Analysis of circuits containing active elements by using modified T - graphs

DALIBOR BIOLEK *) and VIERA BIOLKOVA**)

Department of Telecommunications *) Radioelectronics **)

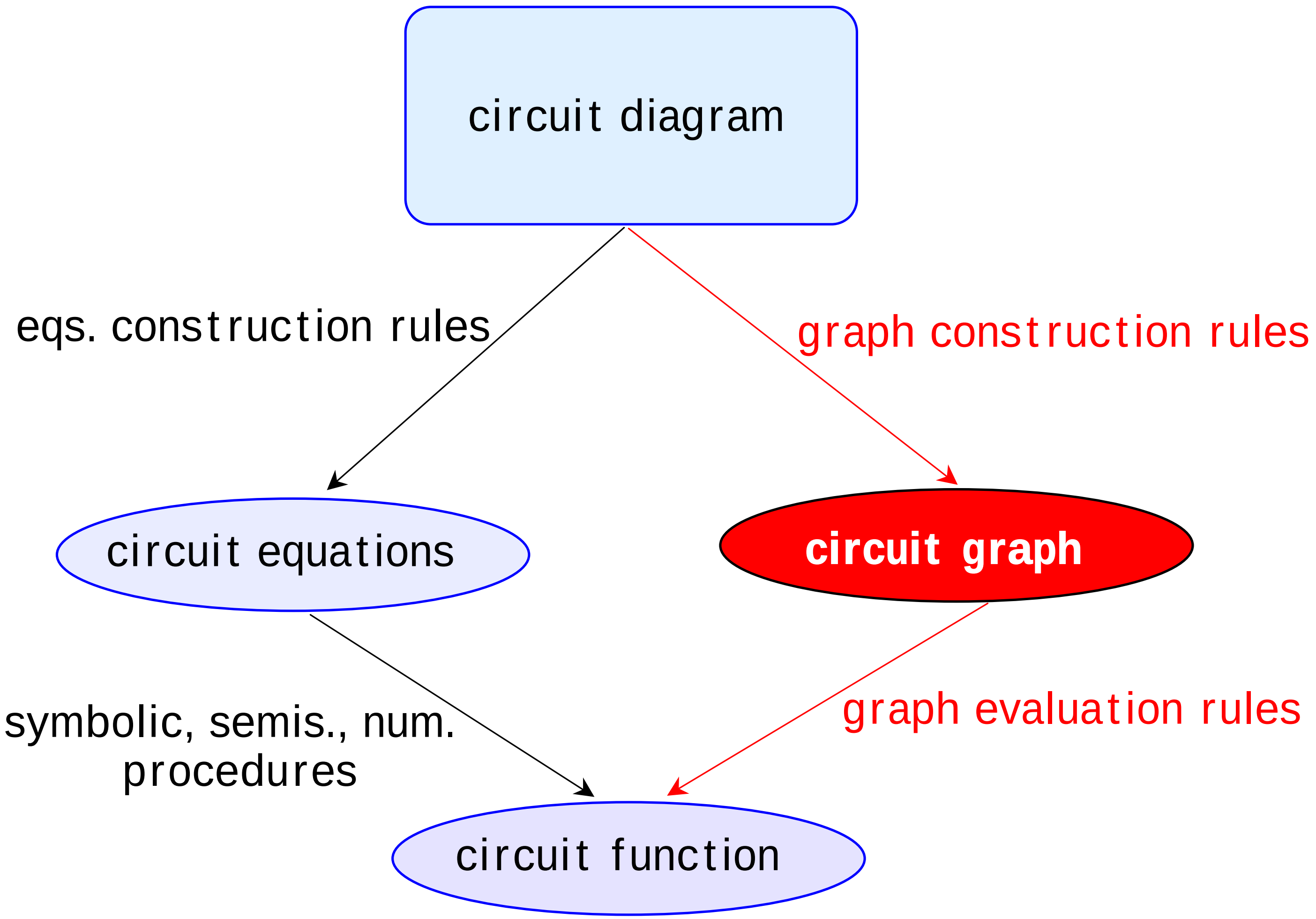
Brno University of Technology

Purkynova 118, 612 00 Brno

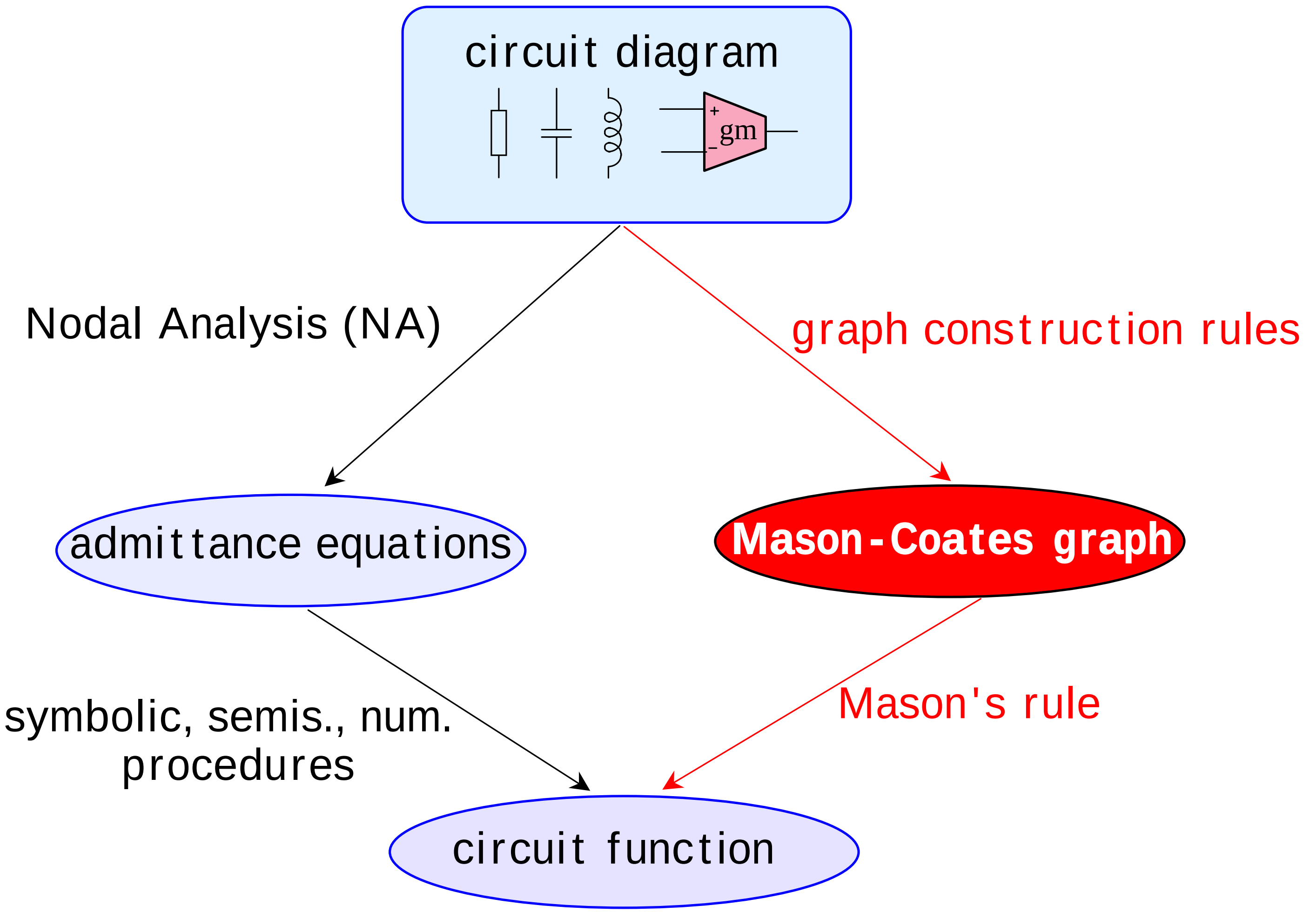
CZECH REPUBLIC

dalibor.biolek@vabo.cz http://www.vabo.cz/stranky/biolek

