The types of interpolation of a signal given by samples and their use by semisymbolic method

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Abstract: *In computer analyses of linear circuits in the time domain we are mainly interested in the system response to a given input signal. We often have only samples of analog signal obtained by sampling in certain intervals instead of the signal itself. Then there is a need to interpolate in order to obtain an analog signal as the output of the analog system tested. A comparison of several types of sampled signal interpolation is given in this article.*

1. Introduction

There are several algorithms for linear-circuit time-domain analysis with the transmission function of the system known, which work on different principles. One of them is, for example, the Inverse Laplace transformation algorithm published by J. Vlach in 1969 [1]. In the course of time this algorithm was improved and expanded in order to simulate circuits with lumped and distributed parameters. At present time, the so-called Vlach-Singhal method is mostly used.

Another method is the time domain semisymbolic analysis, which decomposes the Laplace image into partial fractions. This method is based on finding the analytic equation of system output. First, the Laplace image of the response is numerically decomposed into partial fractions for which the object is then found. The sum of objects of all partial fractions gives the analytic equation of response. The accuracy of this method mostly depends on a correct retrieval of all poles of the Laplace image of system response. In contrast to the first method this one gives accurate results even for long simulation times. In this article the latter method is used for the comparison of different interpolations of a signal given by samples.

2. User signal given by samples

Input signal *v(t)* can be expressed as a sum of its particular segments [4]:

$$v(t) = \sum_{i=-\infty}^{\infty} \delta_{\varepsilon}(t - i\varepsilon)\varepsilon . v(i\varepsilon) \ , \tag{1}$$

where ε is length of sample interval
$v(i\varepsilon)$ is input in time $i\varepsilon$ and
$\delta_\varepsilon(t-i\varepsilon)$ is the shifted Dirac impulse.

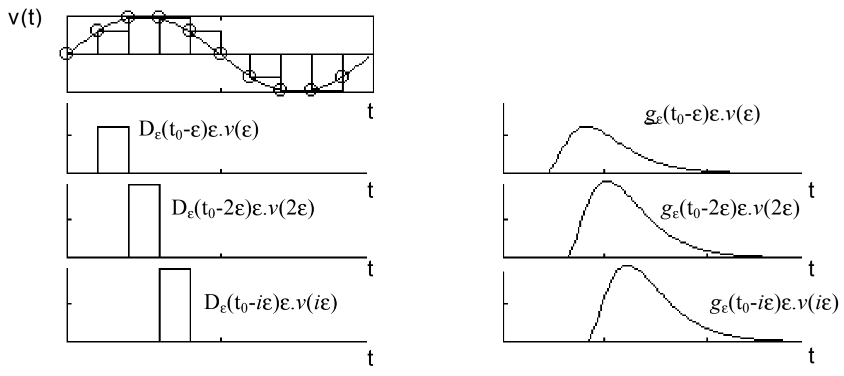

Fig. 1 - Calculation of system response for general input signal

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Output signal *y(t)* is thus a convolution of input signal and system response to the Dirac impulse [4]:

$$y(t) = \sum_{i=-\infty}^{\infty} g_\varepsilon(t - i\varepsilon)\varepsilon . v(i\varepsilon) , \tag{2}$$

where $g_\varepsilon(t-i\varepsilon)$ is the shifted impulse characteristic of system (see Fig. 1).

Four time-domain interpolations of input signal $v(t)$ with step t_i and with period T_i are described in [2].

Impulse interpolation

Samples of signal are replaced by the Dirac impulses, whose bulk is given by the sample size and the respective interval duration. Impulse interpolation is not a classical interpolation because equality $v^{int}(t_i) = v(t_i)$ is not fulfilled.

Time domain: $$v(t_{i-1})T_i\delta(t') \tag{3}$$

Operator domain: $$v(t_{i-1})T_i \tag{4}$$

Step interpolation

The sample is interpolated by the "sample & hold" method, i.e. by a step the size of sample.

Time domain: $$v(t_i) \tag{5}$$

Operator domain: $$v(t_i)/p \tag{6}$$

Linear interpolation

Samples are connected by linear sections.

Time domain: $$v(t_{i-1}) + \frac{v(t_i) - v(t_{i-1})}{T_i} t' \tag{7}$$

Operator domain: $$\frac{v(t_{i-1})}{p} + \frac{v(t_i) - v(t_{i-1})}{T_i p^2} \tag{8}$$

Cubic spline interpolation

Samples are fitted to the cubic spline. The curve formed by cubic spline has the four derivative between two neighbouring fitted points equal to zero, i.e. each section of the waveform is defined by a cubic polynomial [3].

