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MODELING OF PERIODICALLY SWITCHED NETWORKS BY MIXED S-Z DESCRIPTION*

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ABSTRACT

A new description of linear periodically time-varying systems is proposed in this contribution. To describe both continuous-time and discrete-time nature of system phenomenon simultaneously, the *generalized transfer function* of Laplace- and z- operators is introduced. This mixed description includes well-known transfer functions of continuous-time and discrete-time systems as boundary cases. Some computer simulation results are presented.

1. PROBLEM DEFINITION

Some systems from the area of radio electronic (parametric modulators and mixers), analog-to-digital (*Sample-Hold* circuits, *V/F* converters), power electronic (rectifiers, switched transducers, alternators) and telecommunication (switched capacitor filters, *RC* switched networks, current-mode circuits) can be modeled as the systems with periodically varying parameters. By these systems, the effect of spectrum enrichment appears. This fact complicates the well-known system analysis by classical transfer function in the s-domain. The time-varying linear system can be modeled by non stationary transfer function $K(s,t)$. However, this approach is not suitable for computer modeling implementation.

In consequence of periodically varying parameters, the system response contains both continuous- and discrete - time parts. Both aspects can be expressed simultaneously by so-called *generalized transfer function* (*GTF*) $K(s,z)$ as a function of both Laplace operator s and operator of z-transform. In the sense of *equivalent signal* theory by *Tsividis* [1], the frequency response is then obtained after double substitution $s=j\omega$, $z=exp(j\omega T)$, where T is the period of system parameter alternation.

This "hybrid" description includes well-known transfer functions of continuous-time and discrete-time systems as boundary cases. Computer programs, based on this approach, provide good simulation of mixed continuous- and discrete-time systems and switched systems with consistent and inconsistent initial conditions, respectively. It is also possible to compute frequency responses over the *Nyquist's* frequency.

From this point of view, the dynamical properties of time-varying system can be described using pole location of *GTF* in the s- and z-domain simultaneously. It can be proved that the z-domain location is decisive for system stability.

In this contribution, we explain the main ideas of linear time-varying system modeling using *GTFs*. First, the mathematical derivation of *GTF* will be introduced. In this part, some results from [3-9] are utilized. Second, we will consider networks with jump-varying parameters. Third, some simulation results will be presented.

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2. WHAT IS THE GTF?

Consider time-varying system with input $w(t)$, output $v(t)$ and period of parameter alternation T. An example of input/output responses is in Fig.1. Let us sample output signal at instances $kT+\varepsilon T$ for fixed value $\varepsilon \in (0,1\rangle$. The unique continuous-time signal $v_\varepsilon^e(t)$ - so-called *equivalent signal* - can be constructed to keep following properties:

1 represents interpolation function for the sequence $\{v(kT+\varepsilon T)\}\Big|_{k=..-1,0,1,..}$,

2 its spectral components fall to the spectral area of input signal $w(t)$.

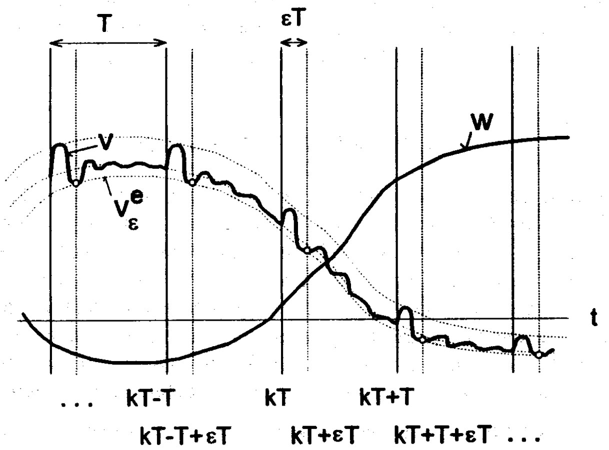

Fig.1. An example of the input (w), output (v) and equivalent (v_ε^e) signals of periodically-varying system.

It follows from the first property that the equivalent and output signals have common points at sampling instances $kT+\varepsilon T$. One can imagine certain envelope of more complicated output signal. Depending on parameter ε, various equivalent signals can represent output signal and create a set of envelopes. The choise of ε depends on the fact at which time instances we are especially interested in the network behaviour.

Second property enables to define so-called *generalized transfer function* (*GTF*) of system with periodically varying parameters as the ratio of *Fourier/Laplace* transformations of equivalent output signal and input signal. Depending on parameter ε, the various *GTF*s can represent single network behaviour.

