Optimization of Linear Systems using Symbolic Modelling

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Abstract

The optimization method of linear circuits discussed is based on the symbolic and semisymbolic analyses of their *s*-domain circuit functions. So-called vertex graph is generated from the symbolic formula. This graph is used as a starting point of modifying the parameters of circuit elements algorithmically to fulfill the optimization criterion. The procedure is explained on examples from the area of continuous-time frequency filters.

Introduction

The following typical tasks are often solved when designing and optimizing linear systems, especially frequency filters [1]:

- Identification of such concrete circuit elements whose parameters influence significantly the monitored properties of the system designed.
- Compensating real properties of elements used by the so-called passive method (i.e. by modifying the parameters of other elements), or by the active method (i.e. by modifying the circuit topology).
- During optimization, the parameters of circuit elements are divided into a set of fixed parameters (SFP) and a set of varying parameters (SVP). The parameters from SFP cannot be changed during the optimization. The optimization procedures often lead to the solution of the following typical problems:

- We significantly change a parameter belonging to SVP. Now we find a procedure how to modify the smallest number of the other parameters from SVP in such a way as to retain certain system properties (e.g. voltage transfer from given input to given output). This task can, but need not have a solution. One example is the replacement of a component by another with considerably different parameters. Without a subsequent compensation it would cause an undesirable modification of the system properties. The objective consists in the total compensation of a given phenomenon by modifying several other system components [2].
- We significantly change a parameter belonging to SVP. Now we find a procedure how to modify the smallest number of the other parameters from SVP in such a way as to retain certain system properties (e.g. frequency response in a given frequency range) within some defined region. Contrary to the foregoing case, it is a partial compensation, which can be performed every time.
- We find a procedure how to modify the smallest number of SVP parameters to preserve certain system parameters and to change other parameters in the desired manner. Note the dynamic range optimization of an active filter as a typical example: We try to modify the voltage level at a given OpAmp output and simultaneously to preserve the voltage levels at the other OpAmp outputs [3].

The problems mentioned above can be advantageously solved by means of algorithms of symbolic analysis of linear systems [4]. The symbolic analysis offers the *s*-domain system function in the rational fraction form. The coefficients of numerator/denominator polynomials are formed by the sums of products of symbols which label the parameters of system elements. The influence of individual parameters on the system function is then visible from the symbolic result. A substitution of numeric values of parameters yields the so-called semisymbolic result, when the individual coefficients of the system function are numbers. This result can be used to investigate the influence of individual parameters on the frequency response, circuit waveforms, zeros and poles of the system function, etc.

The fundamentals of some selected symbolic algorithms and their utilization for the solution of the above problems will be described in the paper. All the symbolic results have been obtained by the SNAP software [5] (Symbolic Network Analysis Program), which includes symbolic algorithms for the optimization.

By reason of problems indicated, we confine ourselves to the methods of full compensation. The approaches of partial compensation will be only outlined.

Opening examples

Consider the active lowpass second-order filter in Fig. 1. The voltage gain in symbolic form is as follows:

$$\frac{V_{out}}{V_{in}} = -\frac{1}{b_0 + b_1 s + b_2 s^2}, \text{ or } \frac{V_{out2}}{V_{in}} = -\frac{1 + a_1 s}{b_0 + b_1 s + b_2 s^2},$$

where

$$a_1 = R_2 C_2 = 250\mu s,\ b_0 = R_1 G_3 = 1,\ b_1 = R_1 C_1 = 10\mu s,\ b_2 = R_1 R_2 C_1 C_2 = 2.5 ns^2. \quad (1)$$

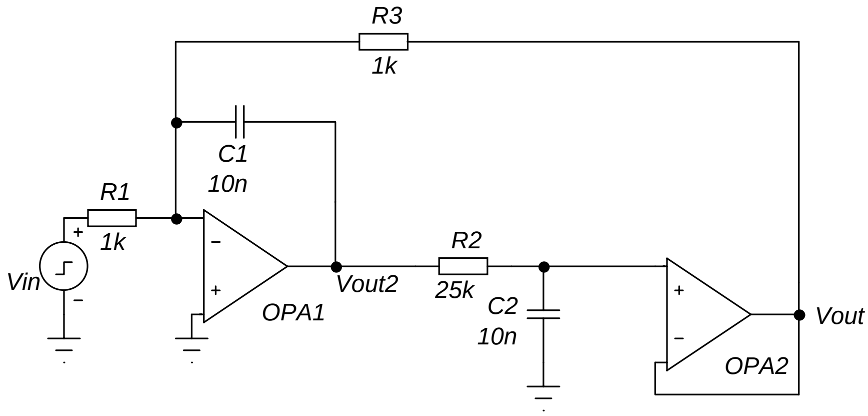

Fig. 1. Example of active 2^{nd} - order filter with parameters f_0=3.2 kHz and Q = 5.

