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RESONANCE TRANSFORMATION AND ITS UTILIZATION FOR MODELING OF SH SYSTEMS

Abstract:

The resonance transformation

$$F(\omega,t)=\int_0^t f(\tau)e^{-j\omega\tau}d\tau \tag{1}$$

has been introduced to the signal theory by Djaïkov in [1]. However, in the period of the development of the theory of resonance transformation, little number of its practical applications has appeared. Today, the necessity of modeling of the special mixed-mode continuous/discrete-time systems as Sample-Hold (*SH*) circuits and some switched filters leads to the possibility to utilize aforementioned charming theory.

First, the basic ideas concerning resonance transformation will be summarized. Second, we apply this theory to the frequency domain analysis of systems with periodically time-varying parameters. This procedure will be explained on the example of the Sample-Hold circuit.

1. INTRODUCTION

Applying classical Fourier transformation to the signal $f(t)$ yields classical spectral function $F(\omega)$ which does not represent time evolution of the signal. Let us consider time-limited signal which evolution starts from the time point 0. In the time instant t>0, spectrum of this signal is given by aforementioned equation defining so-called „flowing spectrum“. Flowing spectrum describes the evolution of spectrum of the real signal from the start point to the actual time using the set of complex functions of independent variable ω and with the parameter t.

On the other hand, equation (1) can be considered as the set of complex functions which the parameter ω where the independent variable is time. Then we talk about *resonance transformation*. From the graphical point of view, we obtain a set of curves („resonance plots“) representing the time functions with the parameter ω.

It should be noted that the terms *flowing spectrum* and *resonance transformation* have arisen in consequence of various understanding of given problem. As obvious from [1], it is more simple to construct resonance plots instead of flowing spectrum plots if we know time-domain signal. Flowing spectrum can be obtained from the resonance plots using so-called time mark points. That details concerning geometrical properties of resonance plots are described in literature [1] with the aim to construct apparatus for drawing of that plots and flowing spectrum. Although toady's computers are able to solve these tasks effectively, some theoretical conclusions are always valuable. For example, the Gibb's phenomenon is described from the mathematical point of view very precisely using the theory of resonance transformation. There exist general relations between resonance transformation and transient phenomena i the electrical resonance circuits. However, description of aforementioned applications is beyond the scope of our contribution.

In following chapter, we define some basic parameters of the resonance plots which will be utilized for the analysis of *SH* networks.

2. BASIC PROPERTIES OF RESONANCE PLOTS

The definition integral of resonance transformation can be considered as the limit of integration sum:

$$F(\omega,t) = \lim_{\Delta\tau \to 0} \sum_{k=0}^{t/\Delta} \Delta\tau f(k\Delta\tau) e^{-j\omega k\Delta\tau}. \tag{2}$$

This equation offers directions of drawing of the resonance plot: The resonance plot is composed of the elementary vectors

$$\lim_{\Delta\tau \to 0} \Delta\tau f(k\Delta\tau) e^{-j\omega k\Delta\tau} = f(\tau) d\tau e^{-j\omega\tau}. \tag{3}$$

The resonance plot is created by addition of these vectors. The elementary increments $f(\tau)d\tau$ are always plotted in the direction determined by the factor $exp(-j\omega\tau)$, i.e. by the angle $-\omega\tau$ with the x-axis.

The moving point which creates the resonance plot is called as **leading point**.

The course of the leading point from the time $\tau = 0$ to the time $\tau = t$ is important quantity called as the **length of the resonance plot**. Using this quantity, we can judge the convergence of the integral (1) for growing time t. It should be noted that the instantaneous length of the resonance plot does not depend on the angular frequency ω and can be evaluated as follows:

$$L_F(t) = \int_0^t |f(\tau)| d\tau \cdot \tag{4}$$

The **density of time marks** is next important quantity. This density is done by the limiting relation of the time interval and the length of corresponding element of resonance plot:

$$D_t(t) = \lim_{\Delta\tau \to 0} \frac{\Delta\tau}{\int_t^{t+\Delta\tau} |f(\tau)| d\tau} = \frac{1}{|f(t)|}. \tag{5}$$

The **speed of the leading point** is inversely proportional to the density of time marks:

$$v(t) = \frac{1}{D_t(t)} = \lim_{\Delta\tau \to 0} \frac{\int_t^{t+\Delta\tau} |f(\tau)| d\tau}{\Delta\tau} = |f(t)| \cdot \tag{6}$$

