SPICE modeling of switched DC-DC converters via generalized model of PWM switch

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Abstract. *A generalization of the model of PWM switch according to Dijk et al. for averaged modeling of switched DC-DC converters is proposed. This model now includes the on-resistance of the active switch, and the forward voltage drop and the differential resistance of diode-type passive switch. The way this model is compiled allows for the influence of parasitic ESRs of the capacitors in the circuit such that the resulting equations are equivalent to the averaged state space equations. The procedure of PSPICE modeling and the results of computer simulation of boost converter are shown.*

Keywords

PWM switch, state space averaging, DC-DC converter, boost converter, line-to-output frequency response, control-to-output frequency response.

1. Introduction

The averaging approach is an effective technique of modeling switched DC-DC converters [1]. It enables an easy analysis of DC and AC relations in the converter via conventional SPICE-like circuit analysis programs. The transient analysis is incomparably faster than similar analyses for the switched-mode model of the converter.

The state space averaging approach [2], [3] is an acknowledged standard of averaged modeling. However, its implementation in conventional simulation programs is problematic. That is why other techniques are used, particularly the averaged modeling of PWM (PulseWidth Modulation) switch [4]. This approach is advantageous because the complete converter, with the exception of the PWM switch, is modeled in the classical way, i.e. by the R, C, and L elements. The model is then complemented with the averaged model of the switch. The original model from [4] is composed of a pair of controlled current and voltage sources. As demonstrated in [5], in some cases, for example if the ESR of the output capacitor cannot be neglected, the averaged modeling of PWM switch does not lead to an absolutely precise description of the circuit, because the resulting equations are not equivalent to the equations of state space averaging. This phenomenon is explained in [5], [6] and the original model of PWM switch is complemented here with an additional resistor whose resistance is obtained directly from the converter topology by inspection on the basis of a simple rule.

Another method of making up a model of the PWM switch is presented in [7]. Both the voltage and the current of the above controlled sources are derived from the state variables of the converter, i.e. from the voltages across the capacitors and from the currents of the inductors, namely in the switching phase when the active switch is closed. It is shown that this method generates results that are equivalent to those obtained by the method of state space averaging.

The above method is easily implemented in SPICE-like circuit analysis programs. The authors suggest that real parameters of converter components can be included in the switch model. However, these procedures are not described.

The on-resistance of the active switch together with the forward voltage drop and the differential resistance of diode-type passive switch belong to significant real parameters of the PWM switch. These parameters can affect the output voltage and converter efficiency. The aim of this paper is to include them in the procedure of generalized modeling of the PWM switch according to [7]. This technique is then illustrated on an example of SPICE simulation of the boost converter.

2. Generalized model of PWM switch

A PWM switch is shown in Fig. 1 (a). It consists of active and passive switches. In general, they can be separated from each other in the converter topology. The active switch, commonly implemented by a transistor, is closed during the time dT_S, where T_S is the switching period, and d is the duty ratio, which can be modulated by another signal. The passive switch is implemented either by a diode or a transistor. Let us assume that the passive switch is closed within the time interval $d'T_S = (1-d)T_S$, otherwise the converter operates in the continuous current mode (CCM).

According to [7], the pair of the active and the passive switches can be modeled by an averaged model in the form of a controlled current source di_a and a controlled voltage source dv_p, as shown in Fig 1 (b). Other components in Fig. 1 (b), modeling the passive switch parameters, will be discussed below.

The current i_a and the voltage v_p can be obtained from the inspection of the converter according to the following rules [7]:

