OPTIMIZATION OF FREQUENCY FILTERS VIA VERTEX GRAPHS

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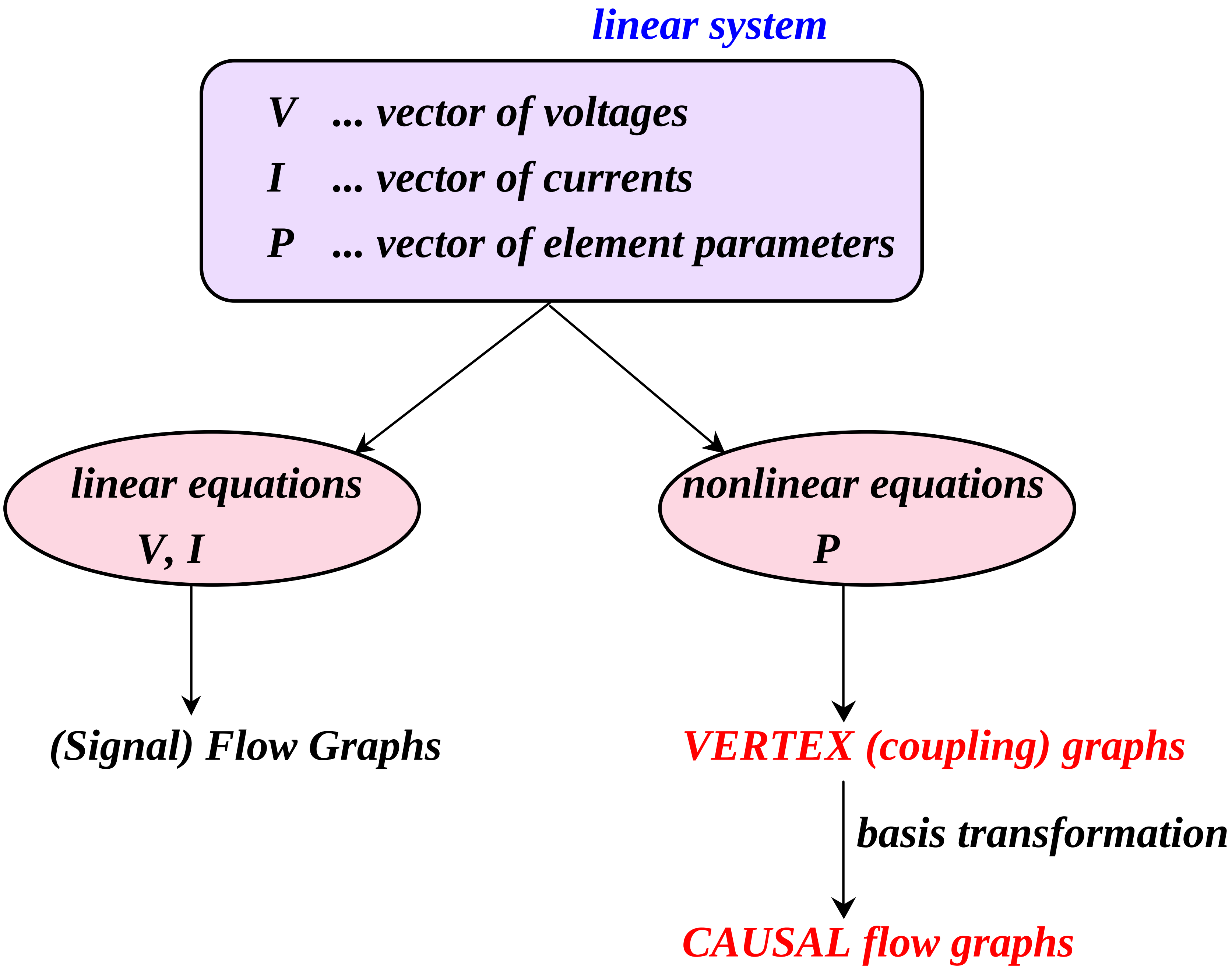
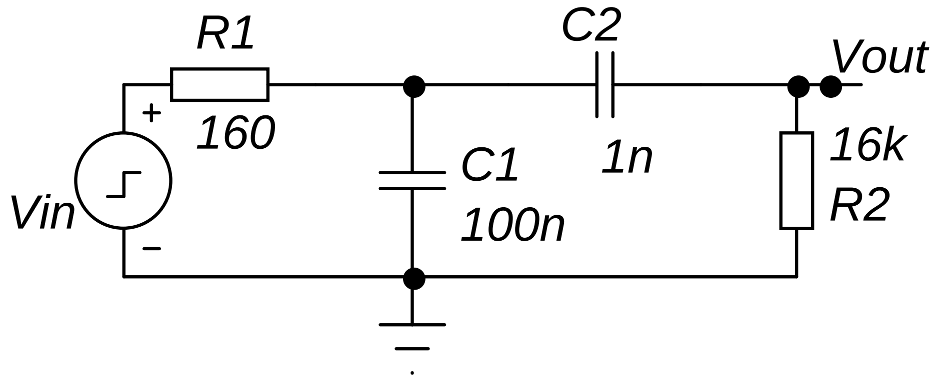
ABSTRACT

A method of systematic investigation of the influence of element parameters on the desired properties of a linear dynamical system –frequency filter in particular – is described. The couplings among the element parameters are examined by means of so-called VERTEX (or coupling) graph with the aim of algorithmic circuit optimization. One of the results is the partition of a set of parameters into disjoint sets. Within these sets, it is possible to vary the parameters independently to obtain the required circuit behavior.


linear system
V ... vector of voltages
I ... vector of currents
P ... vector of element parameters
linear equations
V, I
nonlinear equations
P
(Signal) Flow Graphs
VERTEX (coupling) graphs
basis transformation
CAUSAL flow graphs