Assume that capacitor C_1 is replaced by another with C_1 = 330pF. Our task is to modify a minimum number of the other elements such that the original transfers to outputs OUT and OUT_2 remain unchanged.

Capacitance C_1 is decreased by 10nF/330pF, i.e. 30.3 times. The trivial solution, which can be applied any time, consists in multiplying impedances of **all** circuit elements. Moreover, other solutions can exist while modifying a smaller number of elements. The existence of such prospective solutions depends on the concrete circuit topology.

Equations (1) represent the coupling conditions among five circuit parameters R_1, R_2, G_3, C_1, and C_2. While modifying these parameters without violating the coupling conditions, all the coefficients of transfer function remain preserved. The coupling conditions can be symbolized by the graph in Fig. 2. Let us call it *coupling graph* or *vertex graph*. More detailed definition will be given below.

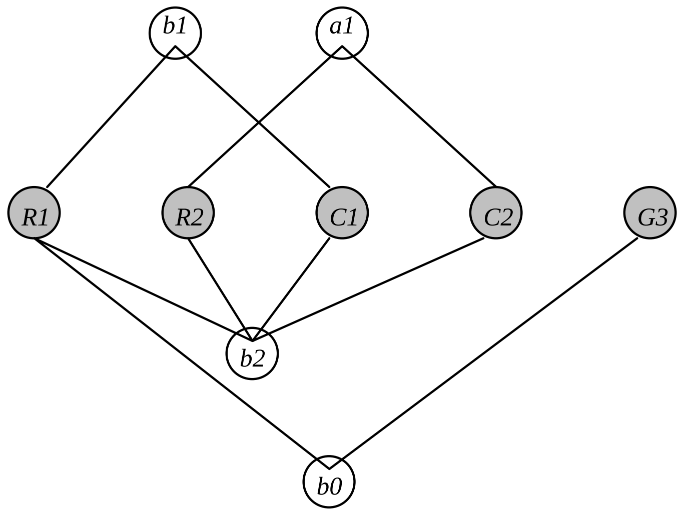

Fig. 2. Coupling (vertex) graph of active filter in Fig. 1.

The symbols of all the parameters which influence the transfer functions appear in the graph. The procedure of generating the coefficients of transfer functions from parameters is then diagrammed.

The following causal relations can be traced up from the coupling graph:
Decreasing capacitance C_1 *a* times, where $a = 30.3$, must be accompanied by increasing resistance R_1 in the same ratio to retain coefficient b_1 unchanged. Increasing R_1 will not affect coefficient b_2 because it depends on the product of R_1 and C_1. However, b_0 will be changed. That is why we must decrease parameter G_3. Modifying G_3 has no more causal effects on the transfer coefficients and the optimization can be terminated. Conclusion: Decreasing C_1 *a* times is fully compensated by increasing R_1 and R_3 *a* times.

We can similarly find out how to fully compensate the modification of C_2: the product R_2C_2 must only be preserved.

Furthermore, consider the passive CR bandpass filter in Fig. 3. The transfer function is in the form

$$\frac{V_{out}}{V_{in}} = \frac{a_1 s}{1 + b_1 s + b_2 s^2},$$

where

$$a_1 = R_2C_2 = 16\mu s,\ b_1 = R_1C_1 + R_2C_2 + R_1C_2 = 32.16\mu s,\ b_2 = R_1R_2C_1C_2 = 256 ps^2. \quad (2)$$

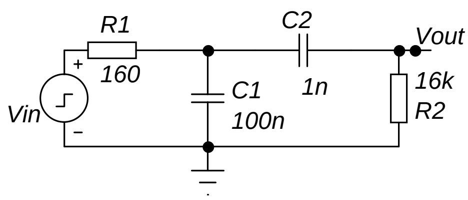

Fig. 3. Example of passive CR bandpass filter with f_0=10kHz.

The corresponding coupling graph is in Fig. 4a.

Now the b_1 coefficient is given as a sum of three elements. This fact is represented in the graph in an appropriate manner. Evaluating the graph, we conclude that no parameter sets can be found excepting the entire set [R_1 C_1 R_2 C_2], which would be modified without changing the transfer function: To preserve all the coefficients of transfer function, it is necessary to modify all parameters simultaneously. The only solution is the well-known impedance scaling.

