Flow Graph Analysis of PWM DC-to-DC Converters

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Abstract: Flow graphs of the pulse-width-modulated (PWM) switch both with the fixed and modulated duty cycle are introduced in the paper. These graphs start from the classical Vorpérian model of PWM switch in the continuous current mode (CCM). The flow graphs are intended for the “Hand-and-Paper” analysis of DC-to-DC converters, utilizing the „averaged modeling“, especially – but not only - for the analysis of DC voltages, currents and powers. The method is illustrated on an example of buck-boost converter.

Keywords: - PWM converter, PWM switch, CCM, flow graph, M-C graph

1 Introduction

Several methods are commonly used for the analysis of switched DC-to-DC converters, mostly the state-space averaging method or the method based on averaged PWM-switch model [1]. Compared with the complete modeling of switching processes in the converter, the average modeling is simple and sufficiently universal, enabling transient, DC steady-state, and small-signal AC analyses.

In DC-to-DC converter analysis, some outputs are often required which conventional Spice-family simulators cannot generate. Let us mention zeros and poles of the control-to-output transfer function, or a formula how the input resistance of the converter depends on the duty ratio. Without some special software for symbolic computation of such switched circuits, the analytical computations have to be performed manually. This is a case when special flow graphs can help us significantly.

In the paper, the so-called Mason-Coates’s (M-C) flow graphs of the PWM switch with a fixed and modulated duty ratio are introduced. These graphs are compiled from equations which model the PWM switch as given in [2]. Advantages of this graph method are demonstrated on several calculations of circuit functions and the voltage, current, and power relations in the buck-boost converter.

2 Operator model of the PWM switch

The well-known schematic of PWM switch is in Fig. 1. The switching period T is divided into two phases: The *active* switch is in ON state (a and c terminals are connected) for a time interval DT, the *passive* switch is ON (p and c terminals connected) during a time interval $D'T = (1-D)T$. Here $D \in (0,1)$ is the duty cycle.

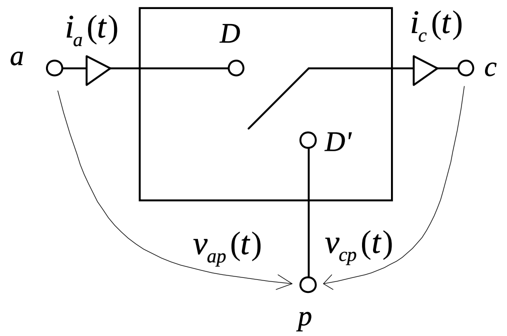


Fig. 1: PWM switch with ***a*** (active), ***p*** (passive) and ***c*** (common) terminals.

Due to periodical switching, the average values of the switch voltages and currents will be dependent on D. In addition, modulation of the duty ratio around a bias value leads to complicated nonlinear effects.

In [2], mathematical models of a switch are given which use average values of branch voltages and currents. The models differ depending on whether instantaneous values are averaged above each switching period or whether their perturbations around the bias values are considered, and whether the duty ratio is fixed or modulated.

A universal operator description of the PWM switch, generalizing the results from [2], is as follows:

$$I_a = D_0 I_c + D I_{c0}, \tag{1}$$

$$V_{cp} = D_0 V_{ap} - D_0 D'_0 r_e I_c + V_{D0} D, \tag{2}$$

where

$$V_{D0} = V_{ap0} + (D_0 - D'_0) r_e I_{c0}. \tag{3}$$

An auxiliary resistor r_e in (2) and (3) is generally a function of lossy resistance R_c (ESR) of the capacitor and the load resistance R. For instance, for the boost and buck-boost converters, r_e is given by their parallel combination, and for the Cuk converter, r_e is directly equal to R_c.

The quantities denoted by the suffix $_0$ are coordinates of the DC operating point.

The symbol D means the duty ratio perturbation around the bias value D_0 or the operator form of this perturbation. Omitting the terms containing D yields equations of the PWM switch with fixed duty ratio.

The remaining symbols in equations for fixed duty ratio can mean either instantaneous averaged voltages and currents or steady-state values or their perturbations around the bias values or the operator form of these signals. For the modulated duty ratio, these symbols can represent either the perturbations due to variations of D or their operator descriptions.

Eqs. (1)-(3) are thus universal. Their equivalent flow graphs of the PWM switch will then represent a powerful tool for the analysis of steady-states, and transient and frequency responses of switched DC-to-DC converters.

3 M-C flow graph of PWM switch

The M-C flow graphs are graphs with undirected self-loops [3]. The node V with self-loop Y, into which a branch with the gain Y_x is directed from node V_x, represents the equation

$$YV = Y_x V_x . \qquad (4)$$

Let us consider the operator model of PWM switch in Fig. 2. Eqs. (1)-(3) are true for voltages and currents.