OPENING EXAMPLE (1): passive RCCR bandpass filter



$$\frac{V_{out}}{V_{in}} = \frac{a_1 s}{1 + b_1 s + b_2 s^2} \ ,$$

where

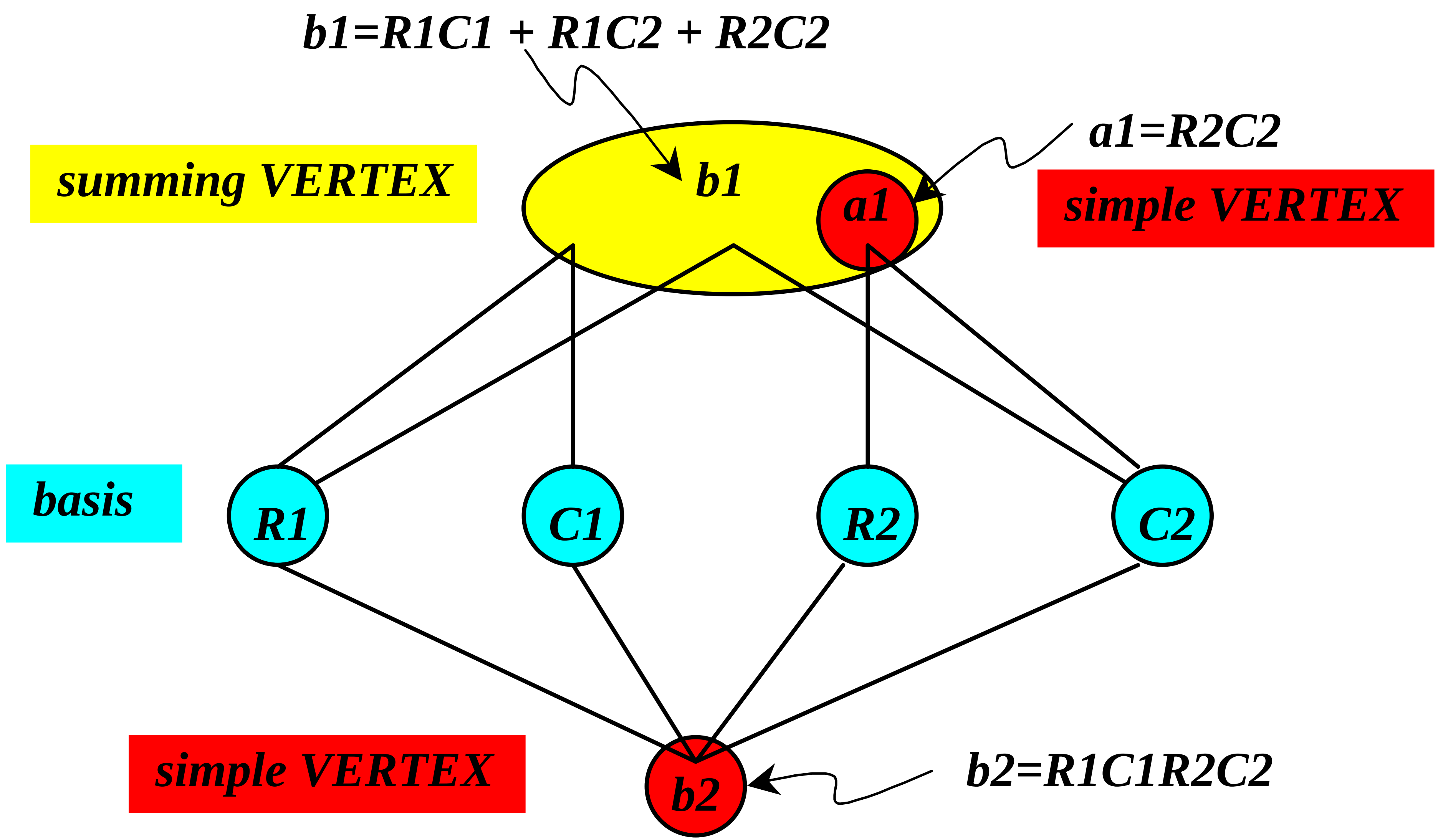
$$a_1 = R_2 C_2 = 16 \mu s$$

$$b_1 = R_1 C_1 + R_2 C_2 + R_1 C_2 = 32.16 \mu s$$

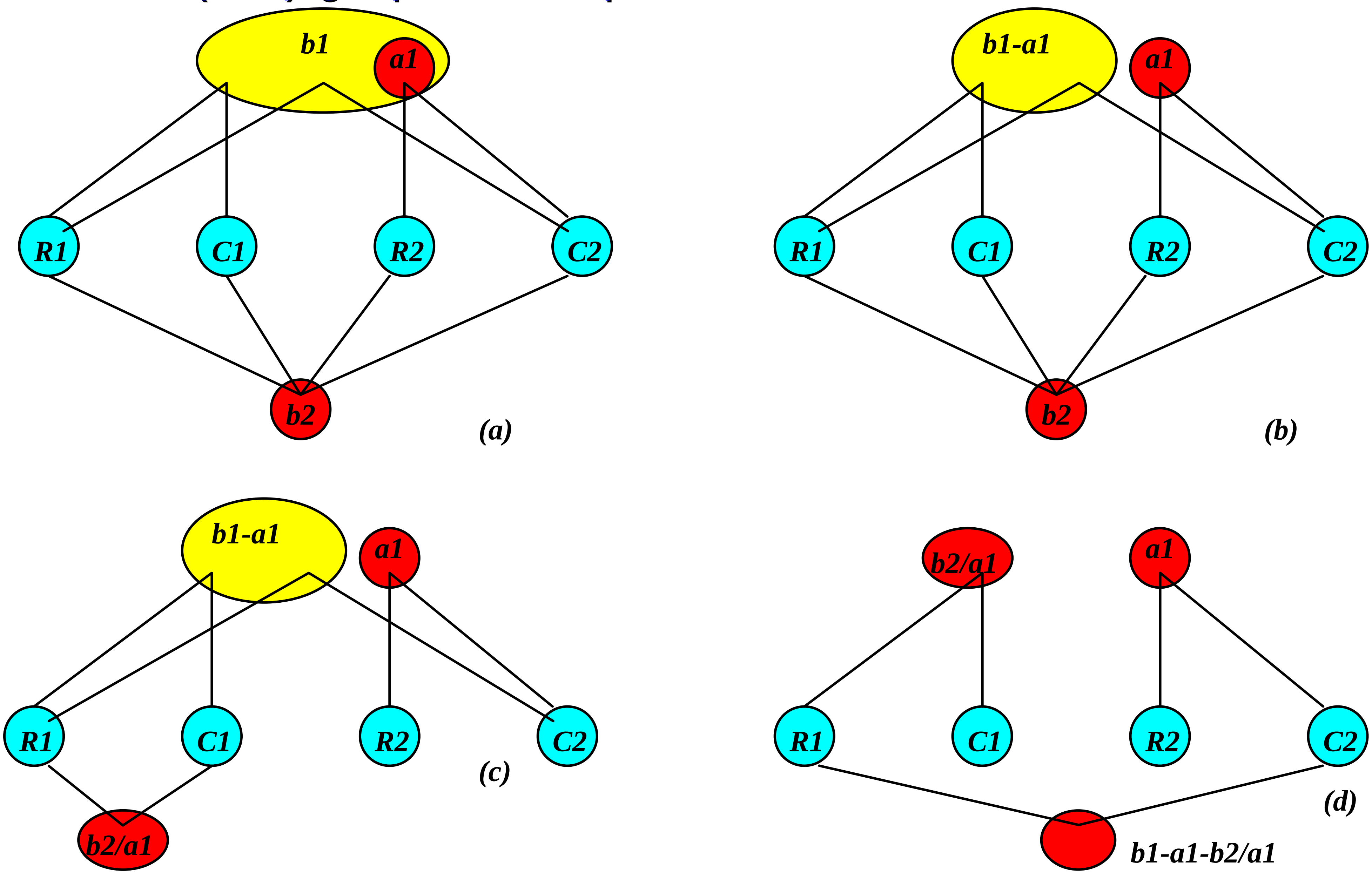
$$b_2 = R_1 R_2 C_1 C_2 = 256 \, ps^2$$

Set of 3 nonlinear equations with 4 unknown variables

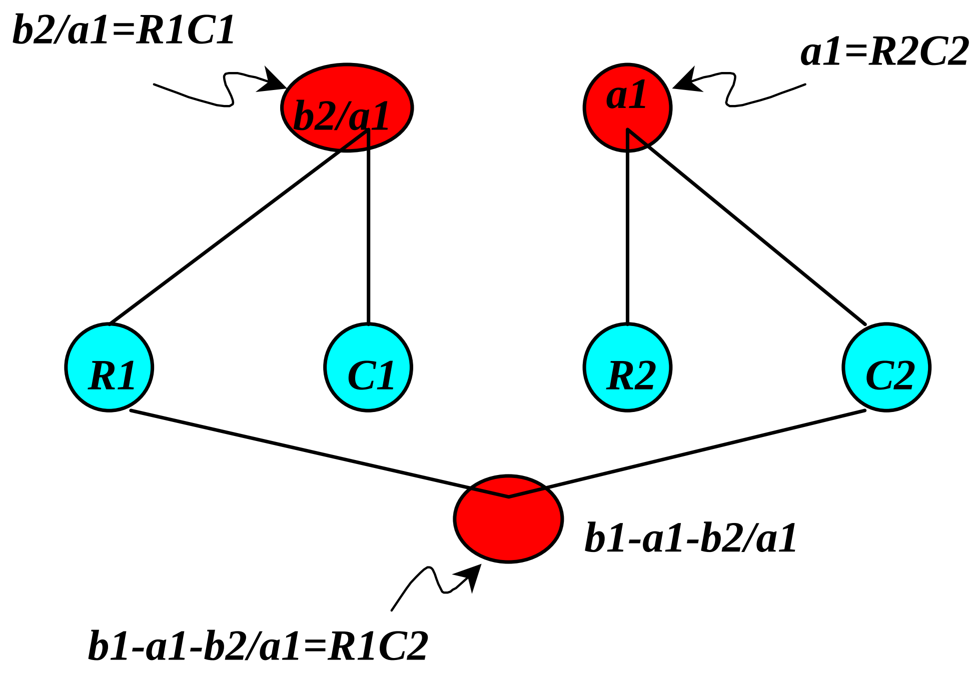
VERTEX graph



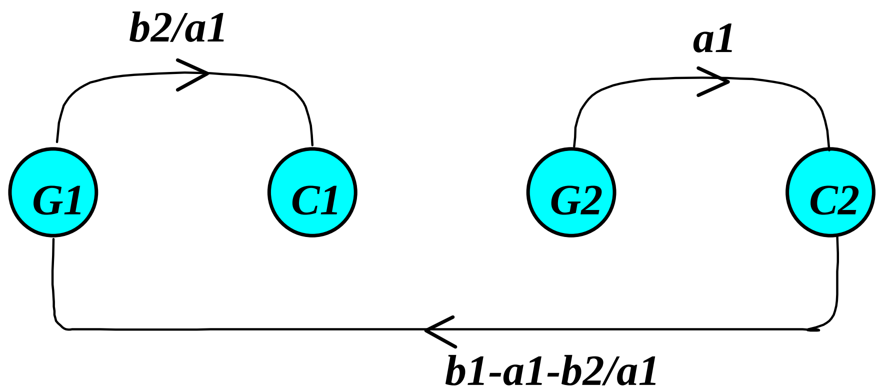
VERTEX (full) graph decomposition



CAUSAL graph from VERTEX graph



VERTEX graph

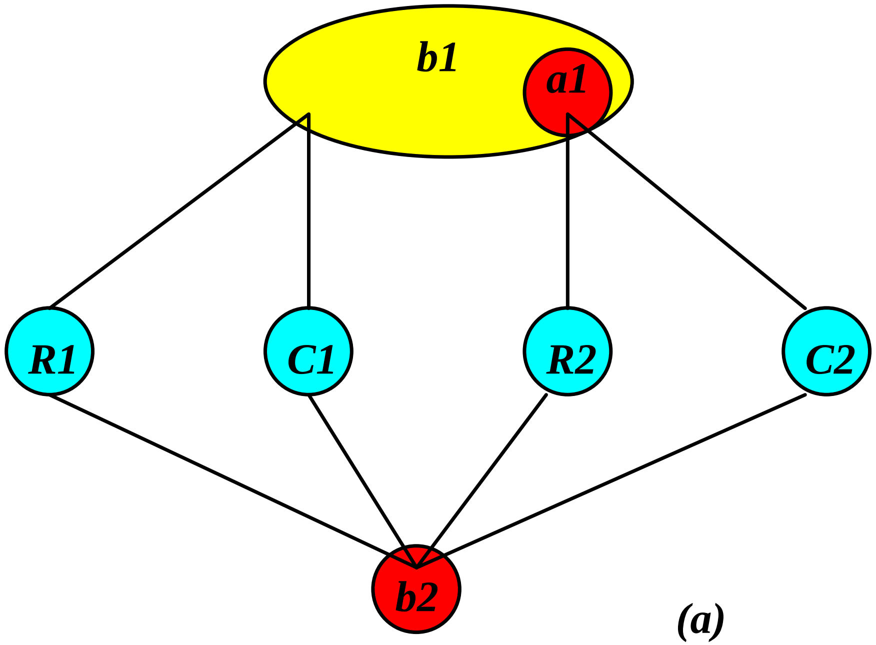


CAUSAL flow graph