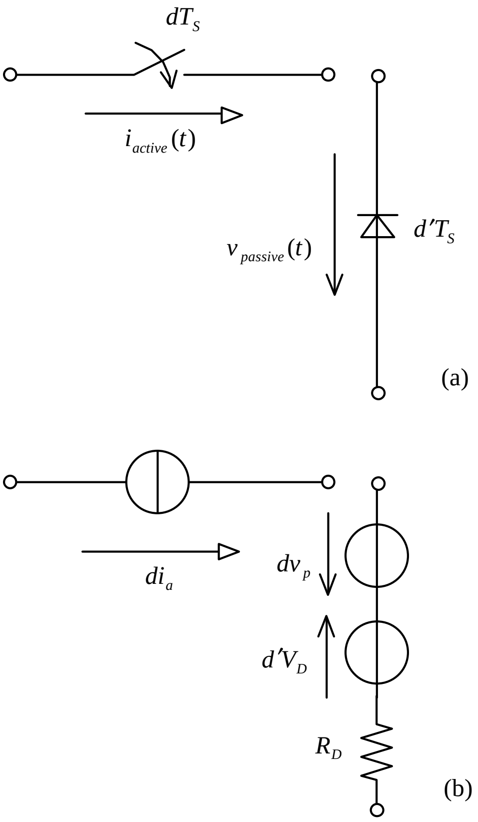

Fig. 1. (a) Pair of active and passive switches, (b) their averaged model.

Rule 1: i_a equals the current through the closed active switch during a time interval dT_S, written as a function of the state and input variables.

Rule 2: v_p equals the voltage across the open passive switch during a time interval dT_S, written as a function of the state and input variables.

Note that the symbols v_p and i_a represent averaged values of voltage $v_{passive}(t)$ and current $i_{active}(t)$ within the switching period T_S.

In Fig. 1 (b) this model is complemented with the influence of forward voltage drop V_D and differential resistance R_D of diode. The corresponding derivation is given in [8].

The information about the on-resistance R_A of the active switch is seemingly not present in the model. Actually, this resistance affects the parameters of controlled sources. The following section demonstrates a procedure of compiling the averaged model of boost converter.

3. Demonstration of modeling the boost converter

The boost converter, analyzed in [7], is shown in Fig. 2 (a). Fig. 2 (b) demonstrates its modeling according to the algorithm from Fig. 1 (b). The coil and capacitor ESRs, the on-resistance R_A of the active switch, and the parameters V_D and R_D of the passive switch are included in the model. The duty ratio is determined by the voltage-controlled source E3. Its DC attribute defines the bias value of the duty ratio, whereas its AC value serves for the modeling of control-to-output frequency response.

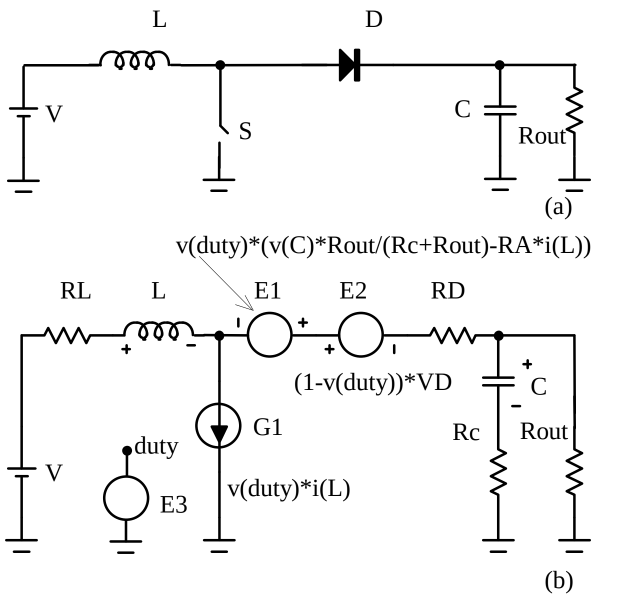

Fig. 2. (a) The boost converter, (b) its averaged model, including serial resistances of L and C and switch non-idealities.

The controlled current source G1 is defined in conformity with **Rule 1:** When the active switch is closed, the diode is in the off-state and the total inductance current i(L) flows through the active switch. This current is multiplied by the duty ratio.

The controlled voltage source E1 is determined according to **Rule 2:** When the active switch is closed, the voltage across the non-conducting diode is equal to the difference between the output voltage of the converter and the voltage across the active switch. Since the diode is in off-state, the output voltage is determined by the voltage across the capacitor, which is transformed into the output via the voltage divider R_{out}, R_c (the first part of the expression for the voltage of source E1 in parentheses). The voltage across the active switch is given by the product of the on-resistance and the coil (the second part of the expression).