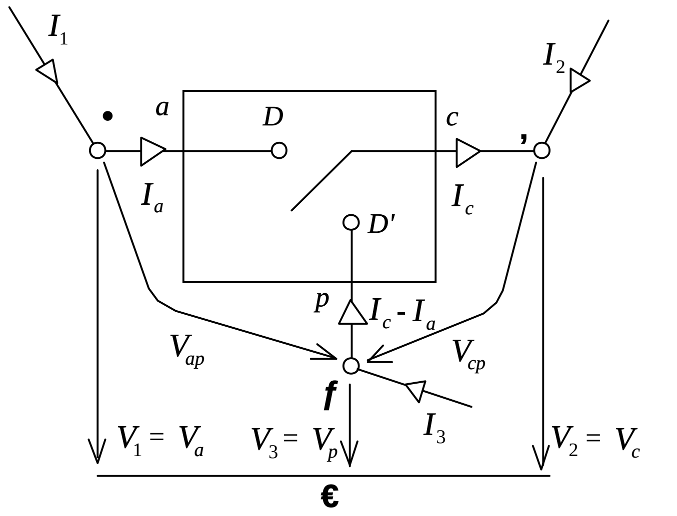


Fig. 2: PWM switch as a floating three-terminal device.

Let the algebraic sum of currents flowing into the node i, i=1, 2, 3, be labeled $I_{\Sigma i}$, i=1, 2, 3. Utilizing Eqs. (1)-(3), the following equations can be derived for the model in Fig. 2. Finally, the modified equation (2) will be appended:

$$0 = I_{\Sigma 1} = I_1 - I_a = I_1 - D_0 I_c - D I_{c0} , \qquad (5)$$

$$0 = I_{\Sigma 2} = I_2 + I_c , \qquad (6)$$

$$0 = I_{\Sigma 3} = I_3 - I_c + I_a = I_3 - D_0' I_c + D I_{c0} , \qquad (7)$$

$$D_0 D_0' r_e I_c = D_0 V_a + D_0' V_p - V_c + V_{D0} D . \qquad (8)$$

The corresponding M-C flow graph is in Fig. 3.

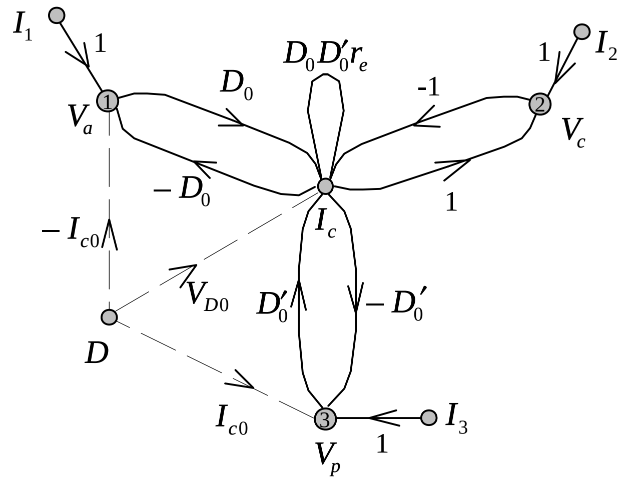


Fig. 3: M-C flow graph of PWM switch.

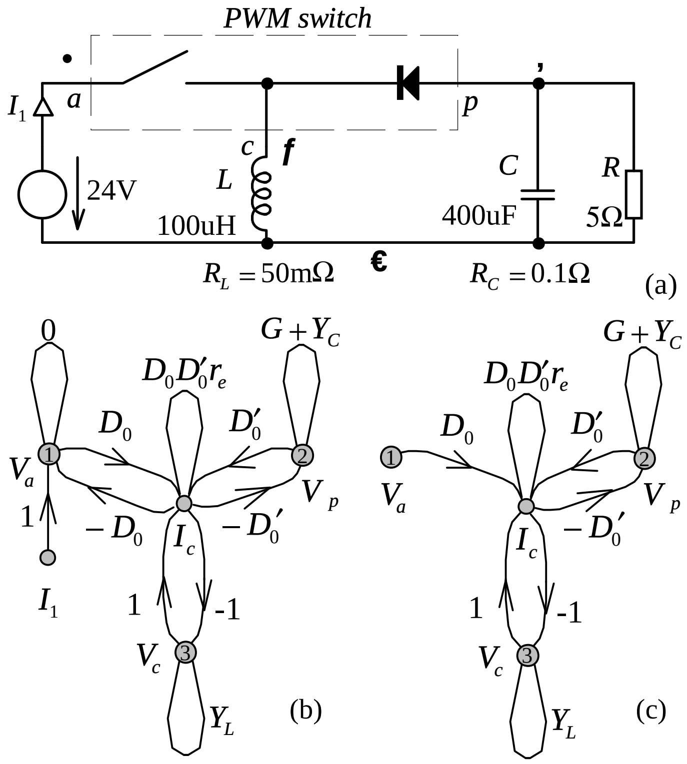
It results from (5)-(7) that nodes V_a, V_p, and V_c in the M-C graph should be complemented with zero-gain undirected self-loops. For simplicity, they are omitted here. According to the theory of M-C graphs, after connecting the switch into a circuit, self-loops will appear in these nodes whose gains will depend on the admittances connected to the nodes.

Node D and the dashed branches are applied only for the switch with modulated duty ratio.