It can be proved that the signal $v(t)$ is identical with its all equivalent signals $v_\varepsilon^e(t)$ on the assumption that the *Nyquist's* condition is true:

$$f_{max} < \frac{1}{2T}, \tag{1}$$

where f_{max} is the maximum frequency of nonzero spectral components of signal $v(t)$.

To explain aforementioned idea in more detail, let us consider the simplified responses of *Sample-Hold* circuit in Fig.2. Due to inertia of circuit in the time interval $(kT, kT+\varepsilon T)$, the dynamic error $\Delta_\varepsilon = v(kT+\varepsilon T) - w(kT+\varepsilon T)$ in the sample definition occurs. This error can be described in the frequency domain using *GTF* theory. The equivalent signal $v_\varepsilon^e(t)$ is defined above the sequence $\{v(kT+\varepsilon T)\}$. In case of precise sample selection without errors, this signal and the input signal are identical. Actually, the error can be investigated by comparison of these signals both in the time and frequency domain. The *GTF* of *SH* circuit which models the aforementioned error can be then defined as follows:

$$K = \frac{L\{v_\varepsilon^e(t)\}}{L\{w_\varepsilon^e(t)\}} = \frac{L\{v_\varepsilon^e(t)\}}{L\{w(t)\}}, \tag{2}$$

where $L\{\}$ is the operator of Laplace transformation. The last equality is true if the signal $w(t)$ fulfills *Nyquist's* condition (1).

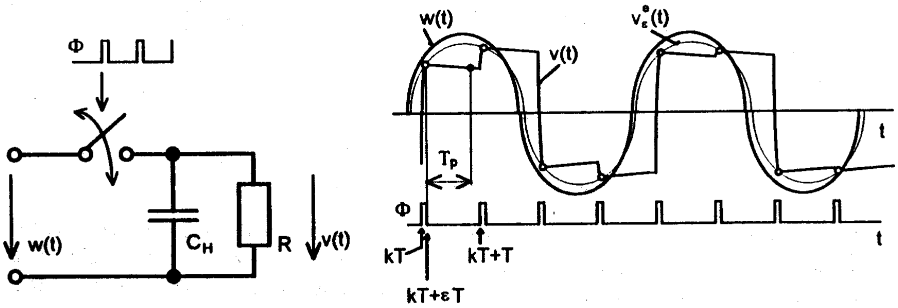

Fig.2. Simplified model of *SH* circuit and corresponding responses including equivalent signal $v_\varepsilon^e(t)$.

Other errors occuring during *SH* process can be modeled using *GTF*s. For example, voltage-drop error during time interval T_p can be investigated, comparing equivalent signals $v_\varepsilon^e(t)$ and $v_{T_p/T}^e(t)$.

Some of the problems mentioned above have been solved in [4]. As described below, the *GTF* is the function of both well-known operators s and z (Laplace- and z- transformation). After its computation, the double substitutions $s = j\omega$ and $z = \exp(j\omega T)$ lead to the frequency response.

Let us examine time response of two-phase second order *SC* filter in Fig.3. Due to switching, the output signal contains high-frequency spectral components. Computing of complete output spectrum is expensive and time consuming. Two continuous-time equivalent signals $v_{5,1}^e$ and $v_{5,2}^e$ create a certain envelopes of impulse signals. Comparing equivalent signals with input signal $v_1 = v_1^e$ for various frequencies, many information on transient properties in both switching phases can be obtained.

