is designated as $\mathbf{E}$. Matrices $\mathbf{g}^*$ and vectors $\mathbf{g}$ of impulse responses can be computed numerically from equation (2) using numerical Laplace inversion [16-18].

It should be noted that transmission properties of switched system are described using functions of both classical operators s and z. In such way, both continuous- and discrete-time character of behavior of switched system are described simultaneously. Aforementioned double substitution yields frequency responses which will not be periodical as in case of well-known discrete-time systems owing to the integrals $\mathbf{G}$.

For the *DC* transmission, s = 0 and equation (6) yields

$$\mathbf{G}_i = \int_0^{T_i}\mathbf{g}_i(\xi)d\xi = \mathbf{h}_i(T_i), \quad i = 1,2 \tag{8}$$

This is the vector of step response at time instance T_i for phase i. For nonzero frequency we obtain special function of both operators s and z. Let us investigate this case.

Vector $\mathbf{g}_i$ denotes vector of impulse response in phase i. During this phase, switched network is stationary system described by one of two equations (2). Impulse response is defined as zero-state response. We can write on the assumption that exciting signal is Dirac impulse:

$$\mathbf{Y}_i\mathbf{V}_i = \mathbf{D}_i, \ i = 1,2, \tag{9}$$

where $\mathbf{V}_i$ is now vector of impulse response in the operator form. In other words, $\mathbf{V}_i$ is the vector of the transfer function of switched network in phase i:

$$\mathbf{V}_i = \mathbf{K}_i, \ i = 1,2. \tag{10}$$

Each element of the vector $\mathbf{K}_i$ is rational fractional function of the operator s. Coefficients of this function can be found, applying *QR* algorithm to the equation (9).

Let us derive integral (6) using aforementioned transfer function. Impulse response $\mathbf{g}_i$ can be calculated as contour integral of transfer function to perform inverse Laplace transformation:

$$\mathbf{g}_i(\xi) = \frac{1}{2\pi j}\oint_c K_i(p)e^{p\xi}dp, \ i = 1,2. \tag{11}$$

Contour c must be chosen suitably to obtain correct result.

Substitution (11) to (6) and some arrangements yield

$$\mathbf{G}_i(s,z) = \frac{1}{2\pi j}\int_0^{T_i}\oint_c \mathbf{K}_i(p)e^{p\xi}e^{-s\xi}dp\,d\xi =$$

$$= \frac{1}{2\pi j}\oint_c \mathbf{K}_i(p)\int_0^{T_i}e^{(p-s)\xi}d\xi\,dp = \frac{1}{2\pi j}\oint_c \mathbf{K}_i(p)\frac{e^{(p-s)T_i}-1}{p-s}dp, i = 1,2$$

Consider that m-th element of $\mathbf{K}_i$(p) is a rational fractional function in the form

$$\frac{A_i(p)}{B_i(p)} = \frac{a_0 + a_1 p + .. + a_{r_i}p^{r_i}}{b_0 + b_1 p + .. + b_{r_i}p^{r_i}}. \tag{12}$$

The denominator is the same for all elements of vector $\mathbf{K}_i$(p) and its roots are poles of switched system in phase i. Number of poles is r_i. Then corresponding m-th element of $\mathbf{G}_i$ is

$$G_i = \frac{1}{2\pi j}\oint_c \frac{A_i(p)}{B_i(p)}\frac{e^{(p-s)T_i}-1}{p-s}dp, i = 1,2. \tag{13}$$

Resulting integral can be solved using residuum calculus. To simplify derivation, consider simple poles of system during phase i. Obtained results will be true also for multiple poles.

For simple poles, equation (12) can be arranged using partial fractional expansion:

$$\frac{A_i(p)}{B_i(p)} = \frac{a_0 + a_1 p + .. + a_{r_i}p^{r_i}}{b_0 + b_1 p + .. + b_{r_i}p^{r_i}} = \sum_{k=1}^{r_i}\frac{R_k}{p - p_k} \tag{14}$$

where p_k, $k = 1,2,..,r_i$ are poles and R_k corresponding residuum.