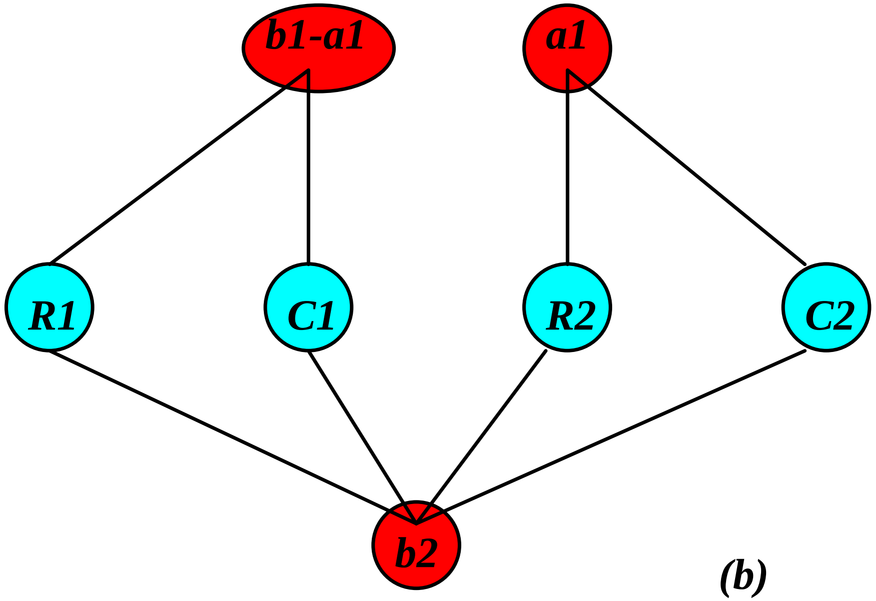
If arbitrary element is changed, we must change all the elements to retain transfer function.

(proportional variation)

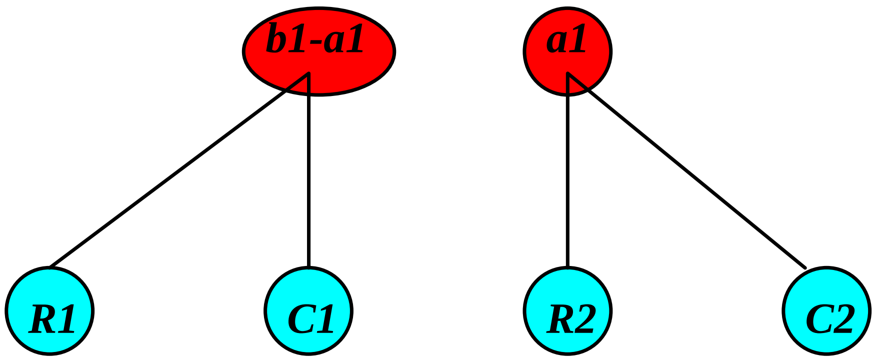
VERTEX (simplified) graph decomposition



(a)



(b)

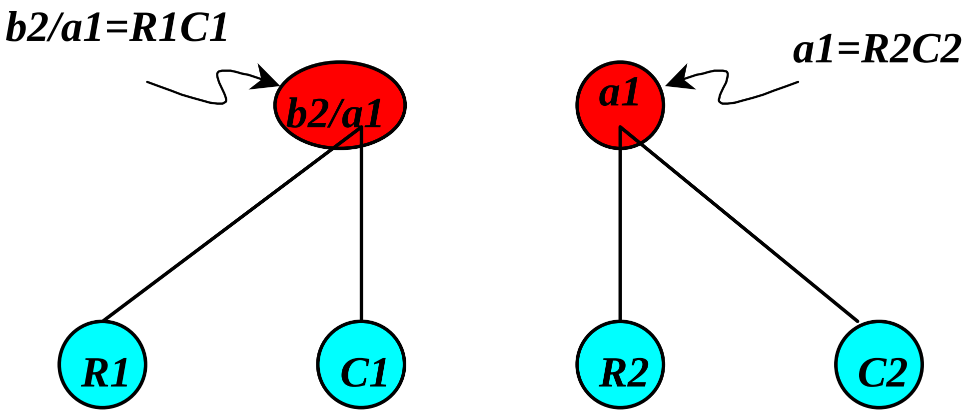


(c)

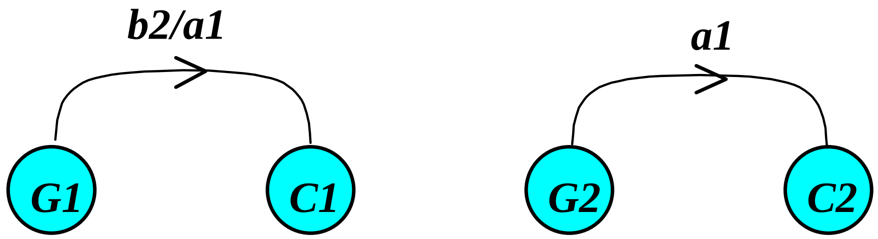
Simplification (a) is done under the condition

b1=R1C1+R2C2+R1C2 ~ R1C1+R2C2

CAUSAL graph from simplified VERTEX graph



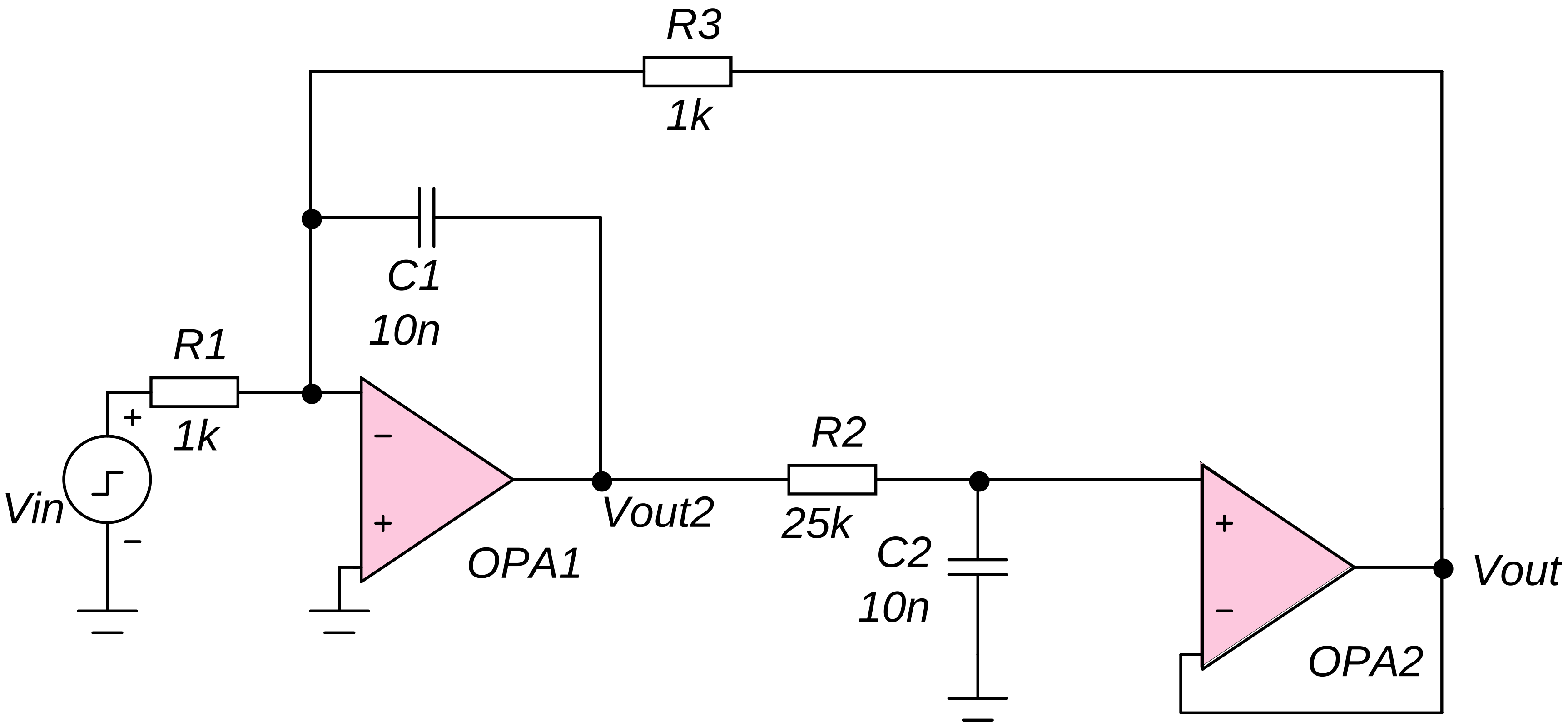
VERTEX graph



CAUSAL flow graph

There are two sets of parameters that can be varied independently (under the conditions of graph simplifying).