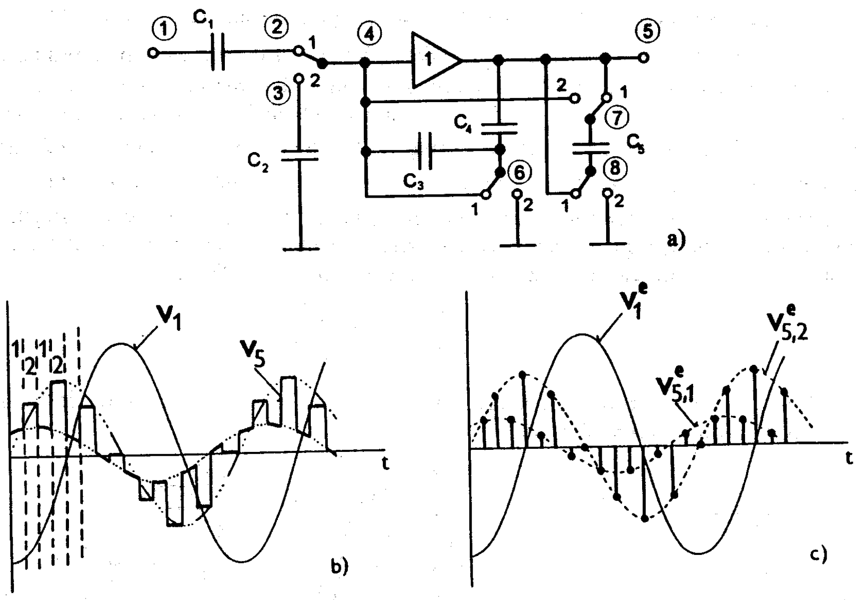

Fig.3. a) SC filter, b) real and c) equivalent signals.

From this point of view, this system with time-varying parameters can be described using relation of *Fourier/Laplace* transformation of certain equivalent output signal and *Fourier/Laplace* transformation of input signal. In this way, the set of *GTFs* can be obtained for single network, taking into account various types of output/input regimes. The *GTFs* can be defined as follows:

$$K_1 = \frac{L\{v_{5,1}^e\}}{L\{v_1^e\}},$$

$$K_2 = \frac{L\{v_{5,2}^e\}}{L\{v_1^e\}}.$$

In following part, the mathematical description of *GTF*s will be presented.

3. MIXED S-Z DESCRIPTION OF PERIODICALLY VARYING SYSTEMS

Let us consider linear time-varying system with period T of parameters alternation. For simplicity, single input will be assumed. The modified nodal approach (*MNA*) [2] leads to the matrix differential equation

$$\mathbf{G}(t)\mathbf{v}(t) + \frac{d}{dt}[\mathbf{C}(t)\mathbf{v}(t)] = \mathbf{D}(t)w(t), \tag{3}$$

where **G**,**C**,**D** and **v** are conductance and reactance modified matrices, an incidence vector and nodal variable vector, respectively; w is the input signal.

Solving Eq.(3) for $t\in(kT+\varepsilon T-T, kT+\varepsilon T\rangle$, $k=\ldots,-1,0,1,..$ yields

$$\mathbf{v}(t)=\mathbf{g}^*(t,kT+\varepsilon T-T)\mathbf{v}(kT+\varepsilon T-T)+\int\limits_{kT+\varepsilon T-T}^{t}\mathbf{g}(t,\xi)w(\xi)d\xi, \tag{4}$$

where $\mathbf{g}^*(t,kT+\varepsilon T-T)$ and $\mathbf{g}(t,\xi)$ are two-dimensional initial condition response and impulse response matrices, respectively. Considering time instances $t=kT+\varepsilon T$, Eq.(4) can be rewritten as follows:

$$\begin{aligned}&\mathbf{v}(kT+\varepsilon T)=\\&=\mathbf{g}^*(kT+\varepsilon T,kT+\varepsilon T-T)\mathbf{v}(kT+\varepsilon T-T)+\int\limits_0^T\mathbf{g}_x(kT+\varepsilon T,\xi)w(kT+\varepsilon T-\xi)d\xi,\end{aligned} \tag{5}$$

where $\mathbf{g}_x(kT+\varepsilon T,\xi)=\mathbf{g}(kT+\varepsilon T, kT+\varepsilon T-\xi)$ is impulse response at time $kT+\varepsilon T$ on the assumption, that *Dirac* impulse has appeared at time delay ξ before time of observation $kT+\varepsilon T$.

Due to time periodicity of system parameters, Eq.(5) can be simplified:

$$\mathbf{v}(kT+\varepsilon T)=\mathbf{g}^*(\varepsilon T,\varepsilon T-T)\mathbf{v}(kT+\varepsilon T-T)+\int\limits_0^T\mathbf{g}_x(\varepsilon T,\xi)w(kT+\varepsilon T-\xi)d\xi. \tag{6}$$

Eq.(6) is an interesting recurrent formula of periodically varying system. This formula can be solved using the idea of equivalent signals.