Time domain: $$a_0^i + a_1^i t' + a_2^i t'^2 + a_3^i t'^3 \tag{9}$$

Operator domain: $$\frac{a_0^i}{p} + \frac{a_1^i}{p^2} + 2\frac{a_2^i}{p^3} + 6\frac{a_3^i}{p^4} , \tag{10}$$

where $t' = t - t_{i-1}$ is the time from the beginning of step i and
a_0, a_1, a_2 a a_3 are polynomial coefficients

According to [3], for a non-equidistant net it is possible to determine polynomial coefficients $a(t_i)$ at time t_i by equations

$$\begin{aligned} a_0(t_i) &= v(t_{i-1}) \\ a_1(t_i) &= \frac{v(t_i) - v(t_{i-1})}{t'} - \frac{t'}{3}\left(a_2(t_{i+1}) + 2a_2(t_i)\right) \\ a_3(t_i) &= \frac{1}{3}\frac{a_2(t_{i+1}) - a_2(t_i)}{t'} \end{aligned} \tag{11}$$

According to [3], coefficients a_2 are given by a system of equations

$$a_2(t_i)t'_i + 2a_2(t_{i+1})\left(t'_{i+1} + t'_i\right) + a_2(t_{i+2})t'_{i+1} = 3\left(\frac{v(t_{i+1}) - v(t_i)}{t'_{i+1}} - \frac{v(t_i) - v(t_{i-1})}{t'_i}\right) , \tag{12}$$

where t'_i is the time from the beginning of step i

3. Illustrative examples

The first system to be tested is a low-pass filter with a cutoff frequency of 1000 Hz and a slope of 20 dB/dec. The input signal is a sine and a rectangular wave of 100 Hz frequency.

Figs 2 and 3 give sine signal responses of the system tested. On the left are 5 periods of the input signal interpolated with impulse, jump and linear interpolation. On the right are the corresponding output signals. The actual behavior of input and output signal is shown in dotted line. Fig. 2 shows the input and output signal of 500 Hz sample frequency, i.e. 5 samples per period. The input and output signal of 5 kHz sample frequency are in Fig. 3, i.e. 50 samples per period.

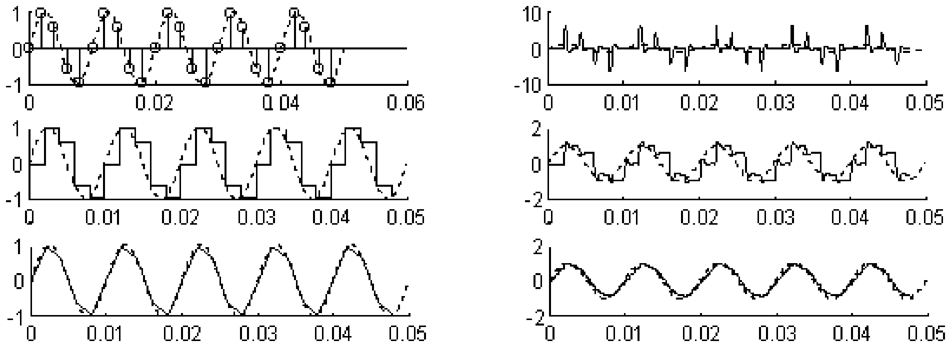

Fig. 2 - Low-pass filter response to interpolated sine wave (f_s=5f)

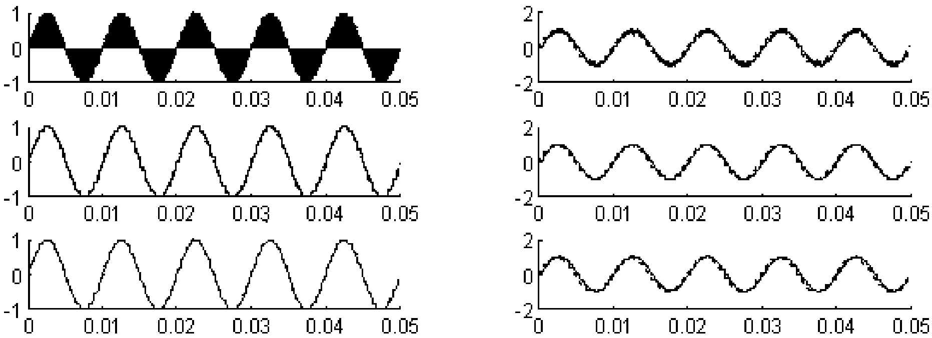

Fig. 3 - Low-pass filter response to interpolated sine wave (f_s=50f)

It can be seen from the Figures that impulse interpolation is unusable with low sample frequency, while a behavior of output signal is almost identical with behavior of actual signal with using linear interpolation. With densely sampled signal all interpolations give an almost identical result.

Figs 4 and 5 give the response of the system tested to a rectangular signal with a keying interval of 0,4. The interpolated input signal is on the left and the corresponding output signal is on the right. The input and output signals for the 500 Hz sampling frequency are in Fig. 4 and for the 5 kHz sampling frequency in Fig. 5.

