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Teaching linear circuit analysis effectively

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ABSTRACT: The authors discuss possible techniques of teaching the analysis of linear circuits. Three pedagogic aims are followed: student's understanding of the function of the circuit analyzed, ability to solve moderate circuits algorithmically, and the basic understanding of the function of professional circuit simulators as an assumption of their later effective utilization.

INTRODUCTION

Methods of linear circuit analysis have had a lengthy evolution. Computer-aided simulation of large electronic systems is a significant accelerator of this process: it has stimulated new and nontraditional approaches to how to speed up analysis and save computer memory. That is why circuit analysis is a wide discipline and poses the question of how to teach it. There are several possibilities. The choice depends on the teaching aims. The following aims of teaching circuit analysis are particularly frequent:

1. To promote an engineering way of thinking while solving electrical circuits, accentuating creative solutions.
2. To master the effective tool - algorithm of "hand and paper" solution of relatively simple circuits.
3. To understand the principle of modern methods of circuit analysis used in computer simulators (and to frame preconditions for mastering them better).

Different methods of analysis and their combinations are optimum for different teaching aims. In the paper, we divide all existing methods of analysis into heuristic (appropriate for aim No. 1) and algorithmic (preferred for aims Nos. 2 and 3). In the first stage of studying Electrical Engineering (EE), it is advisable to lead the student towards creative analysis by using Ohm's and Kirchhoff's laws (K.Ls) and the fundamental principles of theoretical EE. Aim No 1 should thus be pursued by means of heuristic methods, starting in the opening courses of EE. However, we must simultaneously break new ground for the teaching of algorithmic methods. These methods are suitable for those students who have problems with creative thinking. After understanding the algorithm, its application always leads to a conclusion. However, utilizing the algorithm can be "dangerous" because of the solution being separated from the physical background of problem solved.

One of the ways how to reduce this drawback is to inject creative features into the algorithmic method. In other words, in the course of analysis it is necessary to decide on the next procedure. This decision is taken on the basis of physical consideration. For instance, the consideration can be conditional on the student being aware of the physical background of compiled circuit equations. As a typical example, we can mention the compilation of voltage coupling equations given below for one of the variants of modified nodal analysis.

After the student has become acquainted with both the heuristic and the algorithmic methods, the problem of algorithm "dangerousness" becomes less significant. Instead of clear-cut considerations whether to teach by the first or the second group of methods, a more advantageous variant appears: to combine them. In this symbiosis, each method fulfills its function: the heuristic method induces physical thinking, the algorithmic one offers a useful tool of practical solution.

In the following text, we will confine ourselves to algorithmic matrix methods of analysis that proved to be effective in our teaching practice: two special variants of modified nodal analysis (MNA).

SELECTION OF ALGORITHMIC METHOD

In the basic courses of EE, three algorithmic methods are usually discussed: The method of Kirchhoff's and element laws; the method of loop currents; the method of node voltages. In circuit theory, students get acquainted with the method of state variable which fills a special position among the methods of analysis: There exist several algorithms of state equation compilation, but they are not taught due to their elaborateness (and they seem to be out-of-date). Students familiarize themselves with the general principle of state matrix generation, but they use a creative approach to its compilation.

In many schools, some of the variants of MNA are taught. These methods enable analyses irrespective of the type of linear elements. MNA is a promising method of the present and the future. It is implemented in most modern simulators of analog networks. It is thus well algorithmized. With proper modification, it is also suitable for "hand and paper" analysis. For these reasons, the inclusion of MNA in EE curriculum is highly advisable.

The problem is, however, that presently there are numerous MNA variants. Which one should we concentrate on? The answer depends again on the teaching aims.

Various MNA variants always represent a compromise between the "transparency", i.e. simplicity, of the algorithm of equation compilation, and the number of these equations, i.e. the computation complexity. While selecting an appropriate variant, we can move in a space which is bounded by two extremes:

1. The maximum number of circuit equations and a primitive and "easy-to-remember" algorithm of their compilation without the necessity of using the creative approach.
2. The minimum number of circuit equations and a more complicated algorithm of their compilation, utilizing the creative approach.

While teaching modern methods of computer-aided analysis, the given aspects will not be as significant as when we try to give the student an effective tool for "hand and paper" analysis. Then the important factor will be the number of equations and the resultant tendency to drift to extreme No 2.

Next we will describe two MNA variants which do not give a great number of equations and thus they are suitable for "hand and paper" computation. They are especially advantageous for the analysis of circuits containing voltage-controlled voltage sources (VCVSs), but not other types of controlled sources. This fact does not represent any limitation while analyzing circuits with operational amplifiers (OpAmps) or buffers. In the case of other controlled sources, it is better to use a different MNA which increases the number of equations by the number of controlled sources. This method is well-known so that we will not mention it in the following.