The **radii of curvature** of the resonance plot at the point corresponding to the instant t is proportional to the absolute value of the signal and is inversely proportional to the frequency ω:

$$\rho(\omega,t) = -\frac{|f(t)|}{\omega}. \tag{7}$$

The **density of frequency marks** is defined by similar way as in case of time marks:

$$D_f(\omega,t) = \lim_{\Delta\omega \to 0} \frac{\Delta\omega}{|F(\omega + \Delta\omega, t) - F(\omega,t)|} = \frac{1}{\left|\frac{\partial F(\omega,t)}{\partial\omega}\right|}. \tag{8}$$

3. FLOWING SPECTRUM OF IMPULSE RESPONSE OF SH SYSTEM

Let us consider two basic principles of Sample-Hold systems in Fig.1: inertia type without- and with zeroing and integration type with zeroing. Irrespective of used type of sampling network, the result of the operation of *SH* network can be described as the set of samples

$$\{s_2(kT + t_s)\}_{k=-\infty}^{\infty}$$

(see Fig. 2). If the Nyquist's condition is fulfilled for signal s_1, then so-called equivalent signal s_2^e can be reconstructed from samples which will be equal to the input signal s_1 in case of ideal sampling. In consequence of the nonideal operation of *SH* network, some distortion appears which can be investigated using comparison of spectral functions of signals s_2^e and s_1. Relation of both spectral functions is so-called generalized transfer function of *SH* network [2], [3], [4] which can be described as follows [2]:

$$G(\omega,t) = \int_0^t g(\xi) e^{-j\omega\xi} d\zeta F_s\left(e^{j\omega T}\right) \tag{9}$$

where $t = t_s$, g is the impulse response of *SH* network in the sampling phase and F_s is the sampling function which is equal to 1 for *SH* networks with periodical zeroing of storage capacitor. For these cases, flowing spectrum of impulse response fully describes sampling process using generalized transfer function.

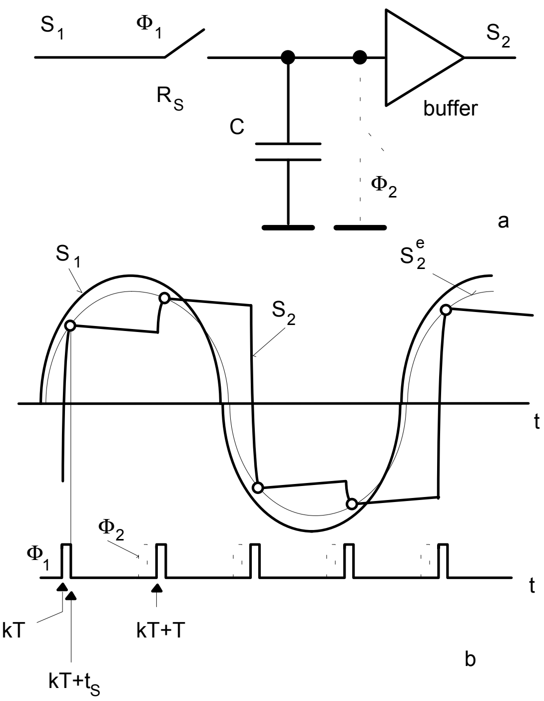

Fig.1. a) Simplified model of inertia-type *SH* networks. Switch controlled by signal Φ_2 serves for periodically discharging of storage capacitor (it need not be used). b) Example of time responses. Signal S_2 is plotted for case if discharging is not applied.

The impulse and step responses of *SH* network in the sampling phase are as follows:

$$g(t) = \frac{K}{\tau} e^{-\frac{t}{\tau}},\ h(t) = K\left(1 - e^{-\frac{t}{\tau}}\right), \tag{10}$$

where amplification $K\approx 1$ and time constant $\tau \approx R_S C$. In reality, both parameters are slightly modified due to influence of the input resistance of buffer.

Impulse response drops to zero for growing time. In this way, its resonant plot converges to a certain point. The entire length of the resonance plot is equal to the amplification K. The density of time marks grows in time:

$$D_t(t) = \frac{\tau}{K} e^{\frac{t}{\tau}}. \tag{11}$$

Then the speed of the leading point will drop:

$$v(t) = \frac{K}{\tau} e^{-\frac{t}{\tau}} \tag{12}$$

The radii of curvature of the resonance plot will drop to zero in time:

$$\rho(\omega, t) = -\frac{K}{\omega\tau} e^{-\frac{t}{\tau}} \tag{13}$$

Everything points to the fact that the resonance plot of the impulse response of *SH* network is some convergent spiral.

