COMPUTER DESIGN OF DISCRETE-TIME FILTERS FROM ANALOG COUNTERPARTS USING SIGNAL INVARIANCES

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A computer algorithm of z-domain transfer function evaluation of discrete-time (DT) system is presented. This system is designed from its continuous-time (CT) counterpart using signal invariances method. The set of two-graph modified nodal approach (2-graph MNA) equations, state-space equations or s-domain transfer function can be considered as the model of analog counterpart. The procedure is based on the algorithm of numerical Laplace inversion.

Introduction

The method of signal invariance is well-known procedure for design of DT systems from their CT counterparts. The initially-at-rest responses on given input signal are then the equivalent for designed DT system and CT system. In this paper, the focus will be concentrated to the impulse, step and linear invariances.

The common design procedure can be described as follows:

1 Computation of the initially-at-rest response of CT counterpart.
2 Sampling of the response to obtain equivalent initially-at-rest response of DT system.
3 Computation of the *z*-transform of sampled response and sampled input signal.
4 Derivation of system function as the ratio of aforementioned *z*-transforms.

This procedure is not convenient for computer implementation. Realization of the first step means the precision time analysis of CT system. The algorithmization of following items is difficult, especially for slowly converging or even diverging responses. In this procedure, there are accuracy limitations for derivation of system function in closed form.

The efficient computer method is described in [1], but on the assumption that the state-space equations of CT system are known. This approach can be also implemented if the design is based on the transfer function $K(s)$, because the construction of state-space equations from $K(s)$ is trivial [2].

However, linear systems are often described by set of differential-algebraic equations. The 2-graph MNA is extended and popular in CAD simulators and design systems for CT applications. To utilise these design tools for the design of DT applications, the 2-graph MNA model of CT counterpart is convenient to be considered.

Algorithm

Let us consider the 2-graph MNA equations of linear system with one input and one output signals w and y, respectively:

$$\mathbf{G}\mathbf{v}(t) + \mathbf{C}\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{v}(t) = \mathbf{D}w(t), \quad y(t) = \mathbf{B}\mathbf{v}(t) \qquad (1),(2)$$

where **v** is a (*nx1*) vector of nodal variables, **B**, **C** are a (*nxn*) matrices and **D** and **B** are (*nx1*) and (*1xn*) vectors, respectively. At least one of the matrices **G**,**C** is often singular.

In spite of this singularity, the differential equation (1) has solution in following form. After time axis discretization into intervals ($kT,kT+T\rangle$, k =...-1,0,1,2,..., T >0, the solution for the end of k-th interval is in the s-domain

$$\begin{aligned}&(\mathbf{G}+s\mathbf{C})\mathbf{V}(s) = \mathbf{C}\mathbf{v}(kT) + \mathbf{D}\mathrm{W}(s) \Rightarrow \\ &\mathbf{V}(s) = (\mathbf{G}+s\mathbf{C})^{-1}\mathbf{C}\mathbf{v}(kT) + (\mathbf{G}+s\mathbf{C})^{-1}\mathbf{D}\mathrm{W}(s),\end{aligned} \qquad (3)$$

and in the time-domain

$$\mathbf{v}(kT+T) = \mathbf{g}^*(T)\mathbf{v}(kT) + \int_0^T \mathbf{g}(\xi)w(kT+T-\xi)d\xi \qquad (4)$$

where $$\mathbf{g}^*(T) = \mathrm{L}^{-1}\{(\mathbf{G}+s\mathbf{C})^{-1}\}\mathbf{C}\big|_{t=T} \qquad (5)$$

is a (*nxn*) matrix of zero-input response for time t=T and

$$\mathbf{g}(\xi) = \mathrm{L}^{-1}\{(\mathbf{G}+s\mathbf{C})^{-1}\}\mathbf{D}\big|_{t=\xi} \qquad (6)$$

is a (*nx1*) vector of impulse response.

As mentioned in (5) and (6), these characteristics can be obtained by using algorithm of numerical Laplace inversion (NLI). This algorithm is described in [2] and [3] and will not be explained in this contribution.

Approximating input signal *w(t)* during given time interval by various terms, the equation (4) can be converted into difference equation for DT system. For approximations mentioned below, the difference equation is in the general form

$$\mathbf{v}(kT+T) = \mathbf{g}^*(T)\mathbf{v}(kT) + \mathbf{G}w(kT) + \mathbf{H}w(kT+T) \qquad (7)$$

It is known that replacing the signal $w(t)$, $t\in(kT,kT+T\rangle$ by Dirac impulse, constant step or linear signal in accordance with the Table 1, the corresponding difference equation (7) describes the DT system, designed from CT model (1) using impulse invariance, step invariance and linear invariance, respectively. The **G** and **H** vectors for all invariances are in the Table 1.

