VERTEX and Causal Graphs of Linear Systems

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Abstract: *The paper deals with connections between the so-called VERTEX and causal graphs for linear dynamic system optimization with a view to active frequency filters. It is known that a certain class of circuit structures exists whose VERTEX graph can be transformed to separated causal graphs. These systems then offer several degrees of freedom for their optimization. Another class of systems is described by only one causal graph, when at most one degree of freedom is available. VERTEX graphs of a number of systems can be transformed to the causal graphs only in part: also VERTEX subgraphs are present which cannot be removed. The connections among the variations of individual parameters are then nonlinear, which exacts special optimization procedures.*

Introduction

Let us consider a general linear system with the nominal parameters p_i of its components, associated in the vector $P=[p_1, p_2,.., p_m]$. The required system behavior is defined by a set of operator system functions, which fix the parameters p_i by conditions of the type

$$F^{j}(P)=f^{j},\ j=1, 2, .., n. \tag{1}$$

Here f^{j} are real constants, e.g. coefficients of transfer functions.

Function F has a special form when analyzing linear circuits symbolically: it is an algebraic sum of N products Π_k (k=1,2,..,N) of M parameters, where N and M are natural numbers. Some products can enter the sum with negative sign, depending on the type of circuit solved:

$$F^{j}=\sum_{k=1}^{N}\left(\pm\Pi_k^{j}\right)=f^{j},\ j=1, 2, .., n. \tag{2}$$

Equations (1) and (2) describe by a general way the couplings among the internal parameters of arbitrary linear system, no matter of its electrical or another nature.
The so-called VERTEX graphs introduced in [1] can be a graphic description of these equations. In the paper, we extend the problem of VERTEX graphs by so-called causal graphs, which will enable to monitor the consequences of variations of individual parameters around their nominal values, as well as the possible ways of compensations of these variations by another parameters.

VERTEX versus causal graphs

The connection between the VERTEX and causal graphs can be shown on an example of filter in Fig. 1. The voltage transfer functions are as follows:

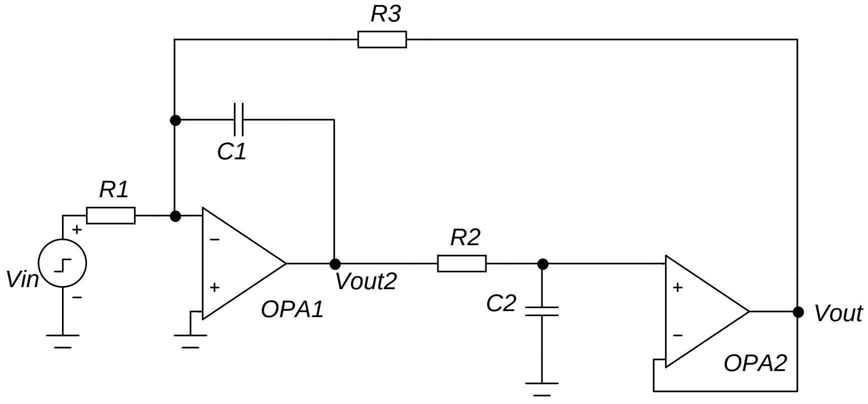

Fig. 1. Example of a 2nd order filter.

$$\frac{V_{out}}{V_{in}} = -\frac{1}{b_0 + b_1 s + b_2 s^2}, \quad \frac{V_{out2}}{V_{in}} = -\frac{1 + a_1 s}{b_0 + b_1 s + b_2 s^2}, \tag{3, 4}$$

where the coefficients are given by the following equations:

$$a_1 = R_2 C_2 = 250\mu s, \; b_0 = R_1 G_3 = 1, \; b_1 = R_1 C_1 = 10\mu s, \; b_2 = R_1 R_2 C_1 C_2 = 2.5 ns^2 \tag{5, 6, 7, 8}$$

The corresponding VERTEX graph is in Fig. 2 (a) [1]. It should be noted that equation (8) results from (5) and (7) and thus it can be omitted. The given simplified VERTEX graph is in Fig. 2 (b).

Let the set of equations (5, 6, 7) be rewritten to the form

$$C_2 = a_1 G_2, \; G_3 = b_0 G_1, \; G_1 = \frac{1}{b_1} C_1, \tag{9, 10, 11}$$

where $G_1=1/R_1$, $G_2=1/R_2$, $G_3=1/R_3$. The corresponding causal graph is in Fig. 2 (c).