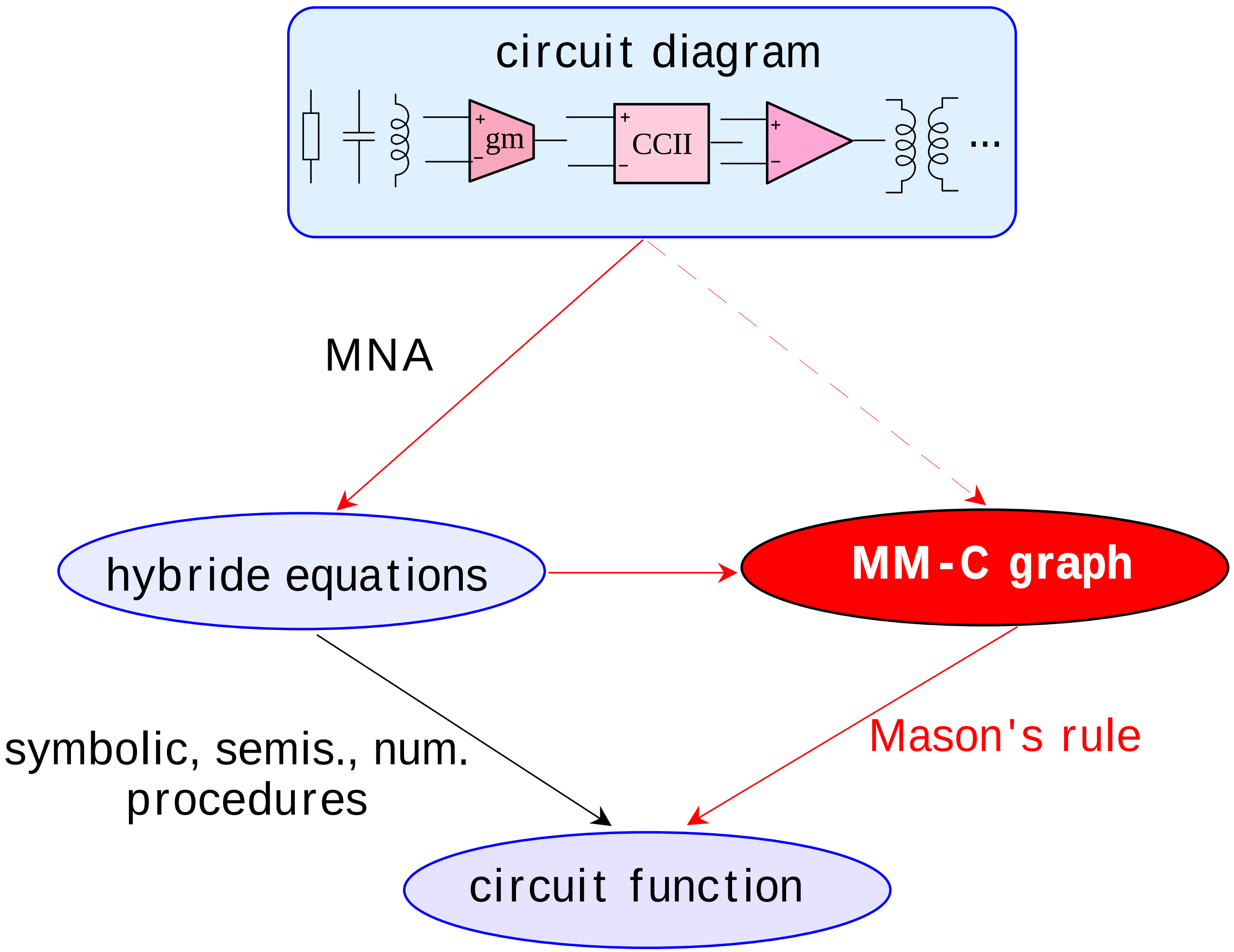
- Nodal Analysis (NA) versus Mason's and Mason-Coates' flow graphs.
- Modified Nodal Analysis (MNA) versus Modified Mason-Coates' flow graphs.
- Voltage and current incidence matrices versus Transform Graphs.
- Method of Modified Transform Graphs = method of Twin Variables & Transform Graphs.
- Conclusions