Numerical evaluation of these vectors can be performed using NLI: $\mathbf{g}(0^+)$ using two-step solution with forward and backward integration, and $\mathbf{h}(T)$ and $\mathbf{q}(T)$ using the rule of Laplace transform of integral [3].

invariance	impulse	step	linear
approximation of $w(t)$ $\mathrm{t} \in (kT, kT+T>$	$Tw(kT+T)\delta(t-kT-T)$	$w(kT)$	$w(kT)+a.(t-kT)$ $a=\left[w(kT+T)-w(kT)\right]/T$
vector **G**	$\mathbf{0}$	$\mathbf{h}(T)=\int_0^T \mathbf{g}(\xi)\mathrm{d}\xi$	$\mathbf{h}(T)-\mathbf{q}(T)$
vector **H**	$\mathbf{g}(0^+)T$	$\mathbf{0}$	$\mathbf{q}(T)=\frac{1}{T}\int_0^T \mathbf{h}(\xi)d\xi$

Tab.1 Various types of invariances and corresponding vectors **G** and **H**.

Applying *z*-transform, the equation (7) can be derived as follows:

$$[z\mathbf{E}-\mathbf{g}^*(T)]\mathbf{V}(z)=(\mathbf{G}+\mathbf{H}z)W(z) \tag{8}$$

where **E** is the identity matrix.

It should be noted, that poles of the designed DT system are the eigenvalues of matrix $\mathbf{g}^*(T)$. Taking account of the output equation (2), the coefficients of *z*-domain transfer function $K(z) = y(z)/w(z)$ can be obtained using Faddejev algorithm or other known procedures.

Obviously, aforementioned algorithm provides right results also on the assumption, that the CT system is described by state-space equations instead of equations (1), because state-space equations are the simplified case of (1) for regular matrix **C**=**E**.

Simulation results

To verify aforementioned theory, the computer program INVAR has been constructed. As an example, the fifth-order Bessel approximation filter is designed. In [4], the continuous-time counterpart has the *s*-domain transfer function as follows:

$$H(s)=\frac{105}{105+105s+45s^2+10s^3+s^4}.$$

The demanded sampling frequencies of discrete-time system are 8 and 16 rad/sec.

Antoniou describes classical procedure to design impulse-invariant discrete-time system in [4]. The results are summarized in the Tab.2 and Tab.3 along with the output of program INVAR. We can see good agreement.

Conclusions

A problem of computer design of discrete-time systems from analogue counterparts using signal invariance approach is presented. The efficient program for the z-domain transfer function computation has been developed. The two-graph modified nodal approach is used to obtain minimum set of equations. Owing to the algorithm of numerical Laplace inversion, this method provides accurate results.

$H(z)=\dfrac{c_1 z+c_2 z^2+c_3 z^3}{d_0+d_1 z+d_2 z^2+d_3 z^3+z^4}$		
	Antoniou	INVAR
c_1	1.4864215566E-2	1.4864213537E-2
c_2	3.2370129187E-1	3.2370127612E-1
c_3	7.2249231396E-1	7.2249235117E-1
d_0	3.8820300061E-4	3.8820320392E-4
d_1	-3.8651370378E-3	-3.8651352789E-3
d_2	1.7060001861E-2	1.70599868556E-2
d_3	2.9420700000E-2	2.9420552873E-2

Tab.2. Results of design procedure: Sampling frequency is 8 rad/sec.

$H(z)=\dfrac{c_1 z+c_2 z^2+c_3 z^3}{d_0+d_1 z+d_2 z^2+d_3 z^3+z^4}$		
	Antoniou	INVAR
c_1	2.0704126099E-2	2.0704115762E-2
c_2	2.0789952444E-1	2.0789948419E-1
c_3	1.4674496376E-1	1.4674501386E-1
d_0	1.9702866951E-2	1.9702872987E-2
d_1	-1.6112205431E-1	-1.6112205302E-1
d_2	5.6070962114E-1	5.6070961592E-1
d_3	-1.0449874000E0	-1.0449874061E0

Tab.3. Results of design procedure: Sampling frequency is 16 rad/sec.

References

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