Consider the equivalent vector signal $\mathbf{v}_\varepsilon^e(t)$,

$$\mathbf{v}_\varepsilon^e(t)\Big|_{t=kT+\varepsilon T}=\mathbf{v}(kT+\varepsilon T) \tag{7}$$

It will be shown that replacing discrete time variable by continuous time according to substitution

$$kT+\varepsilon T\to t, \tag{8}$$

the equation (6) can be rewritten for equivalent signal:

$$\mathbf{v}_\varepsilon^e(t)=\mathbf{g}^*(\varepsilon T,\varepsilon T-T)\mathbf{v}_\varepsilon^e(t-T)+\int\limits_0^T\mathbf{g}_x(\varepsilon T,\xi)w(t-\xi)d\xi. \tag{9}$$

This statement can be verified by comparison of equations (7) and (9) with the properties of equivalent signal given by items 1 and 2 mentioned in chapter 1. Equation (7) is the confirmation of the item 1, that the equivalent and output signals have common points at sampling instances $kT+\varepsilon T$. As the equivalent and input signals in (9) are bound by linear equation, their frequency ranges are identical and item 2 is then fulfilled.

Applying *Laplace* transform to equation (9) yields

$$\mathbf{V}_\varepsilon^e(s)=\mathbf{g}^*(\varepsilon T,\varepsilon T-T)\mathbf{V}_\varepsilon^e(s)e^{-sT}+\int\limits_0^\infty\int\limits_0^T\mathbf{g}_x(\varepsilon T,\xi)w(t-\xi)e^{-st}d\xi\,dt.$$

After short arrangement

$$\mathbf{V}_\varepsilon^*(s,z)=\mathbf{K}_\varepsilon(s,z)\Big|_{z=e^{sT}}W(s),$$

where $\mathbf{K}_\varepsilon(s,z)$ is so-called *generalized transfer function* (*GTF*) of time-varying system

$$\mathbf{K}_\varepsilon(s,z)=\left[\mathbf{E}-\mathbf{g}^*(T+\varepsilon T,\varepsilon T)z^{-1}\right]^{-1}\mathbf{G}_\varepsilon(s) \tag{10}$$

and

$$\mathbf{G}_\varepsilon(s)=\int_0^T \mathbf{g}_x(\varepsilon T,\xi)e^{-s\xi}d\xi \tag{11}$$

is the short-time *Laplace* transform of impulse response. **E** is the symbol for the identity matrix.

The *GTF* (10) consists of two multiplicative parts. First part - function of z - operator, express the time-discretization effect due to periodically varying parameters. Second part - function of s - operator, refers to continuous-time system character. For more details concerning physical interpretation of *GTF* see [3-5].

The dynamic properties of linear time-varying system can be investigated of the s- and z-domain pole location. As regards system stability, the z- domain poles are decisive. As follows from (10), z- domain poles are eigenvalues of the matrix $\mathbf{g}^*(T+\varepsilon T,\varepsilon T)$. The s-domain poles determine the transient process character caused by the variation of system parameters.

For practical simulation, computing of matrices $\mathbf{g}^*$ and $\mathbf{g}$ from *MNA* description is the fundamental problem. The detailed solution is beyond the scope of this contribution. For simplicity, modeling of systems with jump-varying parameters will be discussed. The *switched capacitor*, general *switched filters* and *sample-hold* circuits are typical applications.

On the other hand, this approach can be used for the solution of systems with continuous-varying parameters, if the number N of below mentioned phases is satisfactory large.

4. SYSTEMS WITH JUMP-VARYING PARAMETERS

Let period T is divided to N phases each of length T_i (see Fig.4). Suppose that the system is stationary during each phase. In i-th phase, the *MNA* equations in the s-domain are as follows [5]:

$$\begin{aligned}(\mathbf{G}_1+s\mathbf{C}_1)\mathbf{V}_1(s)&=\mathbf{C}_{1,N}\mathbf{v}_N(kT^-)+\mathbf{D}_1w_1(s)\\(\mathbf{G}_i+s\mathbf{C}_i)\mathbf{V}_i(s)&=\mathbf{C}_{i,i-1}\mathbf{v}_{i-1}(kT+\sigma_{i-1}^-)+\mathbf{D}_iw_i(s),\ i=2,3,..,N,\end{aligned} \tag{12}$$

where $\mathbf{C}_{i,i-1}$ are the matrices of phase-to-phase energy transfer. These equations can be rewritten to the matrix form:

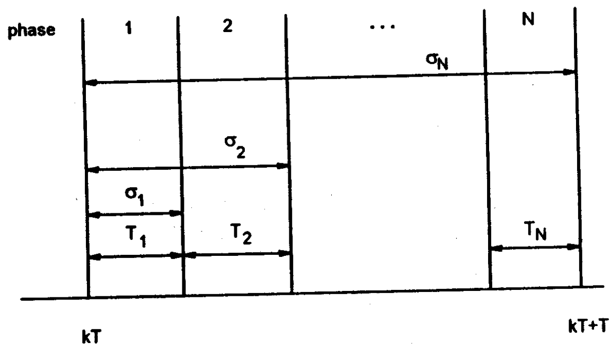

Fig.4. Switching diagram.

$$
\begin{bmatrix}
\mathbf{G}_1+s\mathbf{C}_1 & & & & \\
 & \mathbf{G}_2+s\mathbf{C}_2 & & & \\
 & & \mathbf{G}_3+s\mathbf{C}_3 & & \\
 & & & \cdots\cdots & \\
 & & & & \mathbf{G}_N+s\mathbf{C}_N
\end{bmatrix}
\begin{bmatrix}
\mathbf{V}_1(s) \\ \mathbf{V}_2(s) \\ \mathbf{V}_3(s) \\ \cdots\cdots \\ \mathbf{V}_N(s)
\end{bmatrix}
=
$$

$$
=
\begin{bmatrix}
 & & & & \mathbf{C}_{1,N} \\
\mathbf{C}_{2,1} & & & & \\
 & \mathbf{C}_{3,2} & & & \\
 & & \cdots\cdots & & \\
 & & & \mathbf{C}_{N,N-1} &
\end{bmatrix}
\begin{bmatrix}
\mathbf{v}_1(kT+\sigma_1^-) \\ \mathbf{v}_2(kT+\sigma_2^-) \\ \mathbf{v}_3(kT+\sigma_3^-) \\ \cdots\cdots \\ \mathbf{v}_N(kT^-)
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{D}_1 w_1(s) \\ \mathbf{D}_2 w_2(s) \\ \mathbf{D}_3 w_3(s) \\ \cdots\cdots \\ \mathbf{D}_N w_N(s)
\end{bmatrix}
$$

Applying theory of equivalent signals yields large set of equations

$$
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & -\mathbf{g}_1^\bullet e^{-sT_1} \\
-\mathbf{g}_2^\bullet e^{-sT_2} & 1 & 0 & \cdots & 0 & 0 \\
0 & -\mathbf{g}_3^\bullet e^{-sT_3} & 1 & \cdots & 0 & 0 \\
\cdot & & \cdot & \cdots & \cdot & \cdot \\
0 & 0 & 0 & \cdots & -\mathbf{g}_N^\bullet e^{-sT_N} & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{V}_1^e \\ \mathbf{V}_2^e \\ \mathbf{V}_3^e \\ \cdot \\ \mathbf{V}_N^e
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{G}_1(s)\mathbf{W}_1(s) \\ \mathbf{G}_2(s)\mathbf{W}_2(s) \\ \mathbf{G}_3(s)\mathbf{W}_3(s) \\ \cdot \\ \mathbf{G}_N(s)\mathbf{W}_N(s)
\end{bmatrix}
\qquad (13)
$$

where

$$
\mathbf{g}_i(t) = L^{-1}\{(\mathbf{Y}_i(s))^{-1}\}\mathbf{D}_i, \quad \mathbf{g}_i^\bullet = L^{-1}\{(\mathbf{Y}_i(s))^{-1}\}\mathbf{C}_{i,i-1}, \quad \mathbf{G}_i(s) = \int_0^{T_i} \mathbf{g}_i(\xi)e^{-s\xi}d\xi. \qquad (14)
$$

The Eq.(13) must be solved for unknown vectors $\mathbf{V}_i$, $i=1,2,..,N$ to obtain *generalized transfer functions*. Analytical solution is as follows:

$$\mathbf{V}_N = \sum_{m=1}^{N} \mathbf{K}_m W_m, \tag{15}$$

where

$$\mathbf{K}_m(s,z) = \left[\mathbf{E} - z^{-1}\prod_{i=1}^{N-m-1}\mathbf{g}^{\bullet}_{N-i}\right]^{-1}\prod_{i=0}^{N-m-1}\mathbf{g}^{\bullet}_{N-i}e^{-sT_{N-i}}\mathbf{G}_m(s). \tag{16}$$