EXAMPLE 2: active 2nd - order filter, f_0=3.2 kHz, Q = 5

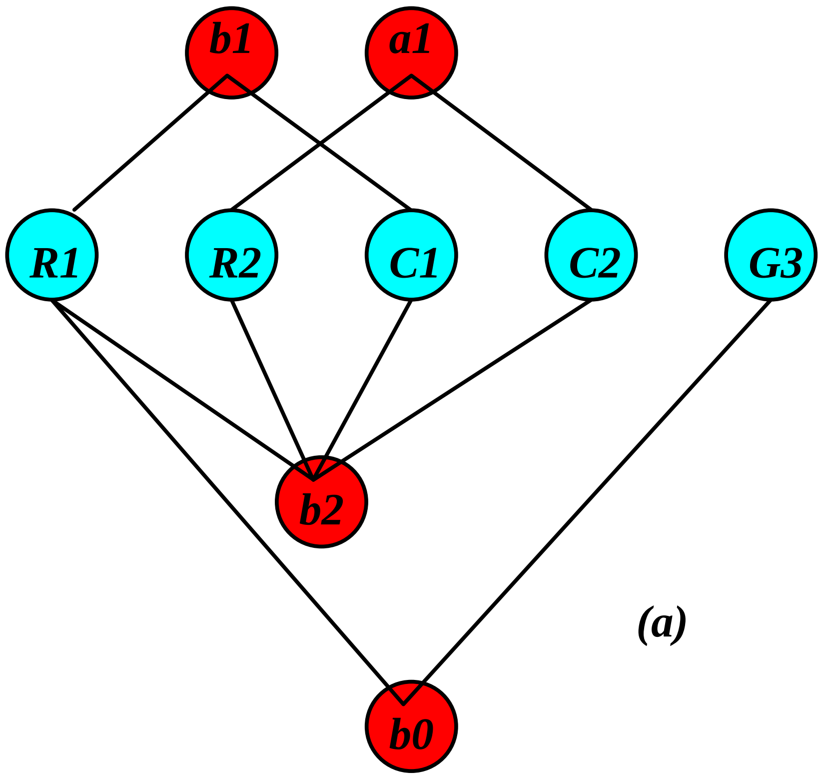


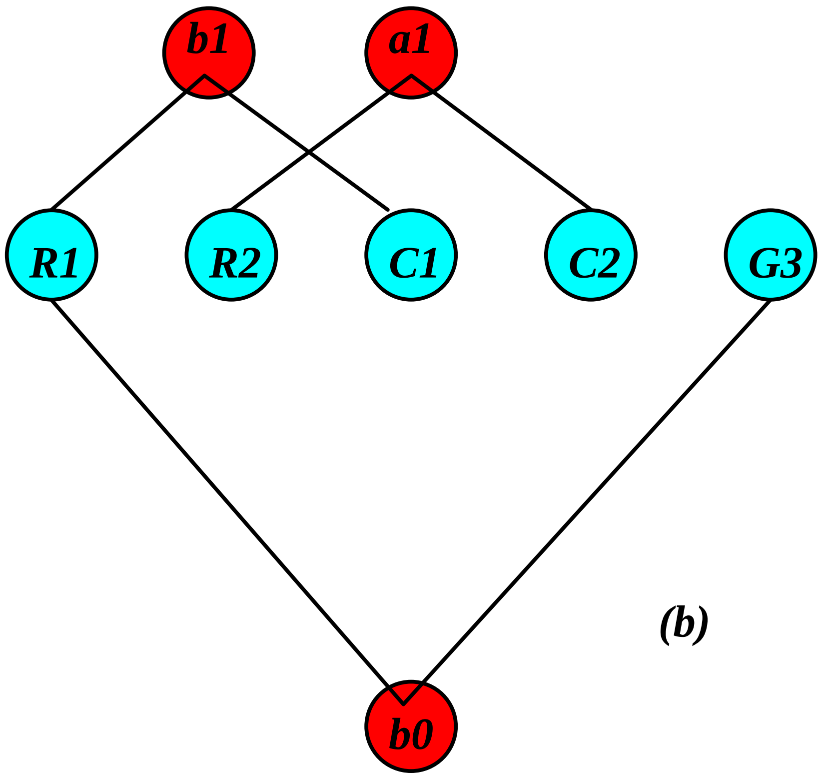
$$\frac{V_{out}}{V_{in}} = -\frac{1}{b_0 + b_1 s + b_2 s^2}\ , \quad \frac{V_{out2}}{V_{in}} = -\frac{1 + a_1 s}{b_0 + b_1 s + b_2 s^2}\ ,$$

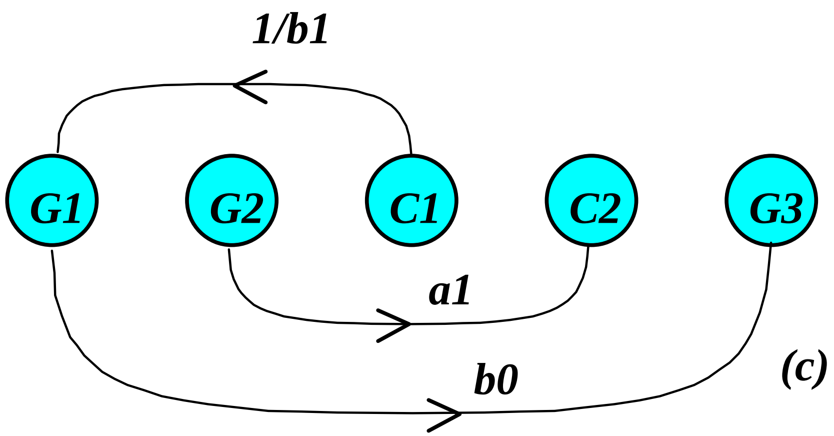
$$a_1 = R_2 C_2 = 250\mu s\ ,\ b_0 = R_1 G_3 = 1\ ,$$

$$b_1 = R_1 C_1 = 10\mu s\ ,\ b_2 = R_1 R_2 C_1 C_2 = 2.5 ns^2$$

Varying the parameters R1, R2, R3, C1, C2 to preserve both transfer functions:





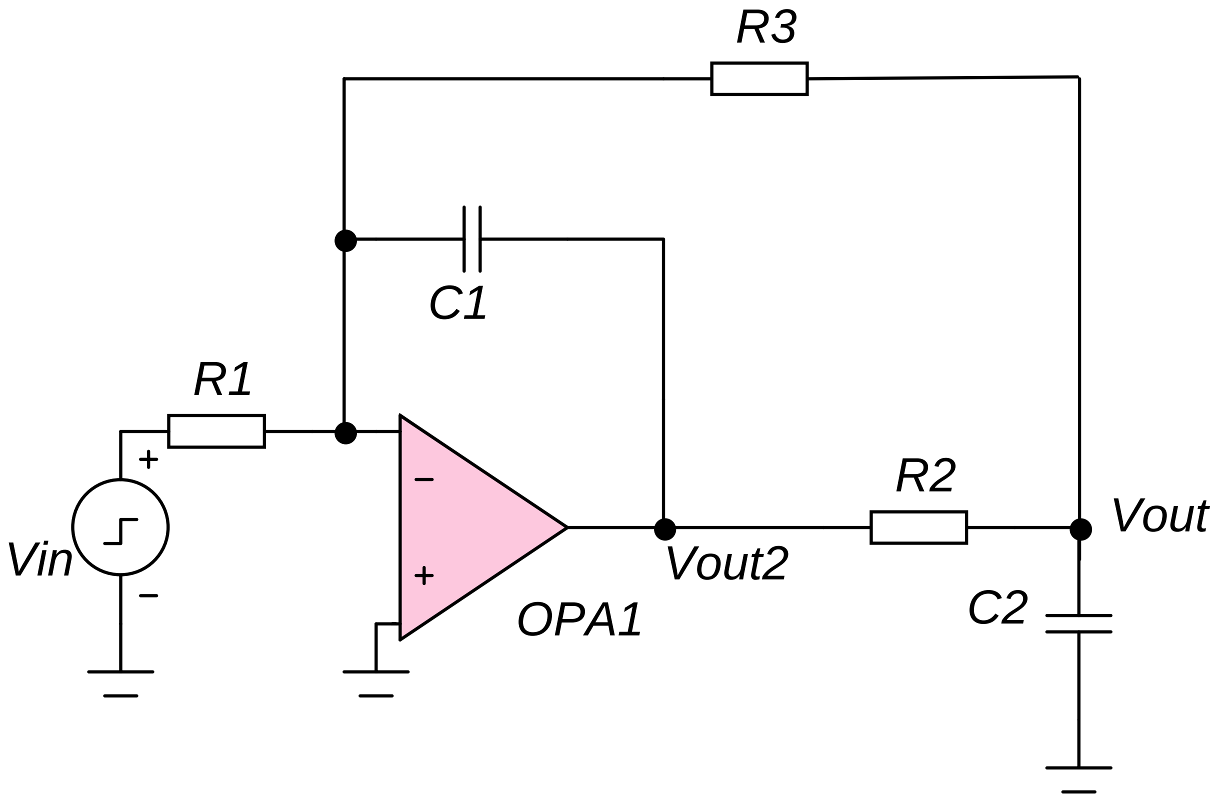


There are two sets of parameters that can be varied independently:

C1-G1-G3, C2-G2

(proportional variation)