Now we choose contour c of integration so that all poles of $B_i(p)$ will be closed by this contour and the complex number s in (13) will be outside. Considering counterclockwise orientation of contour, equation (13) can be simplified:

$$\begin{aligned}
G_i &= \frac{1}{2\pi j}\oint_c \sum_{k=1}^{r_i}\frac{R_k}{p-p_k}\frac{e^{(p-s)T_i}-1}{p-s}dp = \sum_{k=1}^{r_i}R_k\frac{1-e^{(p_k-s)T_i}}{s-p_k} =\\
&= \frac{A_i(s)}{B_i(s)} - e^{-sT_i}\sum_{k=1}^{r_i}\frac{R_k}{s-p_k}e^{p_kT_i} = \frac{A_i(s) - z^{-T_i/T}C_i(s)}{B_i(s)}, i = 1,2.
\end{aligned} \tag{15}$$

Poles p_k and coefficients of polynomials A_i and B_i can be found using *QR* algorithm. Coefficients of polynomial C_i can be obtained using poles and R_k. In this way, we calculate G_i in semi-symbolic form.

SIMULATION EXAMPLES

Example 1

Let us consider simple *SC* lowpass filter in Fig.2. Nonzero switch-on resistances R_1 and R_2 in phases **1** and **2** will be taken into account. Our aim is to obtain input-output transfer function, considering input and output samples in phase **1**.

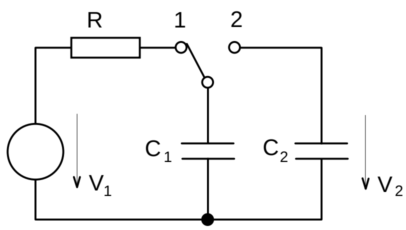

Fig. 2. Example of analyzed network: R = 1MΩ, C_1 = 1nF, C_2 = 10nF, T = 10μs, $T_1 = T_2$ = 5μs, $R_1 = R_2$ = 500Ω (switch-on resistances).

In ideal case $R = R_1 = R_2 = 0$ and this transfer function is completely described in the *z*-domain as

$$K(z)=\frac{C_1}{C_1+C_2}\,\frac{z^{-1}}{1-\dfrac{C_2}{C_1+C_2}z^{-1}} \tag{16}$$

and our system is modeled as discrete-time system of first order with periodical frequency response in Fig.3, line **1**.

For this ideal case, our algorithm gives result

$$K(s,z)=\frac{0{,}090909090909z^{-1}}{1-0{,}909090909091z^{-1}}\;.\underset{\displaystyle G_1(s,z)}{\underset{\uparrow}{1}}=K(z) \tag{17}$$

This result can be verified using (16), considering numerical values of capacitances.

Analysis of *GTF*s for real switched network with all resistances leads to the mixed *s-z* description:

$$K(s,z)=\frac{0{,}090907572573z^{-1}}{1-0{,}945724937524z^{-1}+6{,}728953601465.10^{-6}z^{-2}}\;\underset{\displaystyle G_1(s,z)}{\underset{\uparrow}{\frac{1-0{,}402890321529z^{-1/2}}{1+5{,}5.10^{-6}s}}} \tag{18}$$

We can see that resistors course both increasing of *z*-domain order to 2 and add one *s*-domain pole. Resulting frequency response is not already periodical - see Fig.3, line **2**.