The way of computer implementation is described in [8] and will not be presented here. As an example, let us derive the *GTF* of *SH* circuit described in chapter 2, Fig.2. In this case, the number of switching phases is N=2. Equations (12) are:

$$\begin{aligned}(G+G_s+sC)V_1(s) &= Cv_2(kT^-)+G_s w(s)\\ (G+sC)V_2(s) &= Cv_1(kT+\varepsilon T)\end{aligned},$$

where $G = 1/R$, $G_S = 1/R_s$ is the switch-on conductance, and the numbers of switching phases are designated by subscripts 1 and 2 (1 - switch on, 2 - switch off). The time-domain solution is as follows:

$$\begin{aligned}v_1(kT+\varepsilon T) &= g_1^{\bullet}v_2(kT^-) + \int_0^{\varepsilon T} g(\xi)w(kT+\varepsilon T-\xi)d\xi,\\ v_2(kT^-) &= v_1(kT-T+\varepsilon T^-)g_2^{\bullet},\end{aligned} \tag{17}$$

where

$$g_1^{\bullet} = e^{-\varepsilon T/\tau_1},\quad g_2^{\bullet} = e^{-(T-\varepsilon T)/\tau_2},\quad g(t) = \frac{1}{\tau_1}e^{-t/\tau_1},\quad \tau_1 = \frac{R_s R}{R_s+R}C,\quad \tau_2 = RC.$$

Equations (17) yield the recurrent formula which can be solved using equivalent signals:

$$v_\varepsilon^{\varepsilon}(t) = g_1^{\bullet}g_2^{\bullet}v_\varepsilon^{\varepsilon}(t-T) + \int_0^{\varepsilon T} g(\xi)w(t-\xi)d\xi$$

After *Laplace* transformation and some arrangements the *GTF* (2) is obtained in following closed form:

$$K_\varepsilon(s,z) = \frac{R}{R+R_s}\frac{1}{1-z^{-1}g_1^{\bullet}g_2^{\bullet}}\frac{1-g_1^{\bullet}z^{-\varepsilon}}{1+s\tau_1} \tag{18}$$

It should be noted that this *GTF* models the sample selection dynamic error of *SH* circuit. This transfer function is characterized by s- and z- domain zero-pole location in Tab.1:

	zeros	poles
s-domain	-	$-\frac{1}{\tau_1}$
z-domain	0 $\sqrt[\varepsilon]{g_1^{\bullet}} = e^{-T/\tau_1}$	$g_1^{\bullet}g_2^{\bullet} = e^{-\varepsilon T/\tau_1}e^{-(T-\varepsilon T)/\tau_2}$

Tab.1. s- and z- domain zero-pole location of *SH* circuit.

After double substitution $s=j\omega$ and $z=exp(j\omega T)$, the frequency responses in Fig.5 have been plotted. For comparison, the zero-pole location is attached. However, *s*- and *z*- domain pole connection is very interesting but cannot be investigated in this contribution.