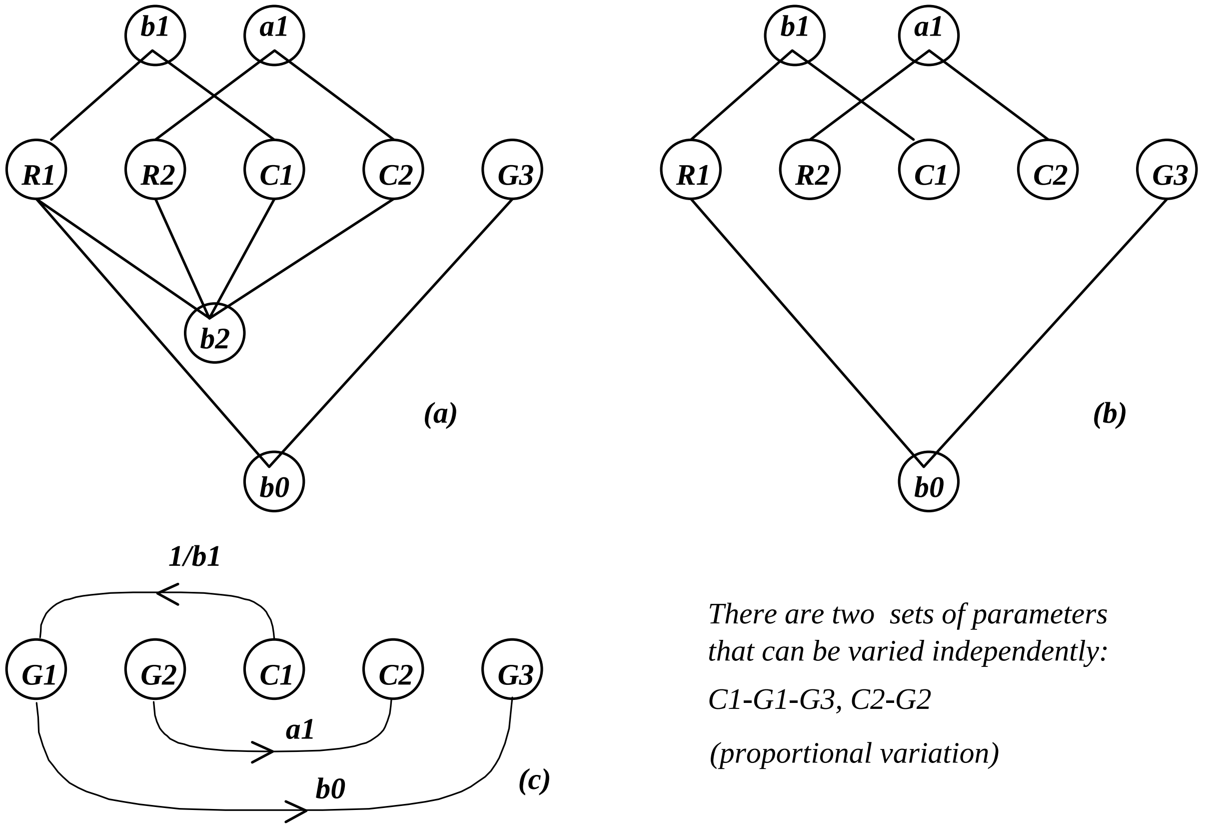

Fig. 2. (a) VERTEX, (b) simplified VERTEX, (c) causal graph of circuit in Fig. 1.

All the three graphs describe the couplings among the parameters of system components, which have to be take into account while varying them to preserve transfer functions (1) and (2). The caption near Fig. 2 (c) explains the meaning of the idea of “proportional variation”, which can be applied independently to two disjunct sets of parameters. For example, it follows from the causal graph that the variation of C_1 cannot be compensated by C_2 even by R_2 in any case. However, the compensation is possible by a concurrent changing R_1 and R_3 .

The well-known „MFB“ filter [2] is in Fig. 2 with the transfer function

$$\frac{V_{out}}{V_{in}} = -\frac{a_0}{b_0 + b_1 s + b_2 s^2} \tag{12}$$

The coefficients are as follows:

$$a_0 = G_1G_3,\ b_0 = G_2G_3,\ b_1 = C_2(G_1 + G_2 + G_3),\ b_2 = C_1C_2 \qquad (13, 14, 15, 16)$$

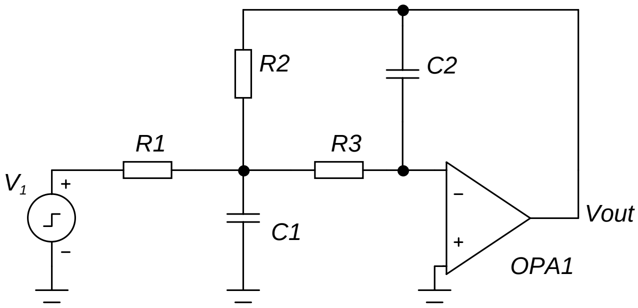

Fig. 3. The „MFB“ filter structure.

