FIR-BL ANALOG FILTERS

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***Abstract*-This paper presents an idea of analog filter design which is based on the discrete-time linear-phase prototype. After the inverse bilinear transformation, the designed linear-phase FIR filter is converted to the continuous-time filter. The phase properties of such structures are discussed.**

I. Introduction

Functional blocks and chains with the constant group delay are required in many signal processing applications. Let us mention anti-aliasing filters, phase equalizers of transmission lines, synthesis of linear delay blocks for pulse compression, some splitter blocks for shift keying modulators, etc. [1], [2].

In general, classical analog filters – both passive and active - have nonlinear phase frequency response, i.e. the group delay depends on frequency. This phenomenon is principal and we can only partly reduce it in a certain frequency band using a suitable approximation of the magnitude response.

The discrete and digital filters offer a solution in the form of the well-known filters with the finite impulse responses (FIR). Fulfilling a certain condition, these filters have an exactly linear phase response over the whole frequency range. Such filters can be realized by nonrecursive structures as a chain of delay registers.

Analog filter design based on its discrete-time (DT) prototype is not a common approach. The reverse procedure is well-known when a discrete-time filter is designed for the consideration of an analog prototype. Below, we describe the method called FIR-BL (Finite-Impulse Response- BiLinear) which models nonrecursive DT structures by recursive analog filters. The basic idea is very simple: We design a DT linear-phase FIR filter and then we transform it into an analog filter using the bilinear transformation. It can be shown that such a filter has interesting phase properties.

II. FIR-BL Design Method and FIR-BL Analog Filters

The filter design can be accomplished in the following steps:

1) Approximation task.

a) Finding the z-domain coefficients of DT linear-phase FIR filter from the frequency response requirements.
b) Conversion to the analog filter transfer function using the bilinear transformation.

2) Realization task.

a) Finding analog filter structure which corresponds to the given transfer function.
b) Analog filter design and optimization.

Step 1a) can be performed by any well-known method in the z-plane. In case of a small separation between the working and sampling frequency, frequency axis prewarping is usually recommended.

For step 2, we choose any known method of analog filter synthesis.

The accomplishment of the s-domain approximation task is a result of the first step. A hitherto unknown approximation arises which can be then compared with the classical one with a good group delay performance (Butterworth, Bessel, Feistel-Unbehauen, TICFU, and others) [3]. However, step 1a) can be performed by a number of different methods (windowing, sampling of the frequency response, Remez algorithm, etc.) [4]. In this way, there can be various approximations. Let us call them globally the “FIR-BL-class approximations” and the filters as “FIR-BL-class filters”.

III. Properties of FIR-BL-Class Filters

Consider a system function of the N^{th} -order DT FIR filter

$$K(z)=\sum_{k=0}^{N} a_k z^{-k}=\frac{\sum_{k=0}^{N} a_k z^{N-k}}{z^N}. \qquad (1)$$

To ensure linear phase, we apply the symmetry and asymmetry conditions to the filter coefficients [5]:

$$a_k = a_{N-k}, \qquad (2)$$

or

$$a_k = -a_{N-k}. \qquad (3)$$

Let us denote a filter with the property (2)/(3) as a symmetrical/asymmetrical filter.

The well-known fact follows from (1): each N^{th} -order FIR filter has an N-multiple pole in 0 and N zeros. Considering the symmetry/asymmetry, the following propositions are true which result from the theory of reciprocal/nonreciprocal algebraic equations [6]:

Asymmetrical filter of the $N \geq 2$ order always has zero $z = 1$. Separating this zero yields a symmetrical N-1 order filter.

Symmetrical filter of the odd $N = 2k+1$ order always has zero $z=-1$. Separating this zero yields an asymmetrical 2k order filter.

If an arbitrary symmetrical or asymmetrical filter has zero $z \neq \pm 1$, then it has simultaneously the zero $1/z$, and for complex zero, also complex conjugate zeros.

These propositions enable us to understand the rules of zero location of an analog filter after backward bilinear transformation.

Equation (4) defines the well-known bilinear (BL) transformation

$$s = a\frac{z-1}{z+1},\ a=\frac{2}{T}=2F_S \qquad (4)$$

where T and F_S are the sampling period and frequency, respectively.

As obvious from (4), N-multiple pole $z = 0$ in the z-plane is transformed using the backward BL transformation to the N-multiple real pole in the s-plane

$$s_p = -a = -2F_S \,. \tag{5}$$

We conclude that an FIR-BL-class analog filter will only have an N-multiple real negative pole and generally N zeros, which are transformed from the z-plane by the backward BL transformation.

An analysis of equation (4) yields:

Zero z_0 = 1/-1 is transformed into the s-plane as a zero $s_0 = 0/\infty$. Zero in the infinity does not affect the transfer function.