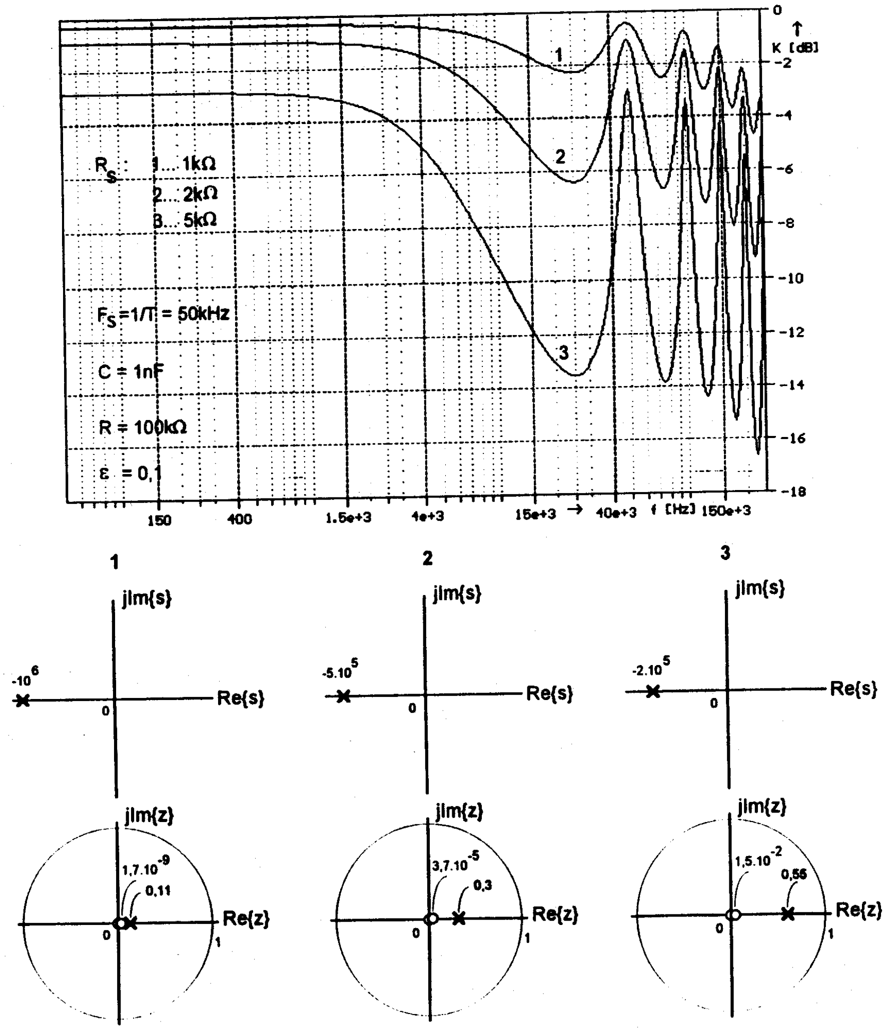

Fig.5. Examples of frequency responses of *SH* circuit computed using *generalized transfer functions* and corresponding *s*- and *z*- domain zero- pole location.

To verified aforementioned theory, computer program *SPIN* for switched networks simulation has been developed [7-9]. In following section, some essential principles of computer simulation are presented.

5. COMPUTER SIMULATION

Aforementioned theory has been verified using computer simulation. Present-day version of program *SPIN* enables to solve linear two-phase networks with external switching, consisting of ideal *OpAmps* and real *OpAmps* with standard gain response, transimpedance *OpAmps*, ideal voltage-controlled voltage sources, capacitors, resistors and zero and nonzero-R_{on} switches. The complexity of analyzed network is limited by available memory (now approximately: 40 nodes, 30 capacitors, 30 resistors, 10 *OpAmps*, 50 switches). The program is in the testing stage. It computes gain and phase responses including the possibility of drawing more shapes by variation of network parameters.

The computer simulation is performed in following steps:

- *Circuit equations creation.*
- *Time-frequency pre-processing of circuit equations.*
- *Semi-symbolic computation of generalized transfer functions.*
- *Frequency analysis.*

Circuit equations creation

The *two-graph modified nodal approach* (*MNA*) [2] has been used as the basic method. To create minimum number of equations, our procedure is as follows:

- We find the set of independent nodal variables in each switching phase:

$$\begin{aligned} \mathbf{v}_1 &= \left[v_1^1, v_1^2, .., v_1^{n_1}\right]^T \text{... phase 1,} \\ \mathbf{v}_2 &= \left[v_2^1, v_2^2, .., v_2^{n_2}\right]^T \text{... phase 2,} \end{aligned} \tag{19}$$

where n_1 and n_2 are numbers of independent variables in phase 1 and phase 2, respectively. The algorithm how to obtain that sets is described in [9].

- It is possible to create the set of differential equations of *MNA* in the operator domain for independent variables:

$$\begin{aligned} \underbrace{\left(\mathbf{G}_1^M + s\mathbf{C}_1^M\right)}_{\mathbf{Y}_1^M}\mathbf{V}_1 &= \mathbf{D}_1 W_1 + \mathbf{C}_{12}^M \mathbf{v}_2(kT^-), \\ \underbrace{\left(\mathbf{G}_2^M + s\mathbf{C}_2^M\right)}_{\mathbf{Y}_2^M}\mathbf{V}_2 &= \mathbf{D}_2 W_2 + \mathbf{C}_{21}^M \mathbf{v}_1(kT + T_1^-), \end{aligned} \tag{20}$$

where $\mathbf{G}_1^M$ and $\mathbf{G}_2^M$ are modified conductance matrices of dimensions ($n_1 x\ n_1$) and ($n_2 x\ n_2$),
$\mathbf{C}_1^M$ and $\mathbf{C}_2^M$ are modified reactance matrices of dimensions ($n_1 x\ n_1$) and ($n_2 x\ n_2$),
$\mathbf{C}_{12}^M$ and $\mathbf{C}_{21}^M$ are modified transitive matrices of dimensions ($n_1 x\ n_2$) and ($n_2 x\ n_1$),
$\mathbf{D}_1$ and $\mathbf{D}_2$ are certain incidence vectors of dimensions ($n_1 x$ 1) and ($n_2 x$ 1),
W_1 and W_2 are Laplace transformations of input signals in phases 1 and 2,
T_1 is length of phase 1.