Utilizing of the definition integral (9) and some arrangements yield description of resonance plot

$$G(\omega, t) = \frac{K}{1 + j\omega\tau} - \frac{K e^{-\frac{t}{\tau}}}{1 + j\omega\tau} e^{-j\omega t} \tag{14}$$

which consists of the time constant vector (first term) and of the vector with the clockwise rotation. Its modulus drops to zero with growing time. For $t = 0$, the resonant plot starts from the origin of coordinates. Vector $G(\omega,\infty)$ represents the convergence point in the Gauss plane for the resonance plot. As a function of frequency, we obtain complex frequency response of the *SH* network in the sampling phase.

As results from (13), the radii of curvature of the resonance plot is $-K/(\omega\tau)$ at time 0 and it decreases with growing time. That is why the complete resonance spiral for $\omega\tau$ = constant is inscribed inside the circle with the radius $K/(\omega\tau)$. The example of resonance plots for various parameters $\omega\tau$ are in Fig. 2. The final points for $t \rightarrow \infty$ lie on the graph of the complex frequency response of *SH* network in the sampling phase (dashed curve).

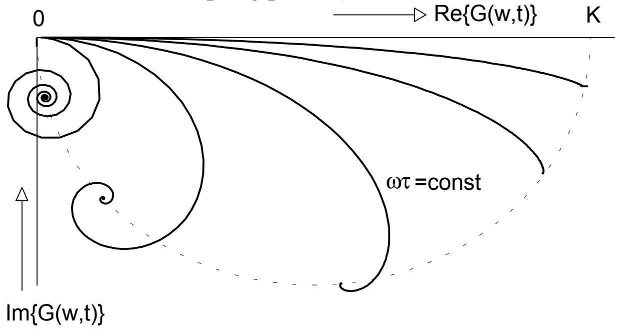

Fig.2. The resonance plots of inertia-type *SH* network.

In a similar way, the points of resonant plots for arbitrary time instant t lie on the graph of complex flowing spectrum of the impulse response of *SH* network with the duration t of sampling phase. For *SH* network