The M-C flow graph of the switch with fixed duty ratio is easy-to-remember: It consists of three directed and one undirected loop. The loop going through the „active node“ V_a („passive node“ V_p) contains branches with gains „D_0“ („D_0'“), with the signs + for going out and – for going in. The loop going through the „common node” V_c contains branches with gains of „1“, i.e. „$D_0 + D_0'$“, with the rule for the signs being opposite to the above case. The undirected loop gain is given by the product of D_0, D_0' and the auxiliary resistance r_e.

4 Demonstration of the graph analysis

The buck-boost converter in Fig. 4 (a) has a set duty ratio D_0=0.4. The theoretical value of voltage transfer is $-D_0 / D_0' = -2/3$, and the output voltage –16V [1]. Let us analyze by means of the M-C graph of the PWM switch the real parameters of the converter, taking into account the parasitic resistances (ESR) of the condenser and the coil.



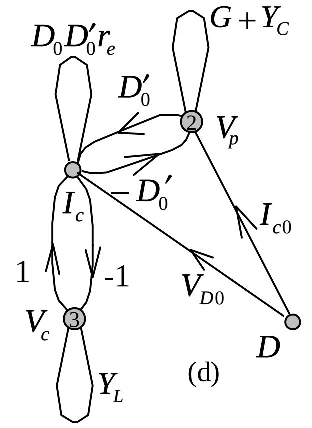


Fig. 4: (a) Buck-boost DC-to-DC converter, (b), (c) M-C graph for fixed *D*, full and shortened, (d) M-C graph for computing control-to-output transfer function.

$$G = 1/R\,,\; Y_L = \frac{1}{R_L + sL}\,,$$

$$Y_C = \frac{1}{R_C + 1/(sC)}\,,$$

$$r_e = R \,\|\, R_C \approx 98\ \text{m}\Omega\,.$$

The M-C graph of the converter can be obtained directly from its schematic. The voltage transfer is analyzed from the shortened graph in Fig. 4 (c) [3]. Its evaluation yields the line-to-output transfer function, which is then simplified for DC steady-state (s = 0):

$$\frac{V_2}{V_1} = \frac{V_p}{V_a} = \frac{-D_0 D_0' Y_L}{D_0 D_0' r_e Y_L (G + Y_C) + D'^2 Y_L + G + Y_C} =$$

$$= \left|s = 0\right| = -\frac{D_0}{D_0'} \frac{1}{1 + \frac{r_e}{R}\frac{D_0}{D_0'} + \frac{R_L}{R D_0'^2}} \approx -0.592.$$

Then the output voltage is –0.592x24=-14.21V, the load current –2.842A, and load power 40.38W.

The input resistance is computed from the input impedance (s = 0), evaluating the full graph in Fig. (c):

$$Z_{in} = \frac{V_a}{I_1} = \frac{D_0 D_0' r_e Y_L (G + Y_C) + D_0'^2 Y_L + G + Y_C}{D_0^2 Y_L (G + Y_C)},$$

$$R_{in} = \frac{D_0'}{D_0} r_e + \frac{R_L}{D_0^2} + \frac{D_0'^2}{D_0^2} R \approx 11.71\Omega\,.$$

The current from the input source will be 2.05A, and the power delivered from this source 49.19W. The corresponding efficiency is 82% (switch-on resistances and other influences are not considered).

The M-C graph in Fig. 4 (d) models the transmission of the perturbation of the duty ratio to the converter output. Evaluating this graph gives the control-to-output transfer function:

$$\frac{V_2}{D} = \frac{V_{D0} D_0' Y_L + I_{C0}(D_0 D_0' r_e Y_L + 1)}{D_0 D_0' r_e Y_L (G + Y_C) + D_0'^2 Y_L + G + Y_c}\,,$$

where I_{C0} =4.892A is the difference of the input and the output current, and V_{D0} = 38.1V results from (3), where V_{ap0} = 38.21V is the difference of the input and the output voltages.

The corresponding frequency response is in Fig 5.

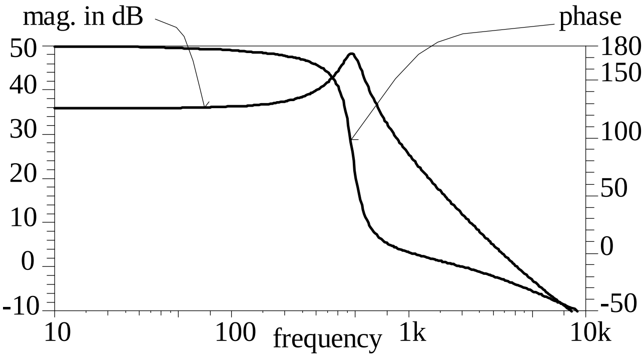


Fig. 5: Control-to-output frequency response of the converter in Fig. 4(a).

5 Conclusion

The present M-C graph of PWM switch combines the Vorpérian models with the fixed and the modulated duty ratio. It enables all the calculations which the averaging technique makes possible. The practical limits depend on the human capability to evaluate the flow graph. That is why the method is especially favorable for less demanding analyses of DC steady states of switched DC-to-DC converters.

References

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