Equations (20) have key significance for computer analysis. It is possible to compile them for arbitrary linear switched network no matter whether the consistent initial conditions occur in the time instances between two switching phases [7]. In practice, the simulator will produce correct results also in case of discontinuously changed nodal variables.

The circuit matrices in (20) are created in several steps:

Insertion of passive components C and R submatrices including non-zero R_{on} switches.
Insertion of own and mutual inductance matrices.
Insertion of real classical and transimpedance OpAmps matrices.
Circuit matrices modification in respect of input voltage sources.

These items are described in [5].

Time-frequency pre-processing of circuit equations

The circuit equations (20) must be pre-processed with the aim to derive equations for *GTF*s in closed form. The general solution is as follows [5]

$$\begin{aligned} \mathbf{V}_1^*(s,z) &= \mathbf{K}_{11}(s,z)W_1^*(s) + \mathbf{K}_{12}(s,z)W_2^*(s), \\ \mathbf{V}_2^*(s,z) &= \mathbf{K}_{21}(s,z)W_1^*(s) + \mathbf{K}_{22}(s,z)W_2^*(s). \end{aligned} \tag{21}$$

The generalized transfer functions $\mathbf{K}_{11}, \mathbf{K}_{12}, \mathbf{K}_{21}$ and $\mathbf{K}_{22}$ depend on the input signal character (*Sample-Hold* response, arbitrary response...). For more details see [5].

Semi-symbolic and frequency analysis

The computer algorithm for evaluation of *GTF*s in semi-symbolic form has been developed. However, its description is beyond the scope of this paper.

The frequency responses can be easily obtained by aforementioned double substitution.

Simulation example

For illustration, the switched filter in Fig.6 is simulated for various sampling frequencies to investigate its tuning ability. It should be noted, that the new tuning principle is used in this circuit: All resistors are fixed and the tuning effect is accomplished using equivalent capacitances alternation in mutators [8].

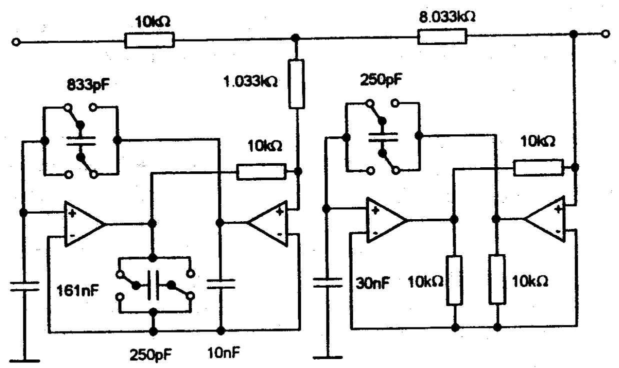

Fig.6. Simulated switched filter; $R_{switch,ON} = 400\Omega$, $GBW_{OZ} = 5MHz$ [8].

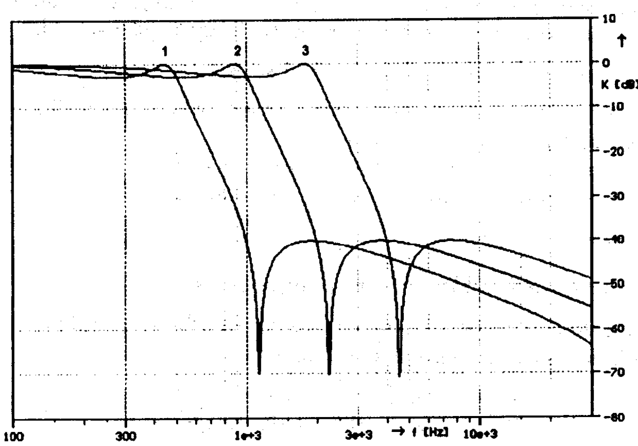

Fig.7. Simulation results - amplitude frequency responses for various sampling frequencies: 1 20kHz, 2 100kHz, 3 200kHz.

6. CONCLUSION

The novel approach to the modeling of periodically varying linear systems is introduced in this contribution. Mixed description both in the *s*- and *z*- domain using the ***generalized transfer function*** is described. Some results of computer simulation illustrate presented theory.

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