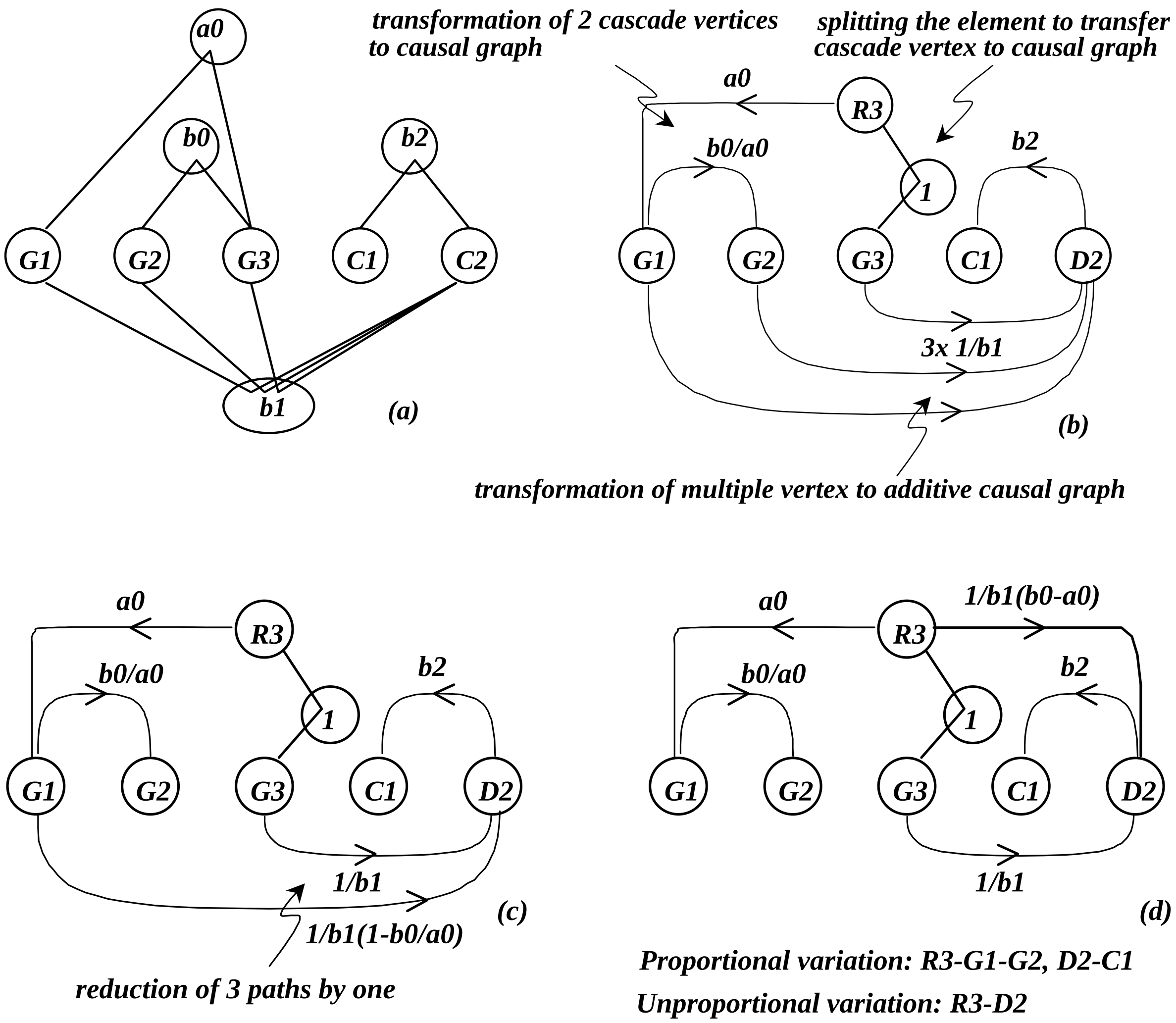

Fig. 4. (a) VERTEX graph, (b)-(d) hybride graphs of circuit in Fig. 3. Here D_2=1/C_2.

The corresponding VERTEX graph is in Fig. 4 (a). It is modified step-by-step (see Figures 4 (b), (c), and (d)) with the aim to obtain a simple causal graph. However, the VERTEX graph above the R_3-G_3 basis could not be removed. The resulting graph shows the subsets of both the proportional and unproportional variation of parameters. For instance, for the variation of R_3 from its nominal value, it is necessary to perform „proportional compensation “ by means of the resistances R_1 and R_2 , and the more complicated – “unproportional compensation” by means of capacitance C_2. Then this variation of C_2 will result in a proportional variation of C_1. In contrast to the filter abobe, all the parameters of circuit elements are mutually coupled.

Generalization

The linear dynamic systems with given operator-domain system functions can be described by VERTEX graphs. Some of these graphs can be fully transformed into the causal graphs, which give directions how to proportionally vary component while preserving required system functions. However, in many cases this transformation into the causal graph is impossible. Then the coupling among given parameters are nonlinear.

The resulting graphs then illustrate couplings among the individual parameters which can be applied to system optimization.

Conclusion

The method described enables the design of algorithms, which explore mutual couplings between the element parameters of a complicated linear dynamic system. These couplings are defined by the features of required circuit functions. Typical examples of application are the optimization of element ratios and of the dynamic range of filters, and the compensation of the influence of undesirable changes in element parameters on the total circuit performance.
The practical application of VERTEX and causal graphs is subject to designing algorithms of automatic composition of VERTEX graphs, and especially algorithms of their step-by-step transformation into the causal graphs. The VERTEX graphs can be generated from the symbolic results while analyzing the system functions. We have developed SNAP program for these purposes [3]. From the mathematical point of view, the VERTEX-to-causal graph transformation represents rather complex problem, which is currently solved.

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