MNA VARIANTS WITH MINIMUM NUMBER OF EQUATIONS

From the point of view of computational complexity, the optimal method is that which leads to the minimum number of circuit equations.

From the pedagogical point of view, the optimal method should represent a well-balanced compromise between the number of circuit equations and the complexity of the algorithm of their compilation.

Opinions on how to realize this compromise differ from one teacher to another.

Depending on the MNA variant used, the VCVSs can increase, maintain or decrease the number of equations with respect to the number of equations of the standard nodal analysis, which is equal to the number of independent nodes. We will describe methods that both maintain and decrease the number of equations, which proved good in the teaching process.

Method maintaining the number of equations (“dead row” method) [1]

Regardless of the number of voltage sources, we compile as many equations as the number of independent nodes. Equations of the first K.L. will be compiled only for those nodes to which no voltage source is connected. For the node where a voltage source is connected we write so-called voltage coupling equation. Its form can be *either node voltage = source voltage* (for classical sources) or *node voltage of controlled source = linear combination of other node voltages* (for controlled sources).

The corresponding algorithm of compilation of the circuit equations is as follows:

We draw a “frame” of matrix equation. By a special symbol, for instance a circle, we mark such rows that correspond to the nodes to which the voltage sources are connected. The voltage coupling equations, corresponding to these sources, will be inscribed into these rows. Equations of the first K.L. for the other nodes will be inscribed into the other rows. Here one can use the well-known rule of thumb when filling element symbols into the main diagonal of the circuit matrix and the other elements outside this diagonal. However, we must not fill the rows marked above (that is why the these rows are called “dead”), otherwise the voltage coupling conditions would be modified.

As an example, let us mention the *Sallen - Key* filter which is excited by the current/voltage source (see Figure 1).

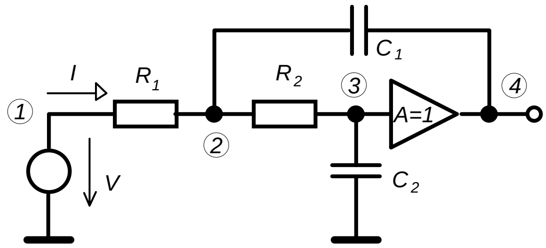

Figure 1. Active filter analyzed.

$$\begin{array}{c|c|c|c|c|c}
 & V_1 & V_2 & V_3 & V_4 & \\
\hline
1 & G_1 & -G_1 & & & \\
2 & -G_1 & G_1+G_2+sC_1 & -G_2 & -sC_1 & \\
3 & & -G_2 & G_2+sC_2 & & \\
4 & & & A & -1 & \bullet
\end{array}
\begin{array}{|c|}\hline V_1\\ \hline V_2\\ \hline V_3\\ \hline V_4\\ \hline\end{array}
=
\begin{array}{|c|}\hline I\\ \hline \\ \hline \\ \hline \\ \hline\end{array}$$

dead rows (1)

$$\begin{array}{c|c|c|c|c|c}
 & V_1 & V_2 & V_3 & V_4 & \\
\hline
1 & 1 & & & & \bullet \\
2 & -G_1 & G_1+G_2+sC_1 & -G_2 & -sC_1 & \\
3 & & -G_2 & G_2+sC_2 & & \\
4 & & & A & -1 & \bullet
\end{array}
\begin{array}{|c|}\hline V_1\\ \hline V_2\\ \hline V_3\\ \hline V_4\\ \hline\end{array}
=
\begin{array}{|c|}\hline V\\ \hline \\ \hline \\ \hline \\ \hline\end{array}$$

Some educators try to avoid the “creative element”, which consists in the necessity to formulate voltage coupling conditions: they define so-called matrices – stamps of controlled sources [2] (concretely, for the given amplifier this matrix contains two nonzero elements A and -1, see equation 1). Students memorize these matrices-stamps. According to our experience, it is preferable to create voltage coupling equations. On the contrary, we show the students that the coupling condition in the dead row can be written by many ways without affecting the analysis result (we can write $AV_3 - V_4 = 0$ as well as $V_3 - V_4/A = 0$, etc.).

For illustration, let us mention the second example – the set of equations for computing the input impedance of the mutator in Figure 2.

For completeness' sake, it should be noted that the algorithm became rather complicated in the case of ungrounded voltage sources. Fortunately those cases are not very frequent.