The converter model is complemented with the source E2 and with the resistance R_D according to the procedure of modeling the passive switch parameters in Fig. 1 (b).

It can be easily shown that the circuit equations of the model in Fig. 2 (b) exactly correspond to the state space averaged equations of the converter.

4. Demonstration of transient, DC, and AC analyses

The converter model in Fig. 2 (b) has been implemented in two analysis programs, namely OrCADPSpice 15.7 [9] and Micro-Cap 9 [10]. To enable a comparative analysis, the switch-level model of the converter has been simultaneously analyzed. The following numerical values from [7] have been used:

V = 60V	Rout = 60Ω
L = 6mH	RL = 3Ω
C = 1000µF	Rc = 1Ω
TS = 100µs	d = 0.25

In addition, the switches (which are modeled as ideal in [7]) have been modeled as follows:

RA = 1Ω, RD = 1Ω, VD = 0.6V.

Results of the transient analysis for the output voltage and inductor current are in Fig. 3. To achieve sufficient accuracy, the Tmax parameter of transient analysis has been chosen to be one thousandth of the switching period. The analysis of the signals in Fig. 3, when the program simultaneously simulated both the switched and the averaged model, was performed on the AMD Athlon ™ Processor, 2.4GHz, 2GB RAM. The simulation time was 33 seconds in Micro-Cap and 46 seconds in PSpice (with the option SOLVER=0). The analysis of the averaged model itself takes only hundredths of a second.

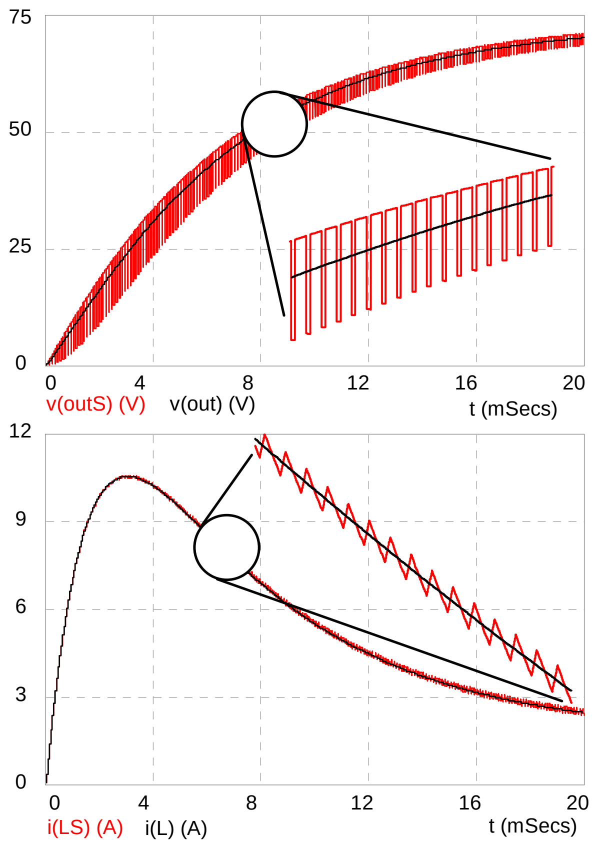

Fig. 3. Comparison of results of switched and averaged modeling of boost converter.

The voltage and the current from the averaged model faithfully trace the switched signals, averaged within the switching period. In the steady state, the following quantities were measured: an output voltage of 70.636V with a peak-to-peak ripple of 1.657V, an inductor current of 1.571A with a peak-to-peak ripple of 223.8mA.

The DC analysis of the averaged model confirms the values from the steady state analysis: an output voltage of 70.642V, an inductor current of 1.570A. In addition, Micro-Cap determined a load power of 83.171W, a power of 94.189W from the input source V, and a corresponding efficiency of 88.3%.