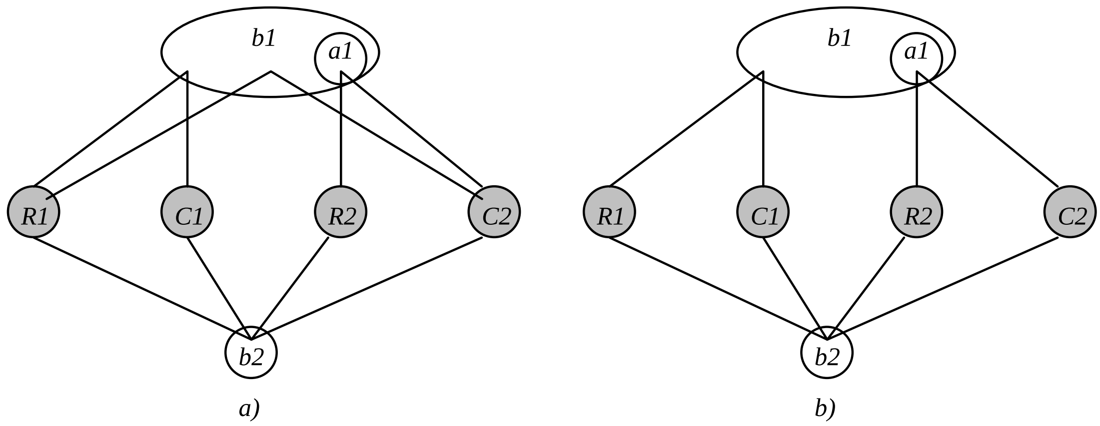

Fig. 4. Coupling graph of circuit in Fig. 3: a total, b simplified after neglecting the "weak" couplings.

Taking into account the numerical values of parameters and neglecting the R_1C_2 term in the formula of coefficient b_1 (error is less than 0.5%) will yield the reduced coupling graph in Fig. 4b. The circuit can then be optimized as two independent CR cells: it is sufficient to keep time constants R_1C_1 and R_2C_2 constant.

Coupling (vertex) graph

Let circuit functions depend on m parameters $P=[p_1, p_2,.., p_m]$. These parameters are coupled by n coupling conditions of the type of

$$F^{j}(P)=f^{j}, j=1, 2, .., n, \tag{3}$$

where f^{j} are constants (for example, transfer function coefficients).

Function F has a special form when analyzing linear circuits symbolically: it is an algebraic sum of N products Π_k (k=1,2,..,N) of M parameters, where N and M are natural numbers. Some products can enter the sum with negative sign, depending on the type of circuit solved:

$$F^{j} = \sum_{k=1}^{N}\left(\pm \Pi_k^j\right) = f^{j}, j=1, 2, .., n. \tag{4}$$

The coupling graph in Fig. 5 corresponds to Equation (4).

The so-called **product paths** start from the individual M parameters and they converge at a point which we call **product vertex**, or simply **vertex**. M parameters form so-called **basis**. From the mathematical point of view, the vertex represents the product of parameters in the basis. In practice, the individual bases can contain some shared parameters.

In a simple case, coupling condition (4) is formed only by one product. Then we speak of the graph with **single vertex** (or the **single graph**). If summing more products, we have a graph with **multiple vertex** (or **multiple graph**), which is sunk into the so-called **summing region**. From the mathematical point of view, the summing region is represented by formula (4). If some products enter the sum with negative sign, then it is labelled near the corresponding vertex (see Fig. 5).

The total coupling graph is a synthesis of individual graphs of all the coupling conditions. Evaluating the graph, we can obtain important information for optimization, for instance, which parameters can be modified independently while preserving the target functions.

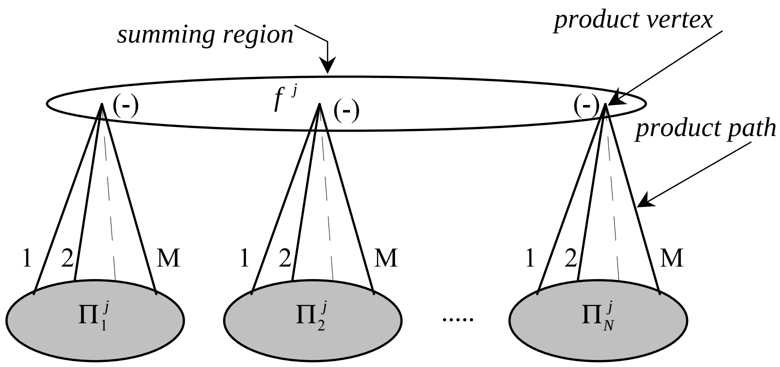

Fig. 5. Definition of the coupling (vertex) graph.

Coupling (vertex) graph evaluation

Coupling graph evaluation is a procedure of splitting all the circuit parameters into individual sets. Within these sets, the parameters can be modified in a manner which ensues from the graph evaluation and which does not modify the monitored parameters of the circuit function.