If a zero z_0 is transformed into the s-plane as s_0, then zero $1/z_0$ is transformed as $-s_0$ and zero z_0^* (complex conjugate) is transformed as s_0^*.

We can conclude that each analog FIR-BL-class filter has an N-multiple negative real pole (5) and the following possible zeros: 0, in pairs s_0, $-s_0$, and in the case of complex zeros there are their complex conjugate counterparts.

Let us assume a general case of such a quaternion of zeros s_0, $-s_0$, s_0^*, $-s_0^*$, where

$$s_0 = re^{j\varphi} = R + jI \,.$$

The corresponding root factor product is as follows:

$$(s-s_0)(s+s_0)(s-s_0^*)(s+s_0^*) = (s^2 - r^2)^2 + (2RI)^2 \,.$$

The simpler case of a couple of real zeros s_0, $-s_0$ leads to the factors

$$(s-s_0)(s+s_0) = s^2 - s_0^2 \,.$$

Setting $s = j\omega$ reveals that the root factors generate only the real part of complex transfer. In other words, they do not contribute to the generation of group delay. The group delay is then determined entirely by the N-multiple pole (5):

$$\tau_g(\omega) = \frac{\tau_0}{1+\left(\dfrac{\omega}{a}\right)^2}, \tau_0 = \frac{N}{a} = N\frac{T}{2} \,. \tag{6}$$

The DT prototype had a constant group delay. After backward BL transformation, the group delay will be frequency dependent according to (6).

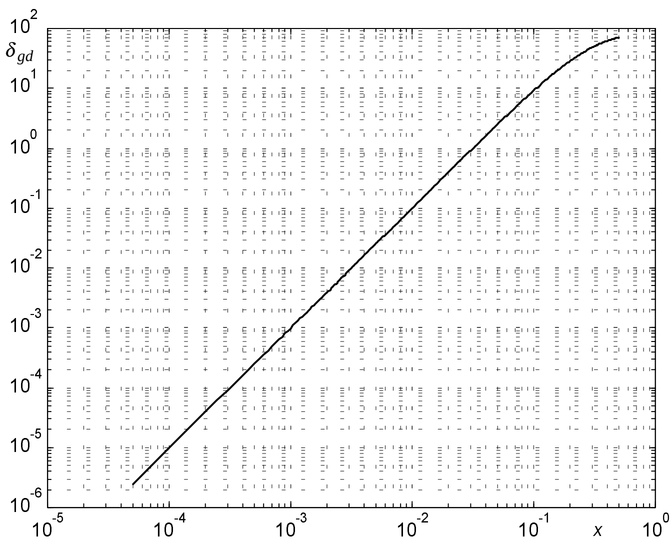

Fig.1. Relation between the percentage group delay drop-off δ_{gd} and the relative frequency $x = f.T = f/F_S$.

The relation between the percentage group delay drop-off δ_{gd} and the product $x = f.T = f/F_S$ is given in Fig. 1. It results from (6) that

$$\delta_{gd} = \left(1 - \frac{1}{1+\pi^2 x^2}\right) 100 \,. \tag{7}$$

As obvious from the graph, for the required group delay ripple up to 0.1%, the relative frequency has to be up to 0.01.

It should be noted that we can expect a good group delay response in the case of a sufficiently large distance between the sampling and working frequencies, which has to be considered during the approximation task. Then the N-multiple real pole represents a cutoff frequency far from the passband. The necessary attenuation is accomplished by the filter zeros. Considering the resulting filter to be a cascade of biquads, then each of them has the natural frequency $a = 2F_S$ and the quality factor 0.5. Due to pole multiplicity, the step response will be aperiodical with critical damping, and we can expect relatively quick settling without overshooting.

IV. Design Example and Comparison to the Classical Approximations

For illustration, a 10th - order DT linear-phase lowpass FIR filter has been designed. The cutoff frequency is 3 kHz and the attenuation 36.5 dB is guaranteed from a frequency of 11kHz. 10 zeros and 10-multiple pole $z = 0$ were converted using the BL transformation to the s-domain. The results are summarized in Tab. 1.

TABLE I

List of S-Domain Zeros and Poles of Designed FIR-BL-Class Filter.

zeros	*poles*
±j7,98054e+5	
±j2,53966e+5	
±j1,38457e+5	*10 x poles -1,1233215e+5*
±j6,06192e+4	
±j8,60374e+4	

To ensure the required maximum group delay drop, the minimum distance of the sampling frequency and the passband can be obtained from equation (7) or the diagram in Fig. 1. The designed filter has the pole $-a$ = -1.1233215e+5 and its order is N = 10. According to (6), its low-frequency group delay is 10/1.1233215e5 = 89µs (see Fig.2). The sampling frequency used for the design is $F_S = a/2$ = 56.166 kHz. Then the relative frequency x = 3/56.166 = 0.0534 corresponds to a cutoff frequency of 3 kHz. According to (7), the percentage drop of group delay will be 2.74% on this frequency compared to an initial value of 89µs. This drop may be lowered by choosing a higher sampling frequency. However, the resulting filter order will then be increased.