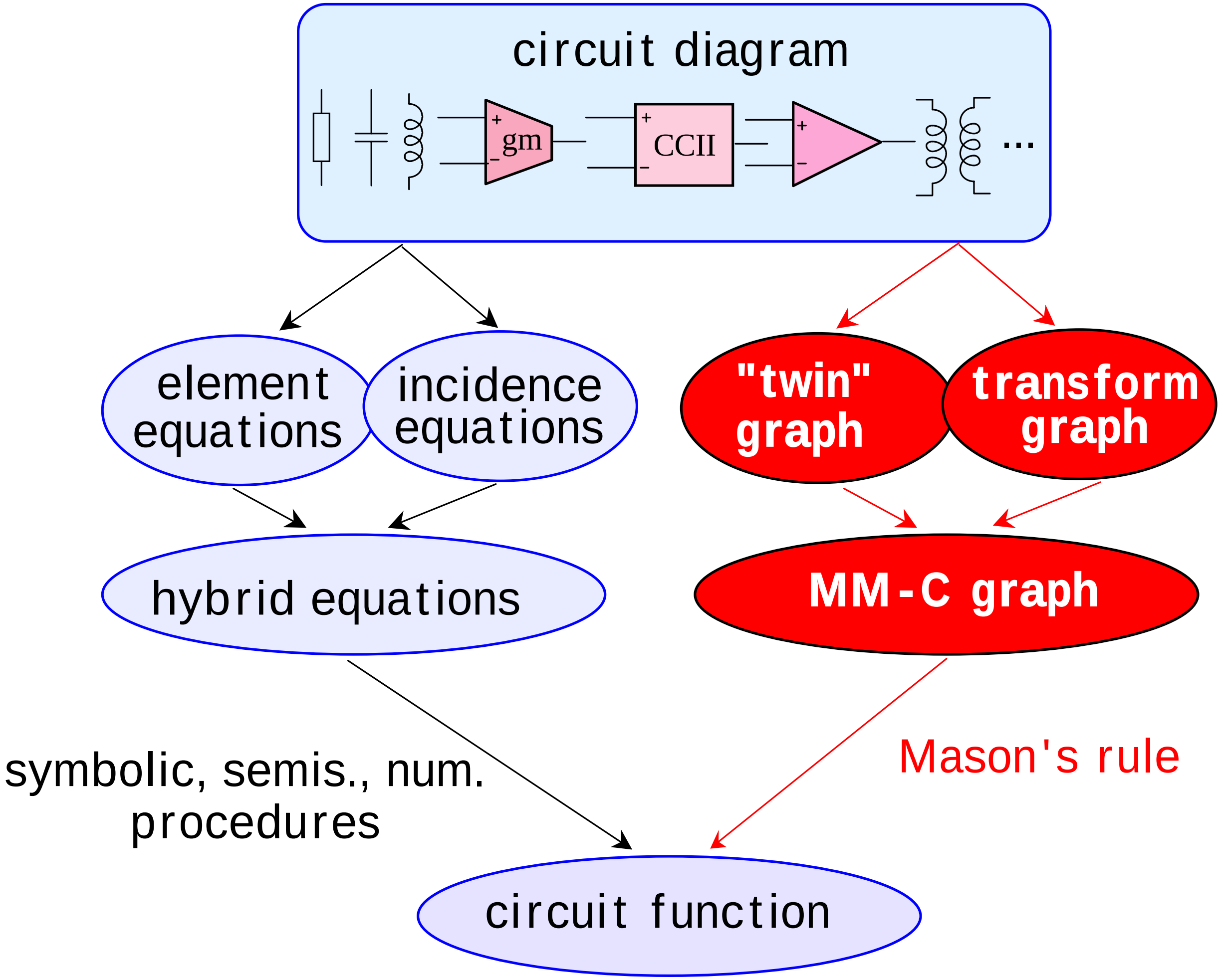
1. Simple rules of graph construction directly from the circuit schematics.
2. Economical graph structure.
3. Simple rules of graph evaluation.
4. Possibility of evaluation both voltage & current gains and immittances.



1. ✓ Simple rules of graph construction directly from the circuit schematics.
2. ✓ Economical graph structure.
3. ✓ Simple rules of graph evaluation.
4. ✓ Possibility of evaluation both voltage & current gains and immittances.



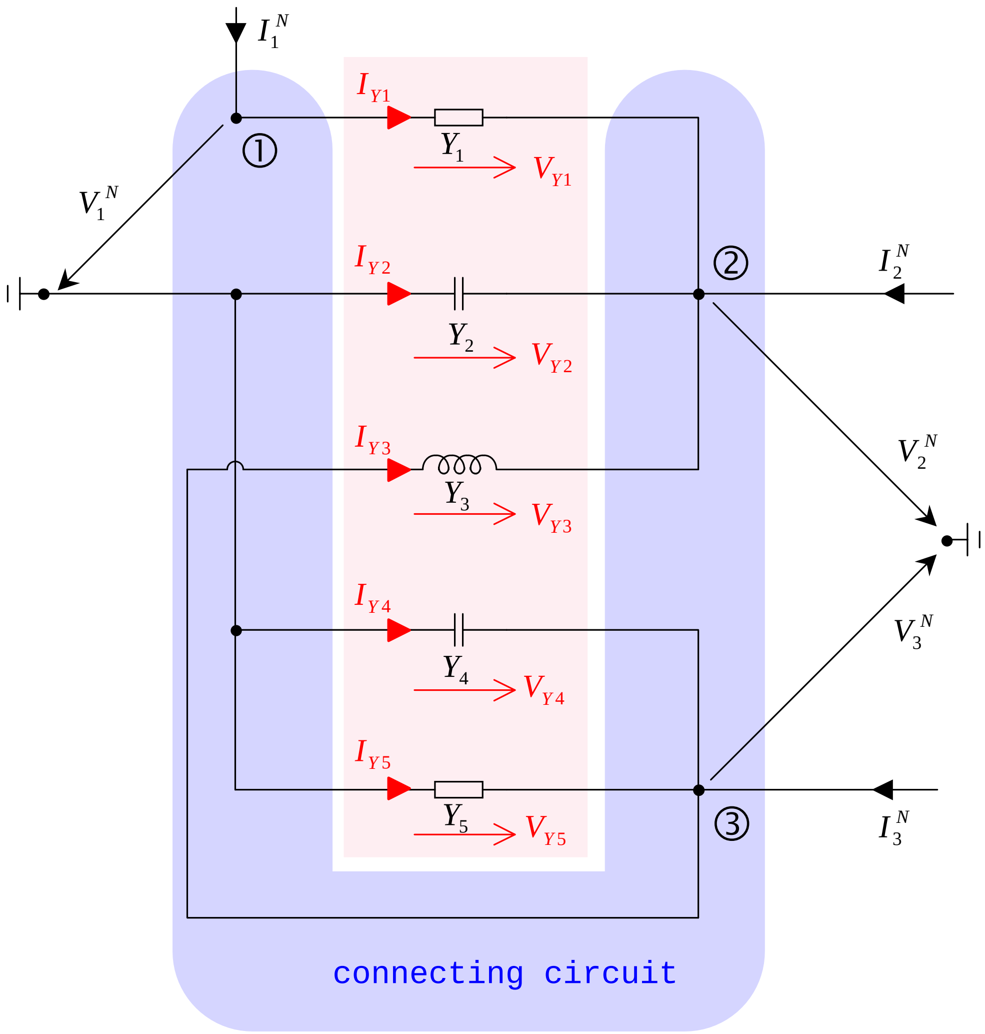
1. ~~Simple rules of graph construction directly from the circuit schematics.~~
2. ✓ Economical graph structure.
3. ✓ Simple rules of graph evaluation.
4. ~~Possibility of evaluation both voltage & current gains and immittances.~~



1. (✓) Simple rules of graph construction directly from the circuit schematics.
2. ✓ Economical graph structure.
3. ✓ Simple rules of graph evaluation.
4. ✓ Possibility of evaluation both voltage & current gains and immittances.