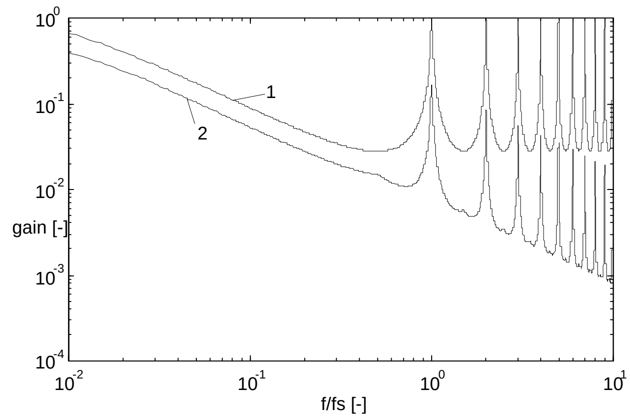

Fig.3. Frequency responses of network in Fig.2; **1** idealized (all resistances are zero), **2** real; f_s is the sampling frequency.

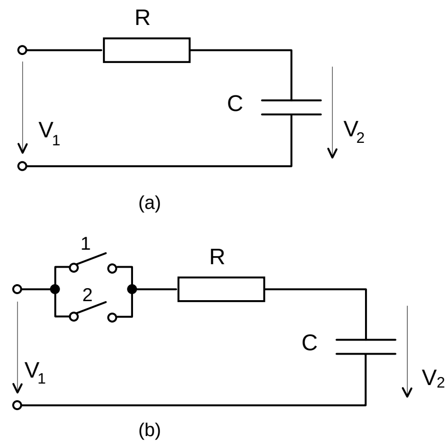

Fig.4. Analyzed unswitched network (a) can be considered as a special case of switched network (b); R = 1kΩ, C = 1μF, T = 1μs, $T_1 = T_2$ = 0,5μs.

Example 2

Following example demonstrates that our algorithm leads to correct result irrespective of the analyzed network is switched or continuous-time. In case of continuous-time network, resulting transfer function will comprise only operator *s*. In accordance with the superposition theorem, excitation of unswitched network can be considered as the superposition of nonoverlapping fragments of input signals in phases **1** and **2**. Transmission of these input fragments to the output is described mathematically using *GTF*s K_{11}, K_{12}, K_{21} and K_{22}, see equation (3). Physical interpretation of equivalent signal [6], [13] leads to the conclusion that classical transfer function of unswitched network can be calculated from partial *GTF*s as follows:

$$K(s)=K_{11}(s,z)+K_{12}(s,z)=K_{21}(s,z)+K_{22}(s,z) \tag{19}$$

Let us analyze simple network *RC* in Fig.4(a). Its *s*-domain transfer function is as follows:

$$K(s) = \frac{1}{1 + sRC} \tag{20}$$

Algorithm based on the *GTF*s requires switching period *T*. However, result (19) will not depend on its value. Switches in Fig. 4(b) operate eliminating one another. They model fragmentation of input signal mentioned above. Algorithm provides following solution:

$$\begin{aligned}
&K_{11}(s,z) = K_{22}(s,z) = \\
&= \frac{1}{1 - 0{,}999000499833z^{-1}} \frac{1 - 0{,}999500124979z^{-1/2}}{1 + 10^{-3}s} \\
&K_{12}(s,z) = K_{21}(s,z) = \\
&= \frac{0{,}999500124979z^{-1/2}}{1 - 0{,}999000499833z^{-1}} \frac{1 - 0{,}999500124979z^{-1/2}}{1 + 10^{-3}s} \\
&K_{11}(s,z) + K_{12}(s,z) = K_{21}(s,z) + K_{22}(s,z) = \\
&= \frac{1}{1 + 10^{-3}s} = K(s).
\end{aligned} \tag{21}$$

CONCLUSION

Novel algorithm of mixed *s*-*z* domain semi-symbolic analysis of general linear networks is presented. This algorithm is based on the theory of generalized transfer functions which includes both classical *s*-domain transfer function of continuous-time networks and *z*-domain ones of discrete-time networks. In case of real switched networks, resulting transfer function includes both operators to model mixed continuous- and discrete-time character simultaneously. In this way, presented algorithm can be used for efficient semi-symbolic analysis of classical or mixed-mode systems.

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