EXAMPLE 3: active 2nd - order filter without buffering

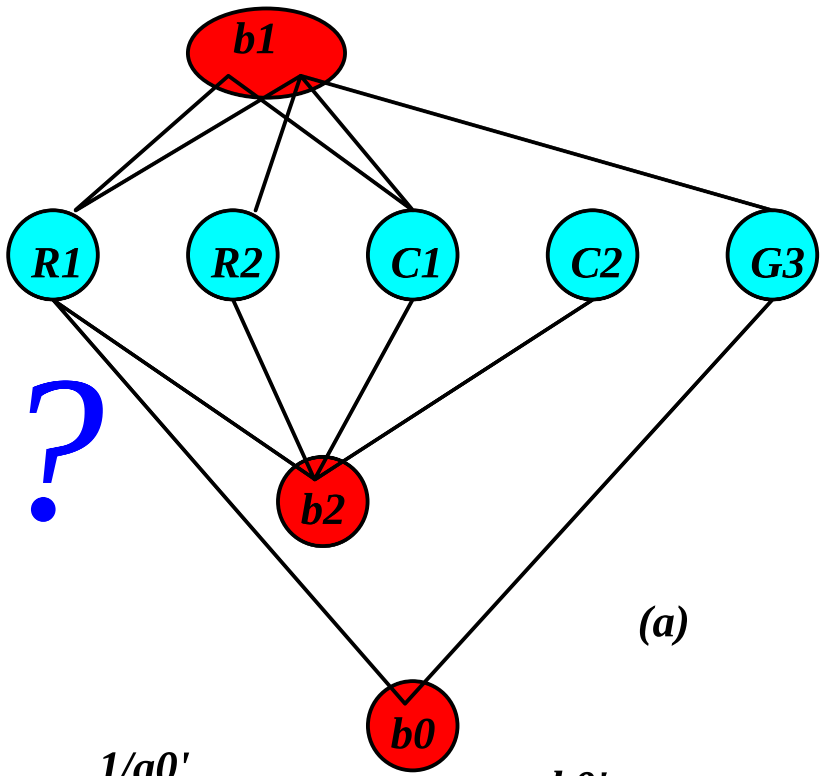


$$\frac{V_{out}}{V_{in}} = -\frac{1}{b_0 + b_1 s + b_2 s^2}, \quad \frac{V_{out2}}{V_{in}} = \frac{a_0 + a_1 s}{b_0 + b_1 s + b_2 s^2},$$

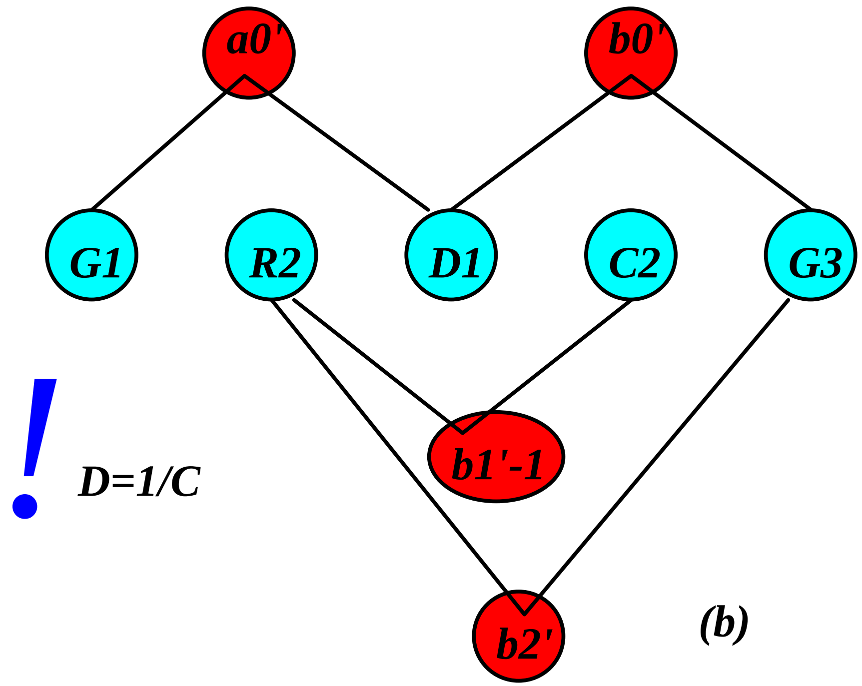
$$a_0 = 1 + R_2 G_3, \quad a_1 = R_2 C_2, \quad b_0 = R_1 G_3,$$

$$b_1 = R_1 C_1 (1 + R_2 G_3), \quad b_2 = R_1 R_2 C_1 C_2$$

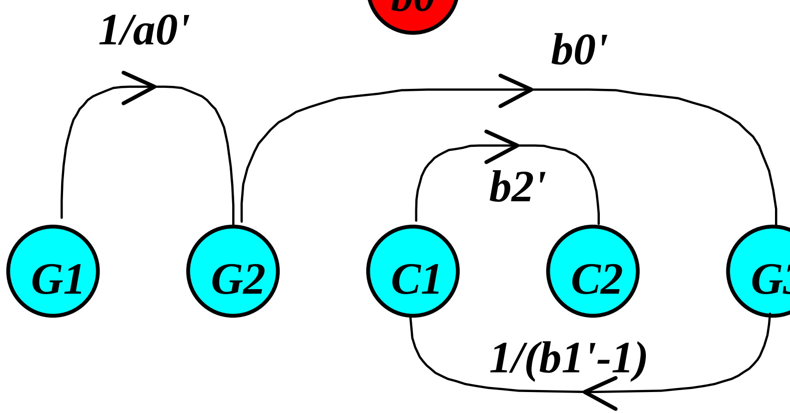
Varying the parameters to preserve **only the first** transfer function:



(a)



(b)

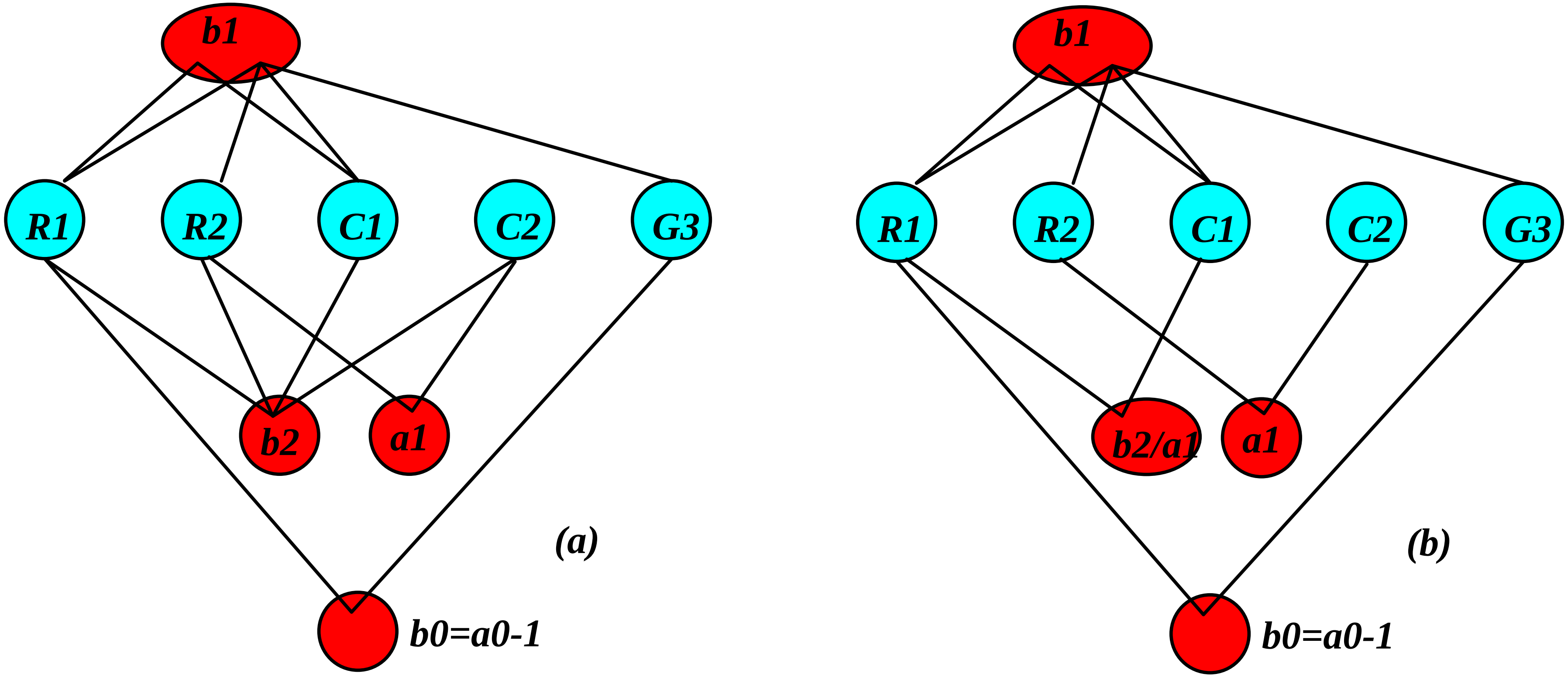


(c)

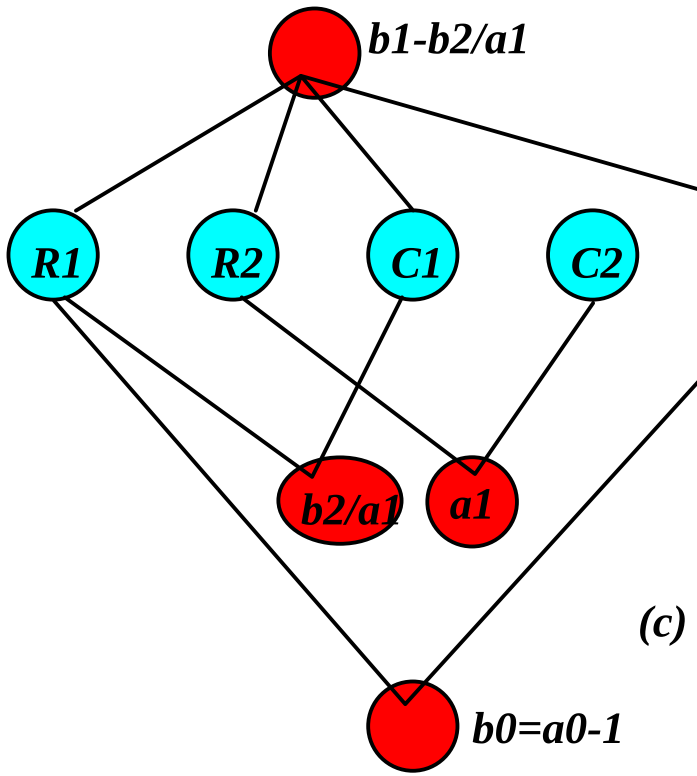
$$\frac{V_{out}}{V_{in}} = -\frac{1}{R_1G_3 + R_1C_1(1+R_2G_3)s + R_1R_2C_1C_2s^2} =$$