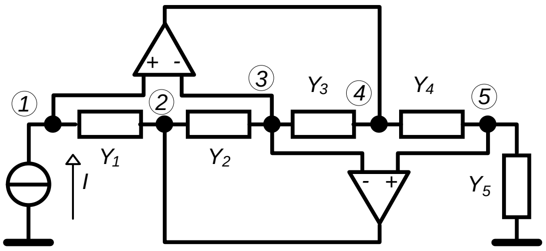

Figure 2. *Antonious mutator*.

$$\begin{array}{c|ccccc|c|c|c}
 & V_1 & V_2 & V_3 & V_4 & V_5 & & & \\
1 & Y_1 & -Y_1 & & & & & V_1 & I \\
2 & & & -1 & & 1 & \bullet & V_2 & \\
3 & & -Y_2 & Y_2+Y_3 & -Y_3 & & & V_3 & = \\
4 & 1 & & -1 & & & \bullet & V_4 & \\
5 & & & & -Y_4 & Y_4+Y_5 & & V_5 &
\end{array} \qquad (2)$$

Method decreasing the number of equations (modified two-graph method) [2]

(a) Reduction of number of equations due to uncontrolled voltage source

Note that in the second variant of Eq's. (1) which is compiled for the voltage input source (see Figure 1), voltage V_1 is included in the vector of unknown variables, although we know it. We can therefore reduce this set of equations to the form

$$\begin{array}{c|ccc|c|c|c}
 & V_2 & V_3 & V_4 & & & V_1 \\
2 & G_1+G_2+sC_1 & -G_2 & -sC_1 & & V_2 & -G_1 \\
3 & -G_2 & G_2+sC_2 & & & V_3 & \\
4 & & A & -1 & \bullet & V_4 &
\end{array} \quad = - \; \cdot V. \qquad (3)$$

The algorithm of compiling the "pseudoadmittance" matrix and the right-side vector is obvious. It is substantial that

1. *the order of pseudoadmittance matrix = number of independent nodes – number of uncontrolled voltage sources*,
2. the right-side vector is marked by "minus".

(b) Reduction of number of equations due to controlled voltage sources

In the set of equations (3), voltages V_3 and V_4 are included in the vector of unknown variables. However, they are mutually dependent according to the relation $V_4 = AV_3$. In our case, A=1. Then we can simply merge the matrix columns marked V_3 and V_4 and omit the voltage coupling condition. The result is this:

$$\begin{array}{c|cc|c|c}
 & V_2 & V_3=V_4 & & V_1 \\
2 & G_1+G_2+sC_1 & -G_2-sC_1 & V_2 & -G_1 \\
3 & -G_2 & G_2+sC_2 & V_3=V_4 &
\end{array} \quad = - \; V. \qquad (4)$$

Note that the circuit in Figure 1 can be described algorithmically by only two equations. Only the equations of the first K.L. are in the set of equations, and only for the nodes to which the voltage sources are not connected (both uncontrolled and controlled). We must take into account all the possible combinations of row and column indexes while filling algorithmically the matrix.

For the mutator in Figure 2, the reduced set of equations is as follows:

$$\begin{array}{c|ccc|c|c}
 & V_1=V_3=V_5 & V_2 & V_4 & & \\
1 & Y_1 & -Y_1 & & V_1=V_3=V_5 & I \\
3 & Y_2+Y_3 & -Y_2 & -Y_3 & V_2 & \\
5 & Y_4+Y_5 & & -Y_4 & V_4 &
\end{array} \quad = \; . \qquad (5)$$

Note: In the case of ideal voltage amplifiers with the gain different from 1, the given reduction can be also realized but using a more complicated procedure [3]. We save one equation indeed, but at the cost of more complicated and difficult-to-remember algorithm. Then the method of dead row is more convenient for such circuits. If both these amplifiers and OpAmps are inside the circuit, the methods can be combined.

CONCLUSION

The choice of a proper type of the taught method of analysis depends on the pedagogical aims. In the first stage of study, the methods should help the student to develop engineering mentality, emphasizing the creative approach. In the second stage, the student should acquire an effective tool of the algorithmic analysis of circuits containing active elements. We prefer two variants of MNA, in particular the method of dead row. On the basis on modern algorithmic methods, the student should understand the global function of professional circuit simulators. This is one prerequisite for mastering them effectively.

REFERENCES

1. MIKULA,J. et al: Theory of linear circuits. Textbook, Military Acedemy Brno, Czech Republic, 1985.
2. VLACH,J.-SINGHAL,K.: Computer methods for circuit analysis and design. Van Nostrand Reinhold Company, New York, 1983.
3. BIOLEK,D.: Fast analysis of circuits with OpAmps. Radioelektronika '91, VUT FE Brno, Czech Republic, 1991, pp. 4-5.