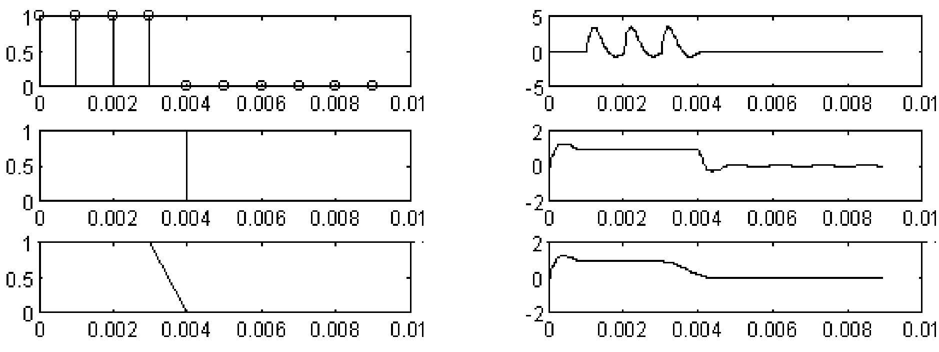

Fig. 4 - Low-pass filter response to interpolated rectangular wave (f_s=5f)

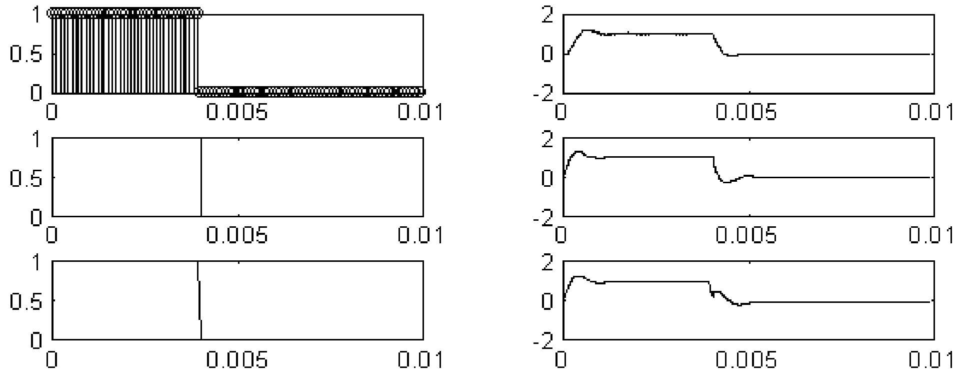

Fig. 5 - Low-pass filter response to interpolated rectangular wave (f_s=50f)

With low sample frequency the impulse interpolation is unusable, the best is jump interpolation. Contrariwise, with densely sampled signal, the worst is linear interpolation. Deviation from the expected course is caused by the prolongation of fall time due to interpolation.

The other tested system is a high-pass filter with cutoff frequency of 1000 Hz and a 20 dB/dec slope. The input signal is a rectangular wave of 100 Hz frequency and 5 kHz sampling frequency. Fig. 6 gives the input and output signal with impulse, jump and linear interpolation.

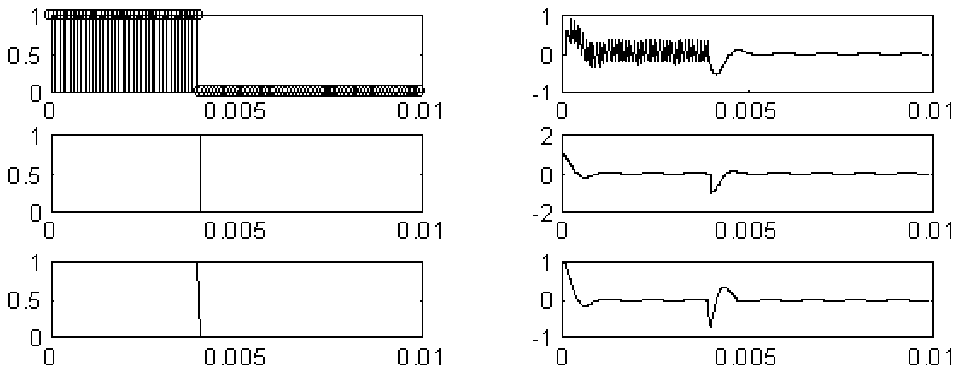

Fig. 6 - High-pass filter response to interpolated rectangular wave (f_s=50f)

The figure shows that in spite of the large density of samples, impulse interpolation has failed and only linear interpolation offers sufficiently accurate results.

5. Conclusion

Examples show that the choice of interpolation type depends on the required accuracy of analysis as well as on actual or expected character of input signal. Impulse and jump interpolations give good results if the sample theorem is fulfilled with sufficient reserve but the system must not be of the high-pass filter nature because fast changes of input signal must not be transferred into the response calculated [2]. These interpolations are suitable if the input signal is of narrow pulse character.

Linear interpolation and cubic spline interpolation are of advantage in the interpolation of user signals given in a non-equidistant net. The cubic spline interpolation is suitable for an accurate analysis providing that the input signal is smooth bud sampled with a relatively low sample frequency. But it is absolutely unsuitable for densely sampled long signals, such as WAV files. Solving the equations of a_3 coefficient considerably prolongs the computing time. In this case, linear interpolation is useful too, as is shown in the first example.

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