Appending the attribute AC=1 to the voltage source V, one can easily determine the line-to-output frequency response via the AC analysis of averaged model. Similarly, the control-to-output frequency response can be analyzed after modeling the AC component of source E3, which determines the instantaneous duty ratio. Analytical formulas for the corresponding transfer functions are derived in [5]. However, neither the switch on-resistances nor the diode forward voltage drop are considered there. That is why the AC analyses on the averaged model were performed with zero values of these parameters.

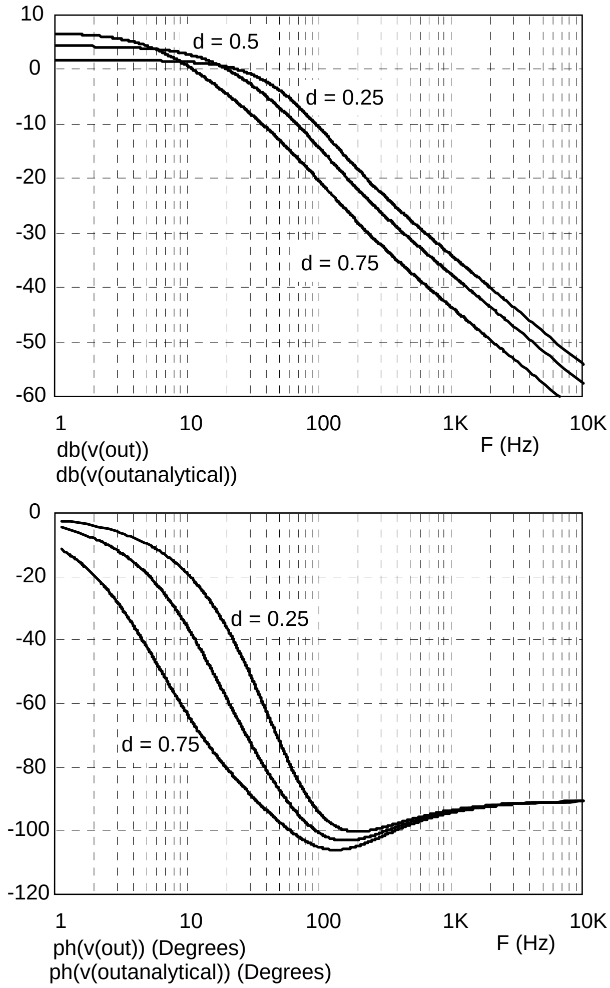

Fig. 4. Line-to-output frequency responses for different duty ratios.

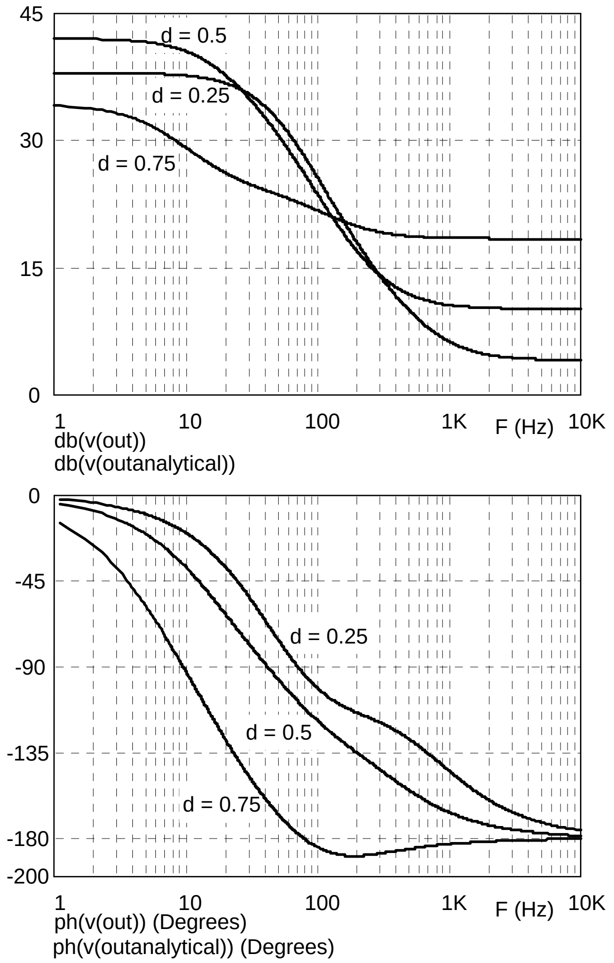

Fig. 5. Control-to-output frequency responses for different duty ratios.