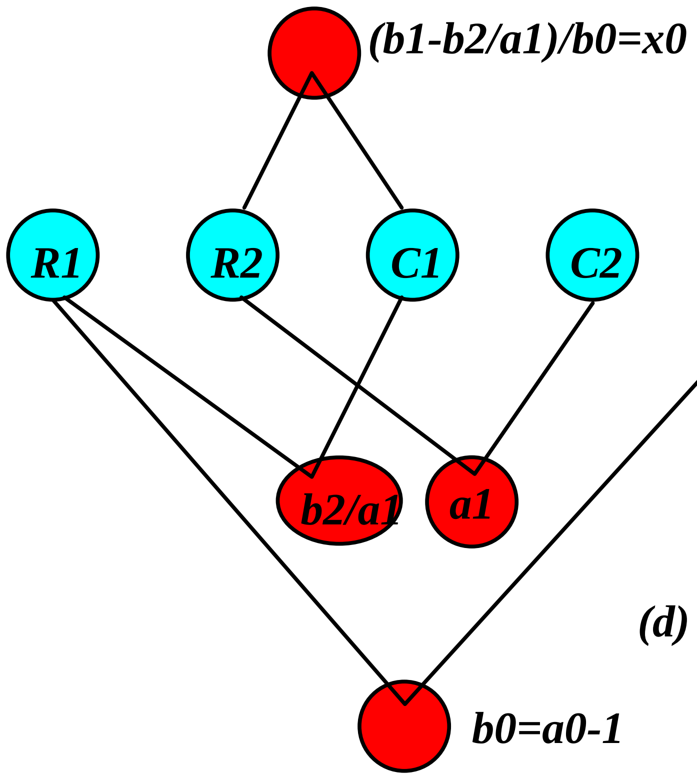
$$= -\frac{\overbrace{1/R_1C_1}^{a0'}}{\underbrace{G_3D_1}_{b0'} + \underbrace{(1+R_2G_3)}_{b1'}s + \underbrace{R_2C_2}_{b2'}s^2}$$

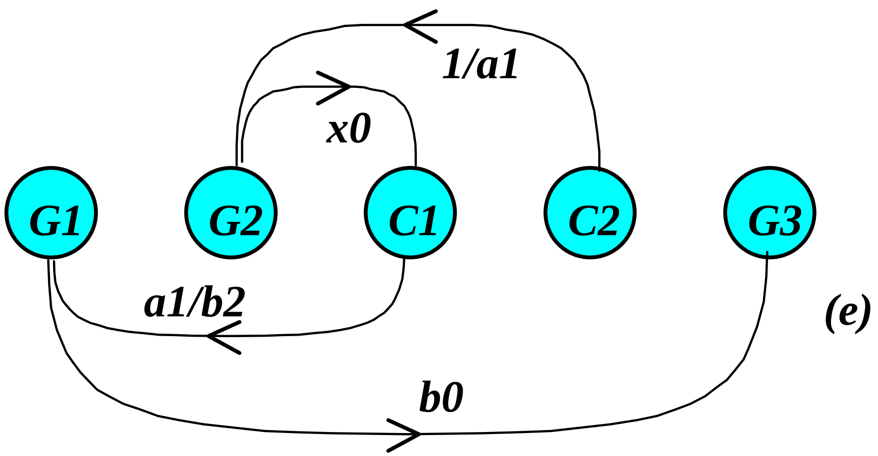
Varying the parameters to preserve **both** transfer functions:



Varying the parameters to preserve **both** transfer functions (cont'):