Incidence matrices, T- and MC-graphs

Passive linear circuit



$$\underbrace{\begin{bmatrix} I_{Y1} \\ I_{Y2} \\ I_{Y3} \\ I_{Y4} \\ I_{Y5} \end{bmatrix}}_{\mathbf{I}^B} = \underbrace{\begin{bmatrix} Y_1 & & & & \\ & Y_2 & & & \\ & & Y_3 & & \\ & & & Y_4 & \\ & & & & Y_5 \end{bmatrix}}_{\mathbf{Y}^B} \underbrace{\begin{bmatrix} V_{Y1} \\ V_{Y2} \\ V_{Y3} \\ V_{Y4} \\ V_{Y5} \end{bmatrix}}_{\mathbf{V}^B}$$

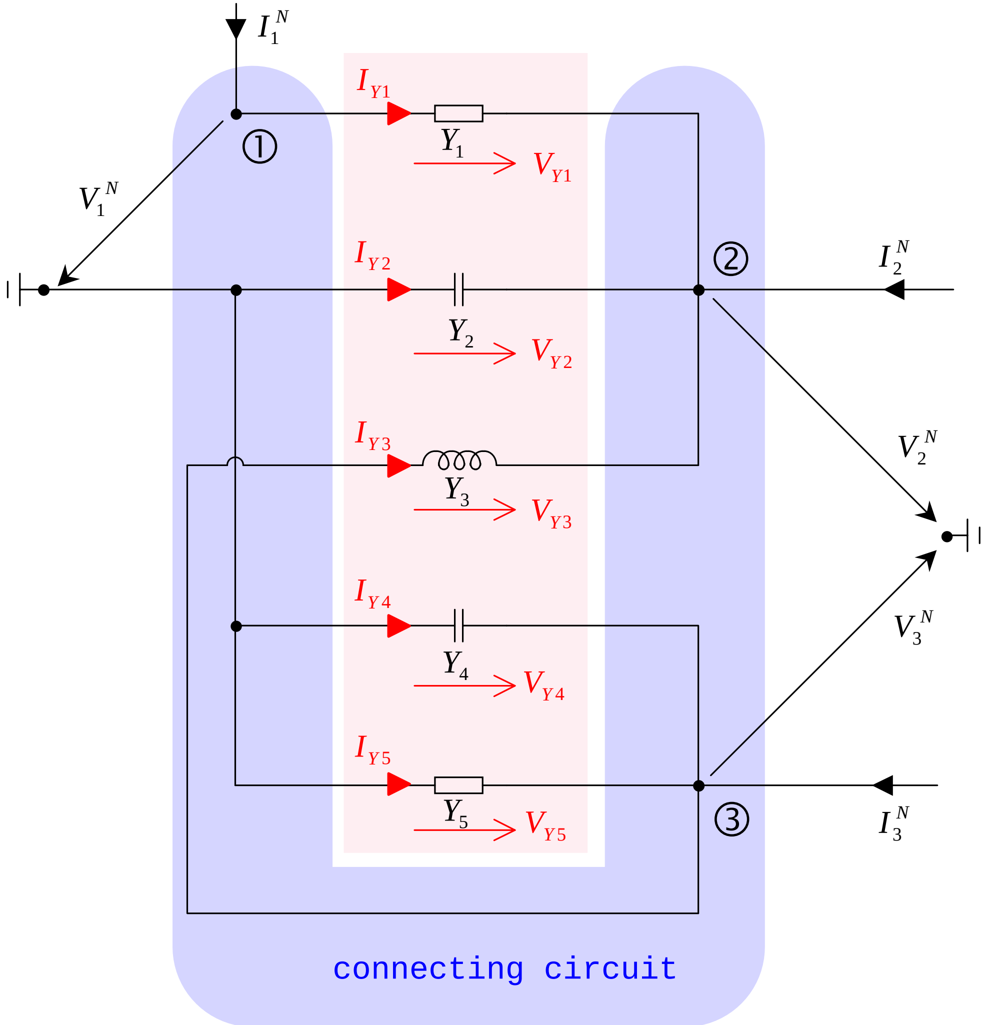
$$\mathbf{I}^B = \mathbf{Y}^B \mathbf{V}^B$$

$$\underbrace{\begin{bmatrix} I_1^N \\ I_2^N \\ I_3^N \end{bmatrix}}_{\mathbf{I}^N} = \underbrace{\begin{bmatrix} 1 & & & & \\ -1 & 1 & 1 & & \\ & & -1 & 1 & 1 \end{bmatrix}}_{\mathbf{A}^I} \underbrace{\begin{bmatrix} I_{Y1} \\ I_{Y2} \\ I_{Y3} \\ I_{Y4} \\ I_{Y5} \end{bmatrix}}_{\mathbf{I}^B}$$

$$\underbrace{\begin{bmatrix} V_{Y1} \\ V_{Y2} \\ V_{Y3} \\ V_{Y4} \\ V_{Y5} \end{bmatrix}}_{\mathbf{V}^B} = \underbrace{\begin{bmatrix} 1 & -1 & \\ & 1 & \\ & 1 & -1 \\ & & 1 \\ & & 1 \end{bmatrix}}_{\mathbf{A}^V} \underbrace{\begin{bmatrix} V_1^N \\ V_2^N \\ V_3^N \end{bmatrix}}_{\mathbf{V}^N}$$