Let us consider the coupling graph in Fig. 4b. While varying the elements, it is necessary to keep constant both the products R_2C_2, $R_1C_1R_2C_2$ and the sum of products $R_1C_1+R_2C_2$. This fact corresponds to the graph decomposition into three partial graphs:

1. simple graph with basis R_2C_2 and vertex a_1,
2. simple graph with basis $R_1C_1R_2C_2$ and vertex b_2,
3. twofold graph with bases R_1C_1 and R_2C_2 and vertex b_1.

For example, the first graph indicates that increasing R_2 has to be compensated implicitly by decreasing C_2. Interaction of graphs No. 2 and 1 implies the necessity of preserving the product R_1C_1. Graph No. 3 does not bring any additional limitation. It only expresses the result of the interaction of graphs No. 1 and 2: the sum of both products must be constant.

In other words, evaluating the graph yields the information on the independent parameter variation inside two sets - [R_1 C_1] and [R_2 C_2].

When optimizing complicated circuits, whose coupling conditions lead to large graphs, it is suitable to utilize algorithms of graph evaluation. They can identify the sets mentioned above automatically. We describe an algorithm which is based on the following assumption: Within each basis, the parameter variations have to occur with both signs. For example, increasing some parameters must be accompanied by decreasing at least one other. It is a necessity for the single graphs, while for the multiple graphs it usually does not represent a significant limitation of optimization. A description of more general algorithms is beyond the scope of the paper.

Algorithm of graph evaluation

The algorithm below is based on the assumption that prospective parameter variations in the graph basis must be of both signs. The positive/negative variation will be labelled as 1/0. All permitted combinations of variations will be written in a special table. For instance, if the basis consists of three parameters, the permitted variations will be

001, 010, 101, 110, 101, 011

(the variations 000 and 111 are not permitted because they are of single direction).

We write all the permitted parameter variations in the bases of all subgraphs.

In the second step, we cut out all combinations that are in logical contradiction.

In the third step we evaluate boolean links among individual parameters. This evaluation enables us to segment parameters into subsets.

As an example, consider the graph in Fig. 2, which corresponds to the filter in Fig. 1. The appropriate table is given below.

No.	R_1	R_2	C_1	C_2	G_3	vertex
1		1		0		a_1
2		0		1		
3	1				0	b_0
4	0				1	
5	1		0			b_1
6	0		1			
7	0	0	0	1		b_2
8	0	0	1	0		
9	0	0	1	1		
10	0	1	0	0		
11	0	1	0	1		
12	0	1	1	0		
13	0	1	1	1		
14	1	0	0	0		
15	1	0	0	1		
16	1	0	1	0		
17	1	0	1	1		
18	1	1	0	0		
19	1	1	0	1		
20	1	1	1	0		

Tab. 1. Table of couplings of graph in Fig. 2.

The gray-marked rows represent the excluded combinations, which cannot occur on principle. For example, the combination [R_1 C_1] = [0 0] in row No. 7 is not permitted on the basis with vertex b_1. After exclusion we have 10 rows for further processing.

Now we find out the relationships among the individual parameter variations. We start with parameter R_1. Comparing data in columns R_1 and R_2 shows the links between R_1 and R_2. As evident, these parameters are in no relation (for instance, [0 0] is in row 7 but [0 1] in row 12). However, parameters R_1 and C_1 are related by inversion, the same as R_1 and G_3. It means that increasing R_1 can be compensated by decreasing C_1 or G_3. One more dependent pair R_2 and C_2 can be identified from the table (inversion too). Our results are in consonance with the tentative circuit analysis in the chapter “Opening examples”.

After repeating the algorithm for the CR passive cell in Fig. 3 with the graph in Fig. 4a, no connection among the parameters would be found. This means that no exact compensation method exists except for the classical impedance scaling. For the simplified graph in Fig. 4b the algorithm will find independent parameter pairs R_1C_1 and R_2C_2.

Partial compensation

Wherever the total compensation is impossible on principle, we apply the partial compensation. First the coupling graph is simplified by leaving out the "weak" couplings. This step is actually important and it should be performed with regard to the expected permissible change in circuit responses (e.g. frequency response). In the first approximation we can omit the product paths of multiple subgraph which influence the total sum in the vertex negligibly. Some constraint rules often pass into the optimization. They can violate the principle of the inverse variations in the basis. Then the algorithm from the previous chapter cannot be used. Nevertheless, the coupling graph can play an important role during the optimization in such cases as well. A description of these procedures is beyond the scope of the paper.

Conclusion

The typical problems while optimizing linear continuous-time circuits are discussed. So-called coupling (vertex) graphs are introduced. These graphs can be generated on the basis of symbolic circuit analysis, and they can be simplified by virtue of semisymbolic analysis. Evaluating the graph offers concrete information on permitted parameter variation which preserves the required target functions.

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