The analysis was run simultaneously with the behavioral analysis using the Laplace-type sources with the modeling of the above analytical formulas of transfer functions. The results are in Figs. 4 and 5 for different values of duty ratios. The curves, coming from the averaged model, practically coincide with the responses of the Laplace sources.

When applying the SPICE TF (Transfer Function) analysis to the averaged model in Fig. 2 (b), the transfer function from the source V to the output must correspond for the given duty ratio to the DC value of line-to-output frequency response in Fig. 4. Similarly, the transfer function from the source E3 to the output must give the DC value of control-to-output frequency response in Fig. 5. In our case, the results of TF and AC analyses are identical with an accuracy to 4 digits.

Note that the control-to-output frequency responses of the analyzed converter were also published in [7]. However, they do not correspond to those in Fig. 5.

The frequency responses were also analyzed for real parameters of switches. A comparison with the results of TF analysis and with the transient analysis of switched-level model again confirmed the correctness of the averaged modeling proposed.

5. Conclusions

A generalization of the averaged model of PWM switch from [7] is presented. Now the model also includes the on-resistance R_A of the active switch, the forward voltage drop V_D, and the differential resistance R_D of the diode-type passive switch. The original model in [7] consists of the controlled current and voltage sources for modeling the active and passive switch. In this new model, the model of the passive switch is extended by a serial connection of a voltage source, whose voltage is given by the product $(1\text{-}d)V_D$, and a resistor with resistance R_D. The rules for determining the parameters of controlled sources from converter state variables remain the same as in [7] with the exception that now all the derivations must respect the nonzero on-resistance of the active switch.

All the comparisons of the results of this averaged modeling with the corresponding results of the analysis of boost converter on the switched level of modeling, with the analytical results from [5], and with the results of SPICE TF analysis indicate the correctness of the proposed method, which is equivalent to the method of state space averaging.

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References

[1] BASSO, C.P. Switch-mode power supply SPICE cookbook. McGraw-Hill, 2001.

[2] MIDDLEBROOK, R.D., ĆUK, S. A general unified approach to modeling switching-converter power stages. *Int. Journal of Electronics*, 1977, vol. 42, no. 6, p. 512-550.

[3] KALOUS, J. Two methods of simulation of DC/DC converters with pulse width modulation. *Sborník Univerzity Obrany*, řada B, 2004, no. 2, p. 96 (In Czech).

[4] TYMERSKI, R.P.E. et al. Nonlinear modeling of the PWM switch. *IEEE Transactions on Power Electronics*, 1989, vol. 4, no. 2, p. 225-233.

[5] VORPÉRIAN, V. Simplified analysis of PWM converters using the model of the PWM switch: Parts I and II. *IEEE Transactions on Aerospace Electronic Systems*, 1990, vol. 26, no. 3, p. 490-505.

[6] VORPÉRIAN, V. Fast analytical techniques for electrical and electronic circuits. *Cambridge University Press*, 2002.

[7] DIJK, E., SPRUIJT, J.N., SULLIVAN, M.O., KLAASSENS, J.B. PWM-switch modeling of DC-DC converters. *IEEE Transactions on Power Electronics*, 1995, vol. 10, no. 6, p. 659-664.

[8] BIOLEK, D., BIOLKOVÁ, V. Generalized modeling of PWM switch in DC-DC converters. Submitted to *IEEE Transactions on Power Electronics.*

[9] OrCADPSpice program, http://www.cadence.com/products/orcad/index.aspx

[10] Micro-Cap 9 simulation program, http://www.spectrum-soft.com.