Incidence matrices, T- and MC-graphs

Passive linear circuit



$$\mathbf{I}^B = \mathbf{Y}^B \mathbf{V}^B$$

$$\mathbf{I}^N = \mathbf{A}^I \mathbf{I}^B$$

$$\mathbf{V}^B = \mathbf{A}^V \mathbf{V}^N$$

$$\mathbf{I}^N = \mathbf{Y}^N \mathbf{V}^N$$

$$\mathbf{Y}^N = \mathbf{A}^I \mathbf{Y}^B \mathbf{A}^V =$$

G_1	$-G_1$	
$-G_1$	G_1+sC_1+1/sL_1	$-1/sL_1$
	$-1/sL_1$	G_2+sC_2+1/sL_1

$\mathbf{Y}^B$

Y_1				
	Y_2			
		Y_3		
			Y_4	
				Y_5

$\mathbf{A}^V$

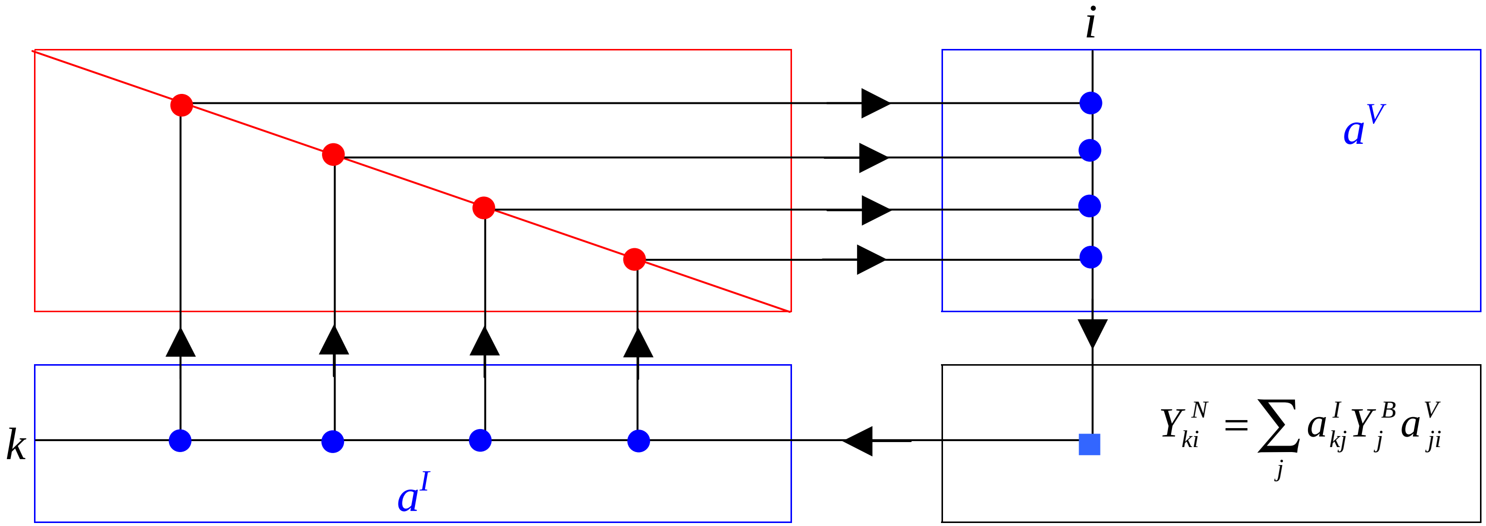
1	-1	
	1	
	1	-1
		1
		1

1				
-1	1	1		
		-1	1	1

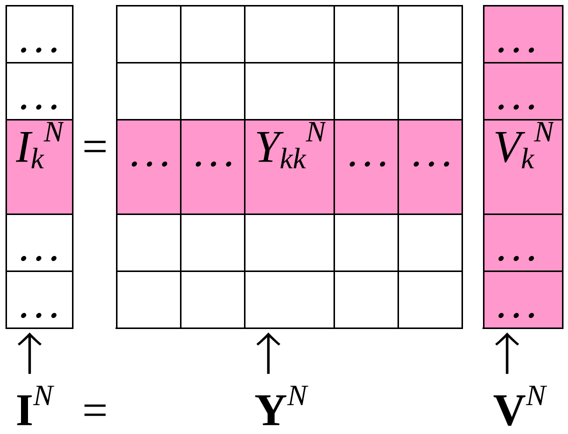
$\mathbf{A}^I$

Y_1	$-Y_1$	
$-Y_1$	$Y_1+Y_2+Y_3$	$-Y_3$
	$-Y_3$	$Y_3+Y_4+Y_5$

$\mathbf{Y}^N$

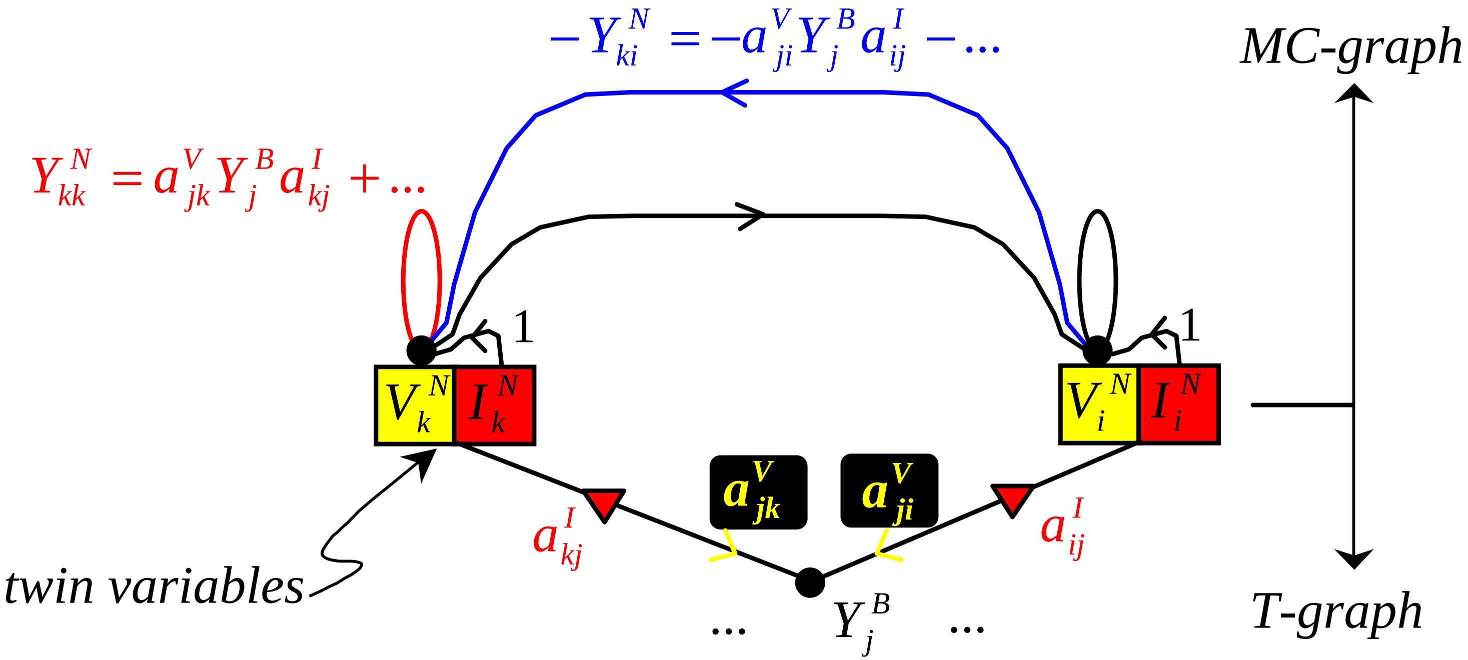


$$Y_{ki}^N = \sum_j a_{kj}^I Y_j^B a_{ji}^V$$

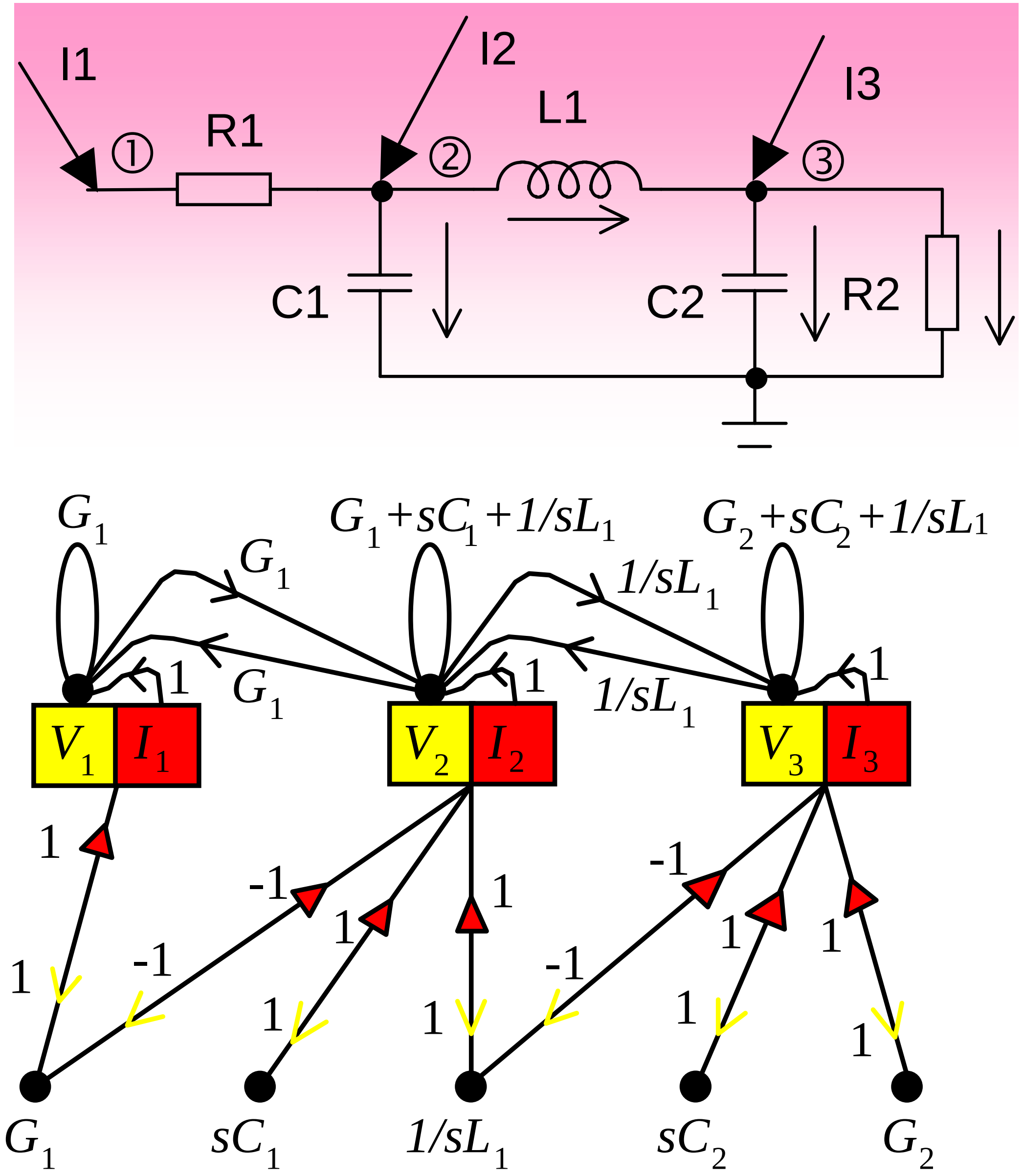