Table 2 compares basic parameters f_0 (natural frequency), f_n (notch frequency) and Q (quality factor) of the first- and second-order building blocks of some filters which fulfill the requirements made on the frequency response. Comparing the Bessel, Feistel-Unbehauen and FIR-BL approximations yields the conclusions that FIR-BL transformation has a minimal Q (0.5) and a maximal

TABLE II

COMPARISON OF BASIC PARAMETERS OF THE SECOND/FIRST-ORDER BLOCKS ACCORDING TO THE INDIVIDUAL APPROXIMATIONS.

F-U 5 MEANS 5^{TH} - ORDER FEISTEL-UNBEHAUEN APPROXIMATION.

approximation	*f_0 [kHz]*	*Q*	*f_n [kHz]*	*f_n/f_0*
Bessel 6	*6,486*	*1,023*	-	-
	5,752	*0,611*		
	5,462	*0,510*		
Bessel 10	*8,345*	*1,415*	-	-
	7,505	*0,810*		
	7,023	*0,620*		
	6,745	*0,538*		
	6,616	*0,504*		
F-U 5	*7,113*	*0,917*	*8,900*	*1,251*
	6,306	*0,564*	*15,128*	*2,399*
	6,087	-	-	-
FIR-BL 10	*17,878*	*0,5*	*127,014*	*7,104*
	17,878	*0,5*	*40,420*	*2,261*
	17,878	*0,5*	*22,036*	*1,233*
	17,878	*0,5*	*13,693*	*0,766*
	17,878	*0,5*	*9,648*	*0,540*

frequency f_0 which is the same for all blocks. The notch frequencies for the FIR-BL approximation have a broad spread to either side of frequency f_0.

The frequency responses of designed FIR-BL filter, the filter according to the 6^{th} - and 10^{th} - order Bessel approximations, and to the 5^{th} - order Feistel-Unbehauen (F-U) approximation are compared in Fig. 2. The NAF program [7] has been used for the design of these coefficients. Among all the filters considered, the 10^{th} - order FIR-BL has the minimum group delay, in spite of the fact that its order is twice that of the F-U filter. That is why the step response of FIR-BL is faster than for the F-U filter.

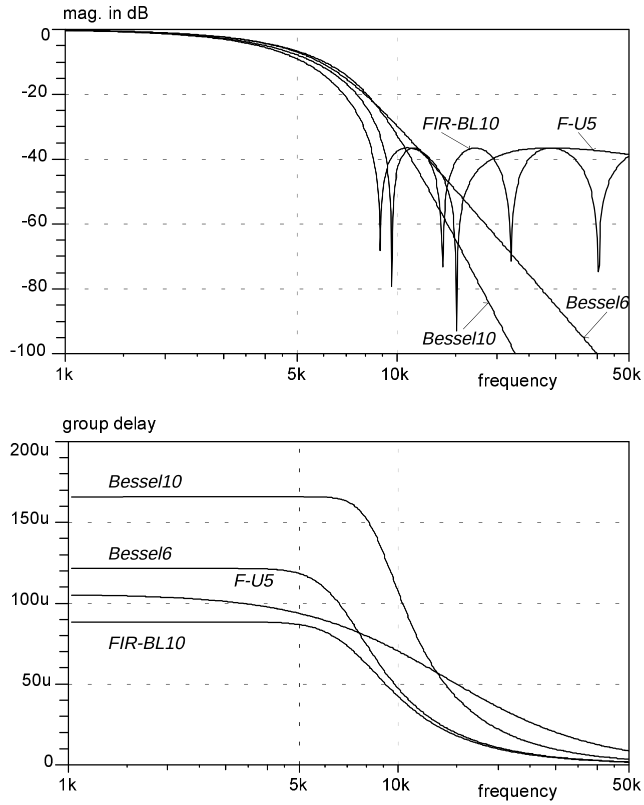

Fig. 2. Amplitude and group delay frequency responses: 6^{th} order Bessel, 10^{th} - order Bessel, 5^{th} - order F-U, 10^{th} -order FIR-BL.

CONCLUSION

The basic idea and properties of analog FIR-BL-class filters are described in this contribution. These filters are derived from the discrete-time linear-phase FIR filters. In principle, an arbitrary small group delay deviation from the ideal passband response can be achieved, but at the cost of relatively high filter order.

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