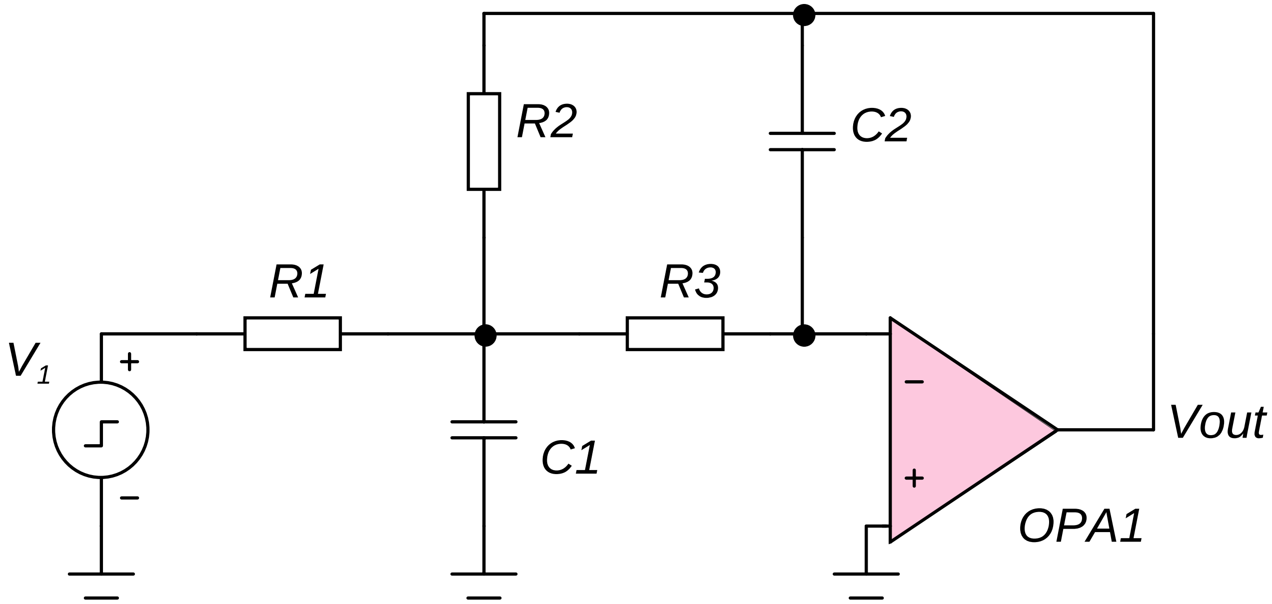






proportial variation

EXAMPLE 4: active 2nd - order "MFB" filter

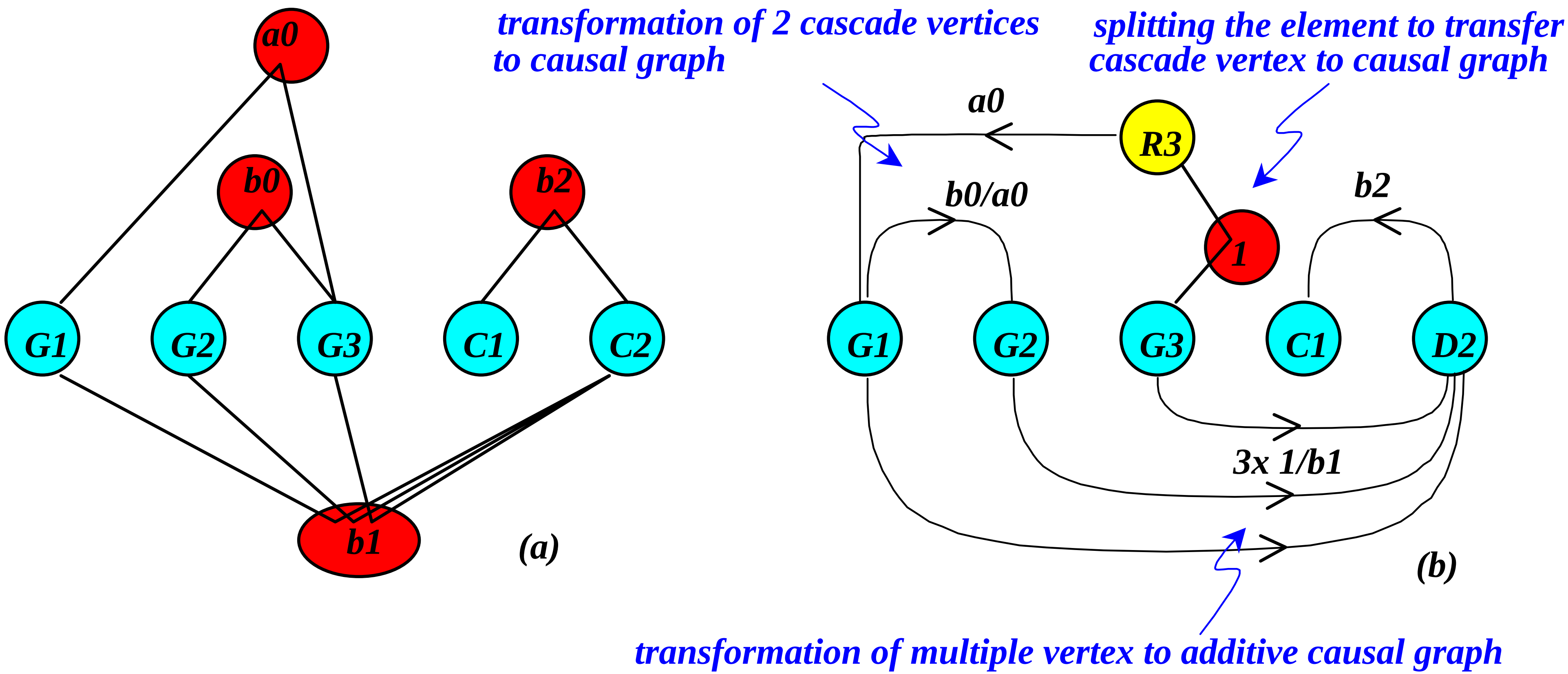


$$\frac{V_{out}}{V_{in}} = -\frac{a_0}{b_0 + b_1 s + b_2 s^2}$$

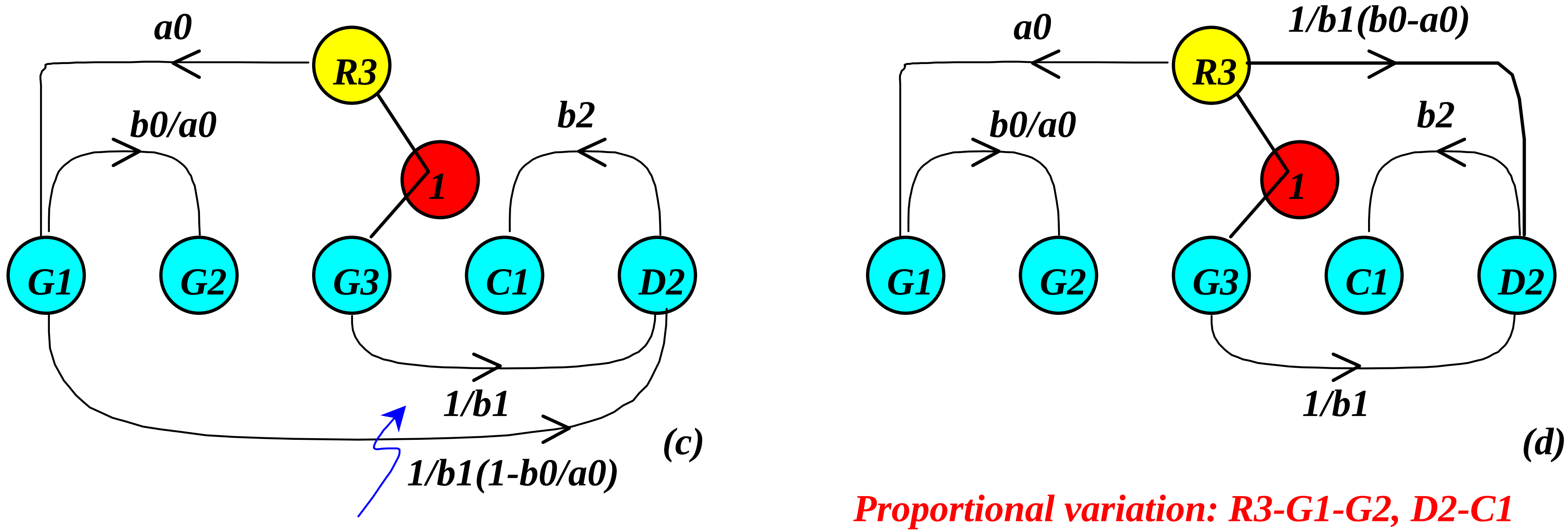
$$a_0 = G_1 G_3\,,\ \ b_0 = G_2 G_3\,,$$

$$b_1 = C_2\left(G_1 + G_2 + G_3\right),\ \ b_2 = C_1 C_2$$

Varying the parameters to preserve transfer function:



Varying the parameters to preserve transfer function (cont'):

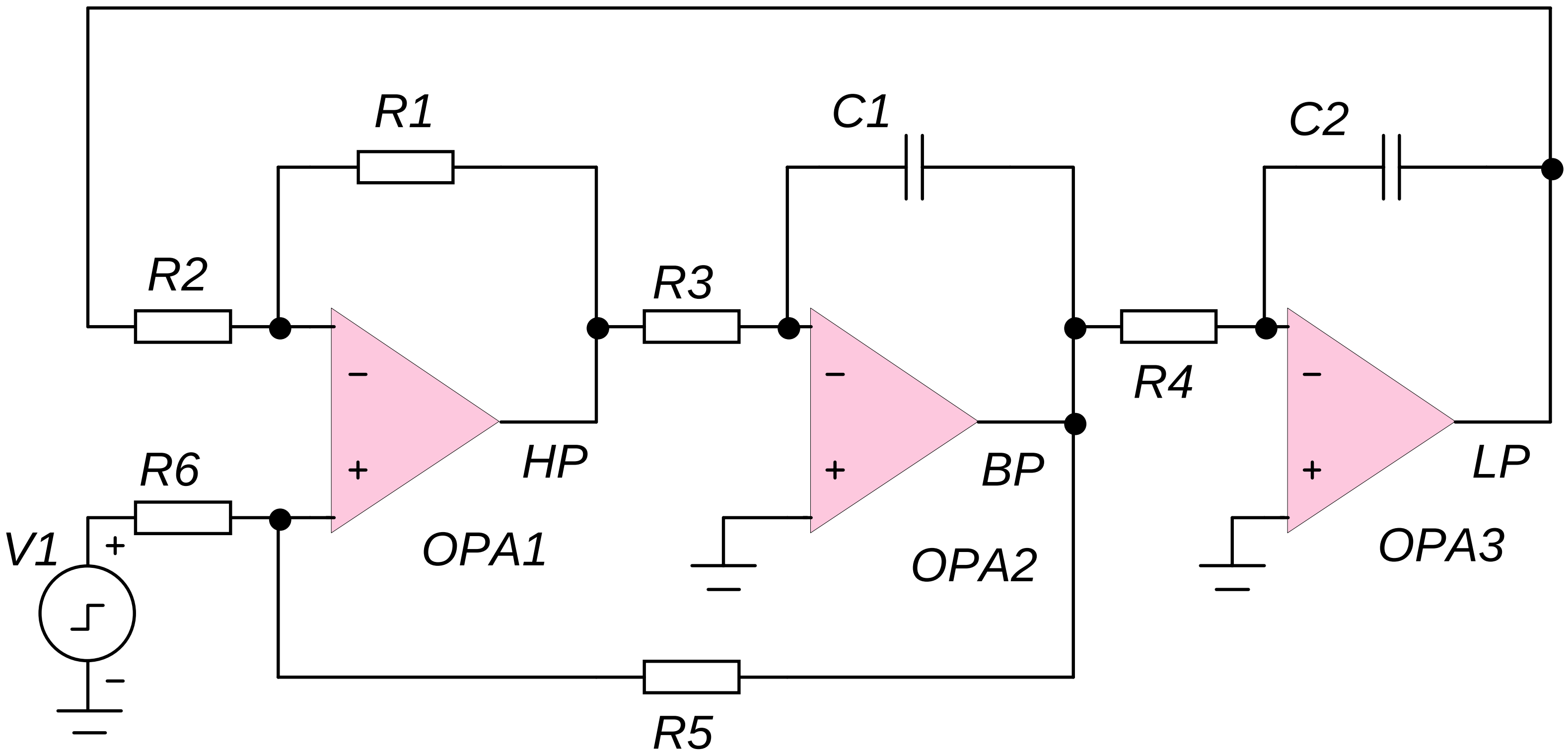


reduction of 3 paths by one

Proportional variation: R3-G1-G2, D2-C1

Unproportional variation: R3-D2

EXAMPLE 5: active "KHN" filter



Denominator of transfer functions

$$b_2 s^2 + b_1 s + b_0 \,,$$

$$b_2 = R_2 R_3 R_4 C_1 C_2 (R_5 + R_6),\; b_1 = R_4 R_6 C_2 (R_1 + R_2),\; b_0 = R_1 (R_5 + R_6).$$

Numerators for HP/BP/LP are

$$a_0 \,,\; -a_1 s \,,\; a_2 s^2 \,, a_0 = R_5 (R_1 + R_2),\; a_1 = R_4 R_5 (R_1 + R_2) C_2 \; a_2 = R_3 R_4 R_5 C_1 C_2 (R_1 + R_2).$$

Objective: to find disjoint sets of element parameters whose proportional variations will have no influence on the above transfer functions.

Original VERTEX graph is complicated $\Rightarrow$ algebraic manipulations (computer program):