$$\to Y^N_{kk}V^N_k = I^N_k - \sum_{i\neq k} Y^N_{ki}V^N_i \to MC\text{ - graph}$$

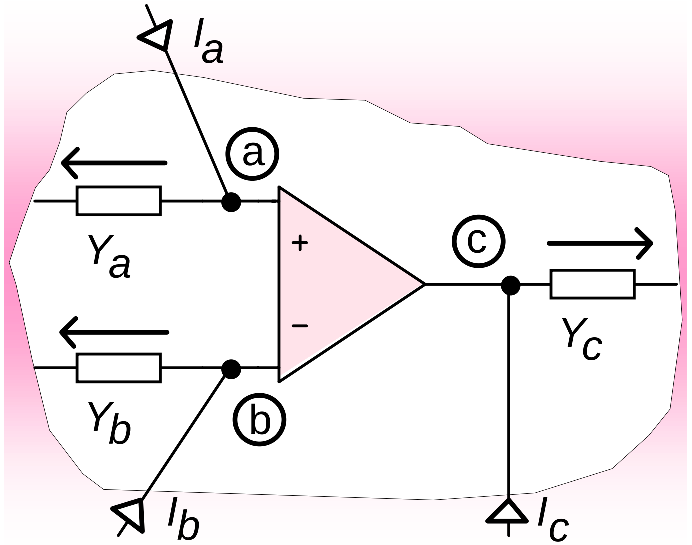
$$Y^N_{ki} = \sum_j a^I_{kj}\, Y^B_j\, a^V_{ji} \to T\text{ - graph}$$



Example – ladder filter



Circuits with ideal OpAmps



MNA:

$$\begin{array}{|c|}\hline I_a \\ \hline I_b \\ \hline \dots \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline Y_a & 0 & \dots \\ \hline Y_b & 0 & \dots \\ \hline \dots & \dots & \dots \\ \hline \end{array} \begin{array}{|c|}\hline V_a{=}V_b \\ \hline V_c \\ \hline \dots \\ \hline \end{array} \Rightarrow$$

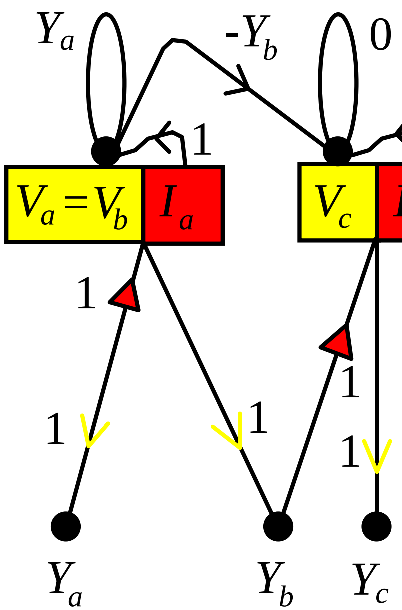
$\Downarrow$

1. $Y_a\, V_a = I_a,\ 0.V_c = I_b - Y_b\, V_a,$
2. $Y_b\, V_a = I_b,\ 0.V_c = I_a - Y_a\, V_a$

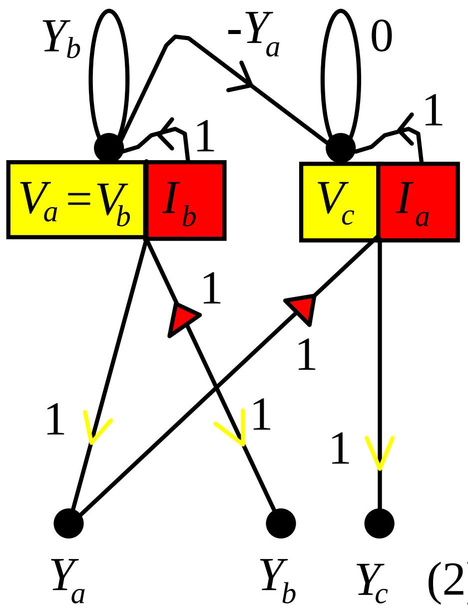
$\Downarrow$

Two possible variants of twin variables:

1. $V_a{=}V_b|I_a,\ V_c|I_b$
2. $V_a{=}V_b|I_b,\ V_c|I_a$

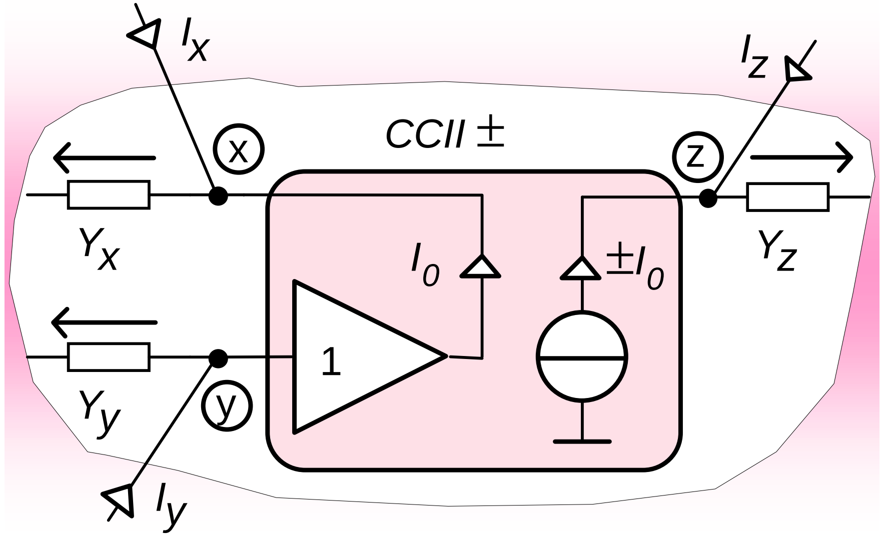


(1)



(2)

Circuits with current conveyors CCII± MNA:

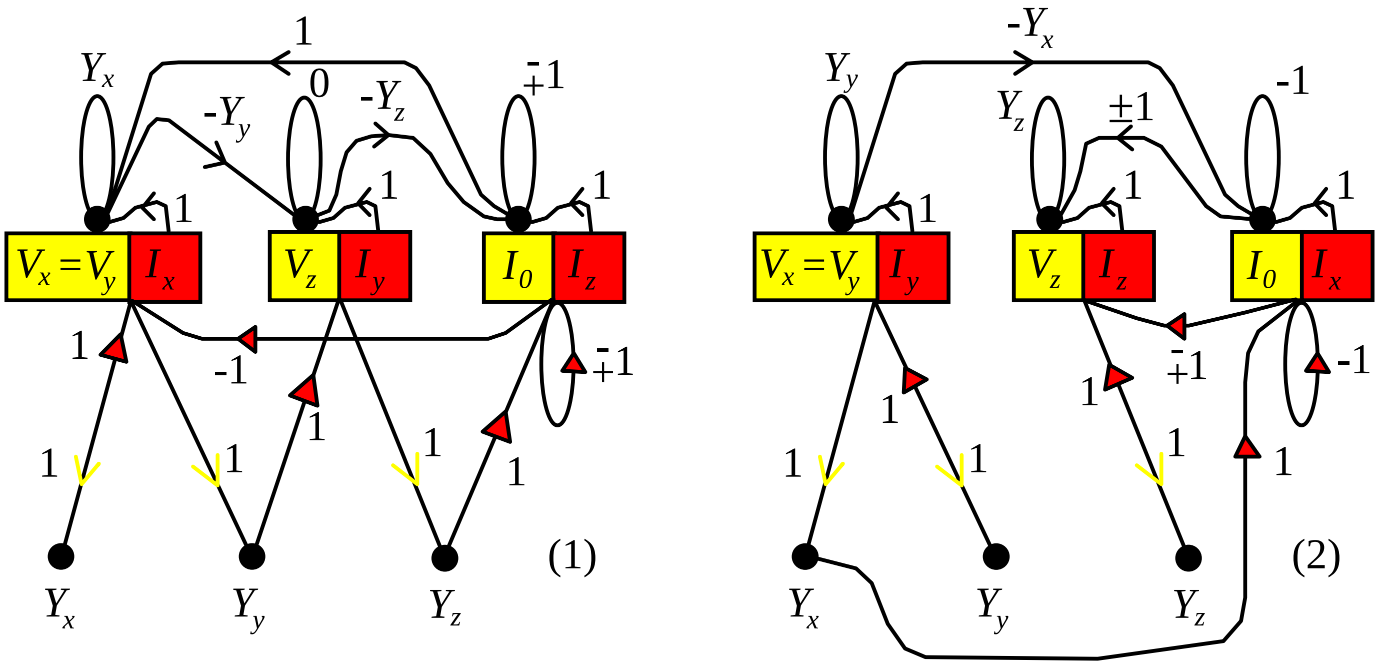


1. $Y_x V_x = I_x + I_0$, $0.V_z = I_y - Y_y V_x$, $\mp I_0 = I_z - Y_z V_z$
2. $Y_y V_x = I_y$, $Y_z.V_z = I_z \pm I_0$, $-I_0 = I_x - Y_x V_x$

⇓

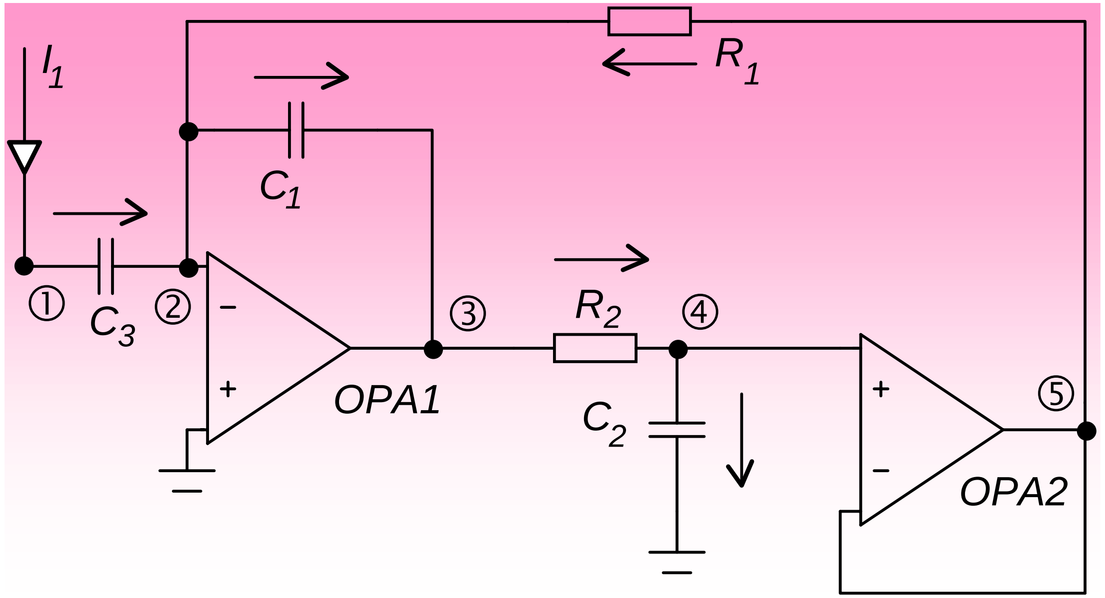
Two possible variants of twin variables:

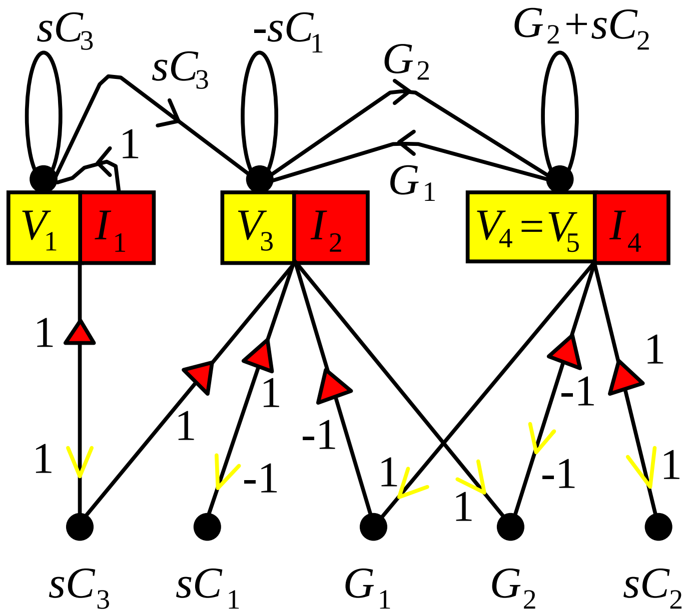
1. $V_x=V_y|I_x$, $V_z|I_y$, $I_0|I_z$
2. $V_x=V_y|I_y$, $V_z|I_z$, $I_0|I_x$



Ilustrative examples

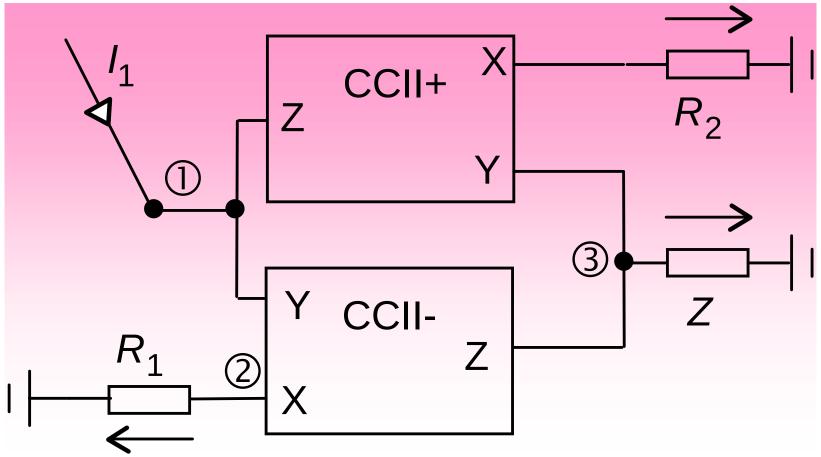
2nd order bandpass filter

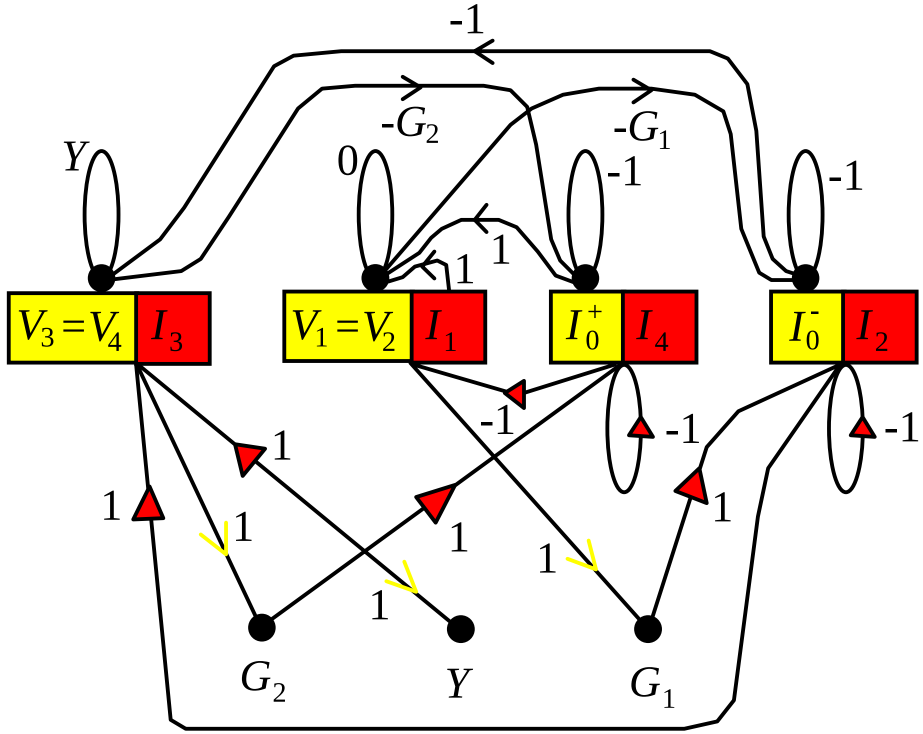




$$\frac{V_5}{V_1}=C_3G_2\frac{s}{-sC_1\left(G_2+sC_2\right)-G_1G_2}=-\frac{sR_1C_3}{s^2R_1R_2C_1C_2+sR_1C_1+1}$$

Impedance converter with CCII+ and CCII- [*].





$$Z_{inp} = \frac{V_1}{I_1} = \frac{1.Z.(-1).(-1)}{0 + G_1 G_2} = \frac{R_1 R_2}{Z}.$$

[*] Tomazou,C., Lidgey,F.J, Haigh,D.G., Analogue IC Design: the Current-Mode Approach. *IEE Circuits and Systems Series 2*, Peter Peregrinus Ltd, London 1993,UK.

Conclusions

- The described method of graph solution of linear systems combines the so-called transform graphs and the method of twin variables.
- As a result, the algorithm of graph construction is relatively simple, and the size of the resulting MC-graph is reduced enough.
- As shown in the final paper, the selection of permitted twin variables must be performed carefully if there is a number of transform elements in the circuit analyzed.
- This method can be used to solve general linear problems, not only in the analysis of electrical circuits.

Acknowledgement: This work is supported by the Grant Agency of the Czech Republic under grants No. 102/01/0432 and 102/00/0907, and by the research programme of BUT „Research of electronic communication systems and technologies".