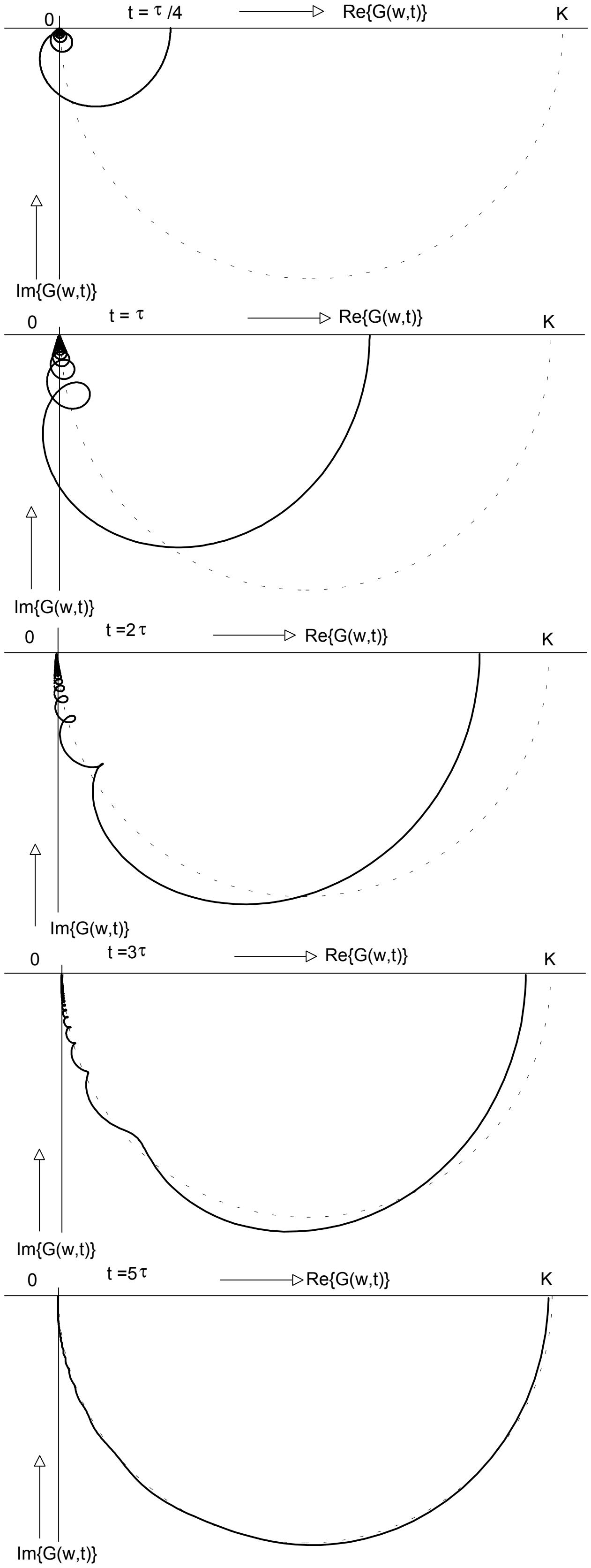

Fig.3. Flowing spectrum of the impulse response of the *SH* network for various time instances. The dashed curve is the classical complex frequency response of *SH* network in the sampling phase.

with zeroing, this flowing spectrum is equal to the generalized transfer function. In Fig.3, some examples are drawn for various time instances. These responses have been obtained using aforementioned resonance plots. We can verify that for growing time duration of sampling phase, the generalized transfer function converges to the classical transfer function of *SH* network in the stationary state (dashed curve).

Corresponding generalized amplitude frequency responses of *SH* network for various relations t/τ are shown in Fig. 4a. As evident, the *DC* gain decreases for short sampling interval. However, the bandwidth increases as shown in Fig. 4b. This fact can be good reason to construct periodically zeroing *SH* networks with relative short sampling interval. Small *DC* gain can be compensated in the following block of signal processing [3].

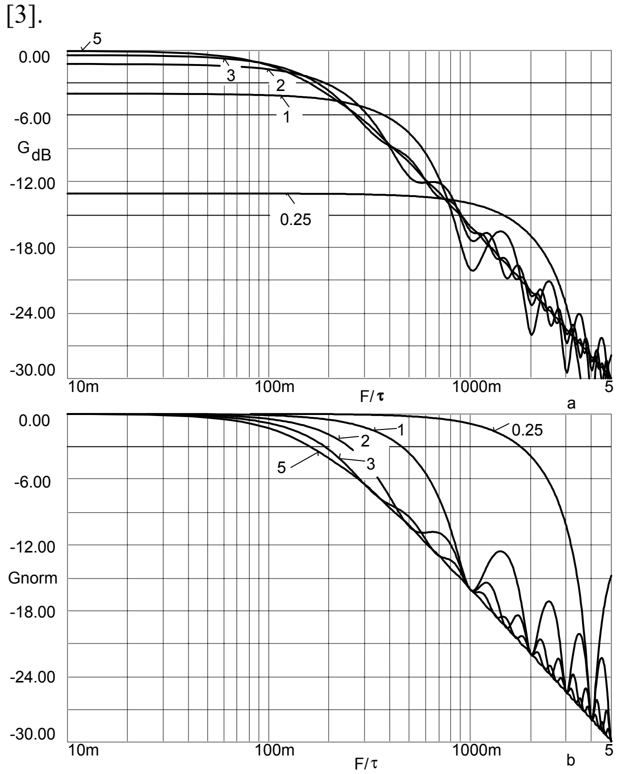

Fig. 4. a) Generalized amplitude frequency responses of *SH* network with zeroing for various relations of t/τ, b) normalized responses for the same *DC* gain.

4. CONCLUSIONS

In this contribution, some possibilities of utilization of so-called resonance transformation for frequency analysis of *Sample-Hold* networks have been demonstrated. It can be shown that *SH* network can be described in the frequency domain using so-called generalized frequency response which consists of two multiplicative parts. First part represents the inertia of the *SH* network during the sampling process and from the mathematically point of view, it is described as the flowing spectrum of the impulse response of the *SH* network in the sampling phase. Second part models influence of nonzero initial conditions inside the *SH* network to the quality of sampling in following sampling phase. This part is described by so-called switching function which depends - among other - on sampling frequency [5], [6]. If the periodically zeroing of the storage capacitor is performed, then the switching function is identically equal to 1 and need not to be considered [2].

As shown in [7], described method is more general and can be extended to the solution of the general case of linear systems with periodically varying parameters. Generalized transfer functions have been applied to the modeling of switched networks, especially switched capacitor filters [5], [6] including computer aided analysis [8], [9].

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