$$a_0 = R_5\left(R_1 + R_2\right),$$

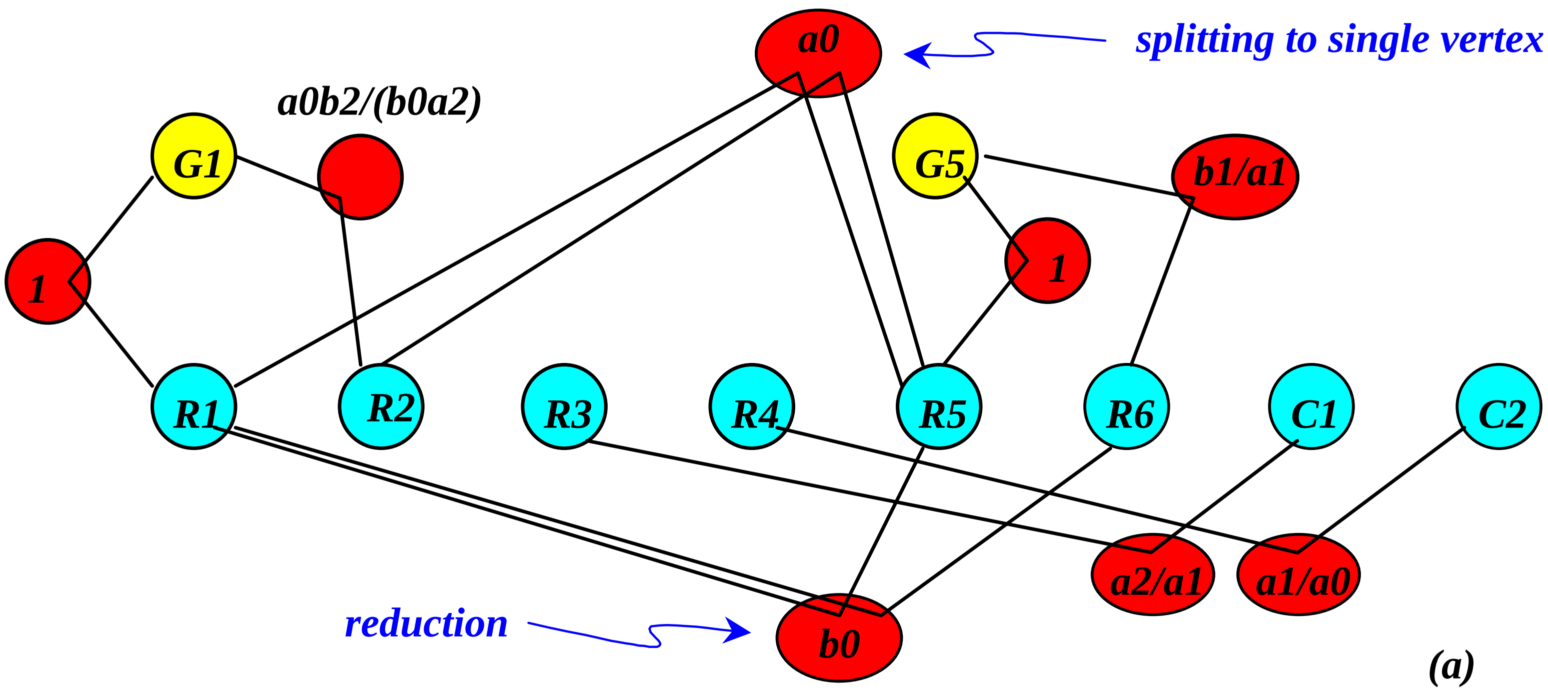
$$a_1 / a_0 = R_4 C_2 ,$$

$$a_2 / a_1 = R_3 C_1 ,$$

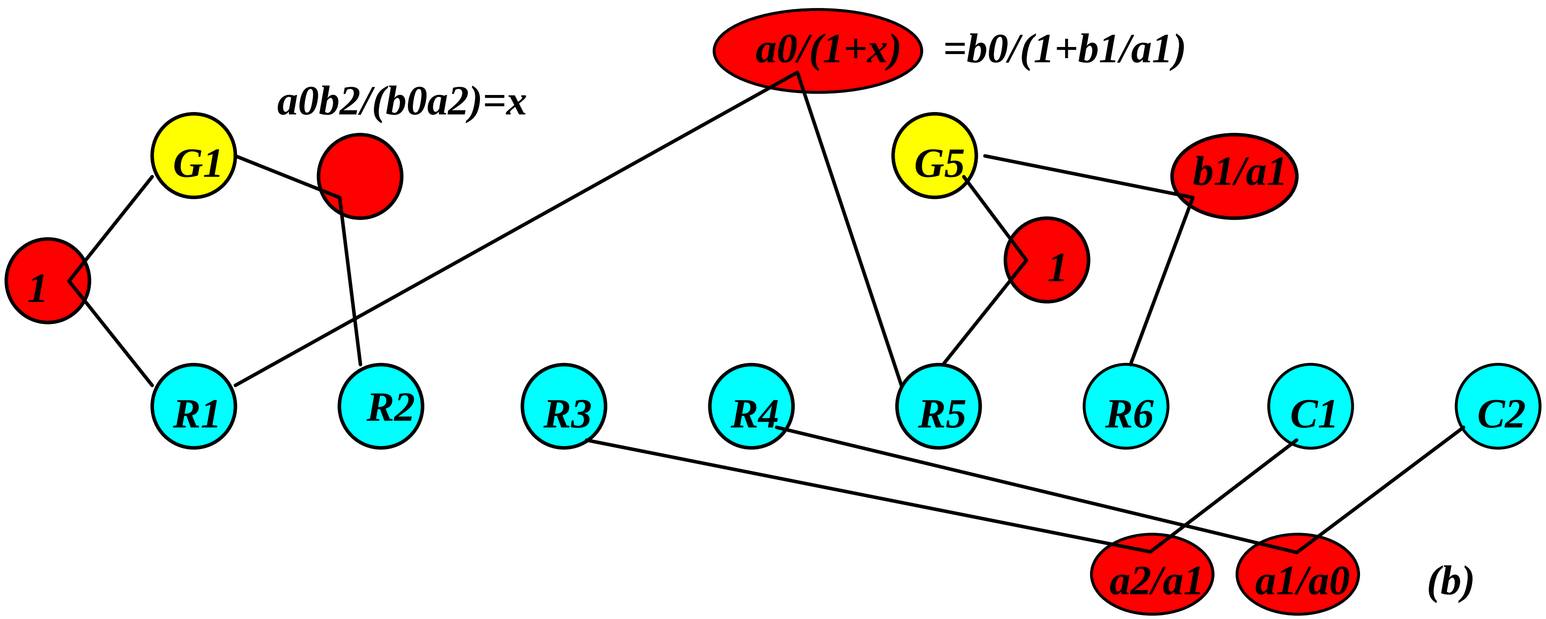
$$b_0 = R_1\left(R_5 + R_6\right),$$

$$b_1 / a_1 = R_6 G_5 ,$$

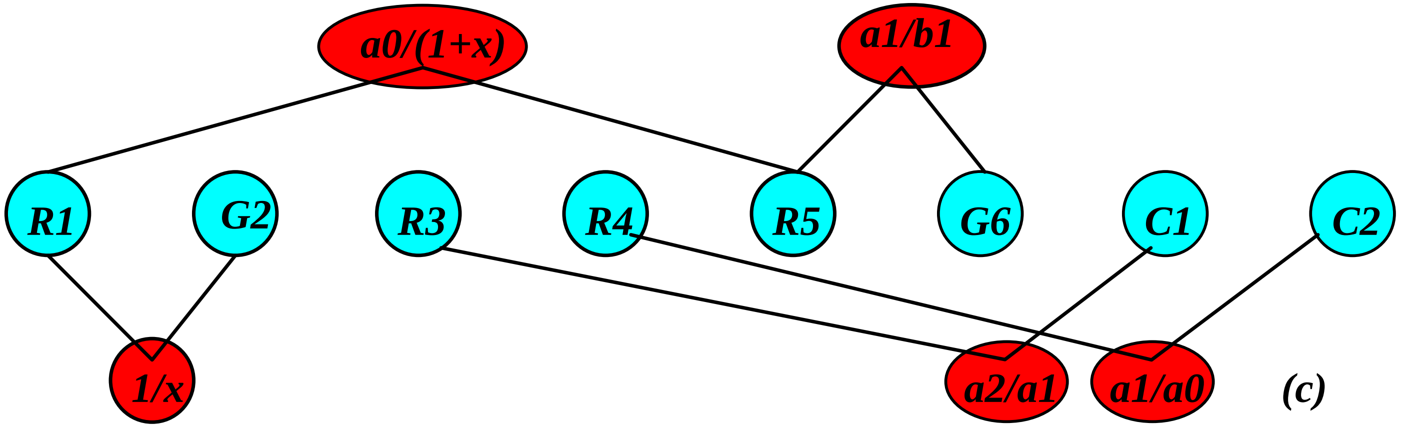
$$a_0 b_2 /(a_2 b_0) = R_2 G_1 .$$


splitting to single vertex
a0
a0b2/(b0a2)
G1
G5
b1/a1
1
1
R1
R2
R3
R4
R5
R6
C1
C2
a2/a1
a1/a0
reduction
b0

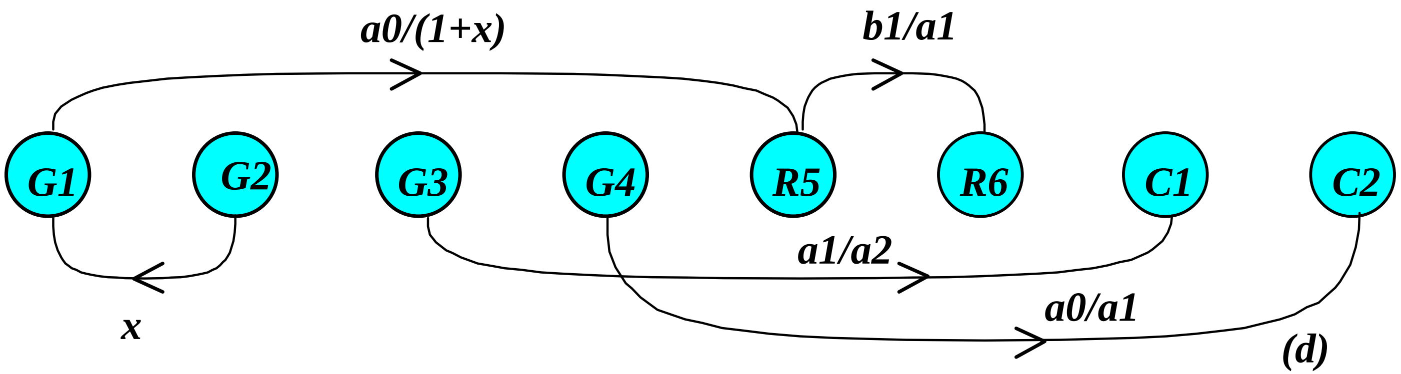
(a)


a0/(1+x)
=b0/(1+b1/a1)
a0b2/(b0a2)=x
G1
G5
b1/a1
1
1
R1
R2
R3
R4
R5
R6
C1
C2
a2/a1
a1/a0

(b)



(c)

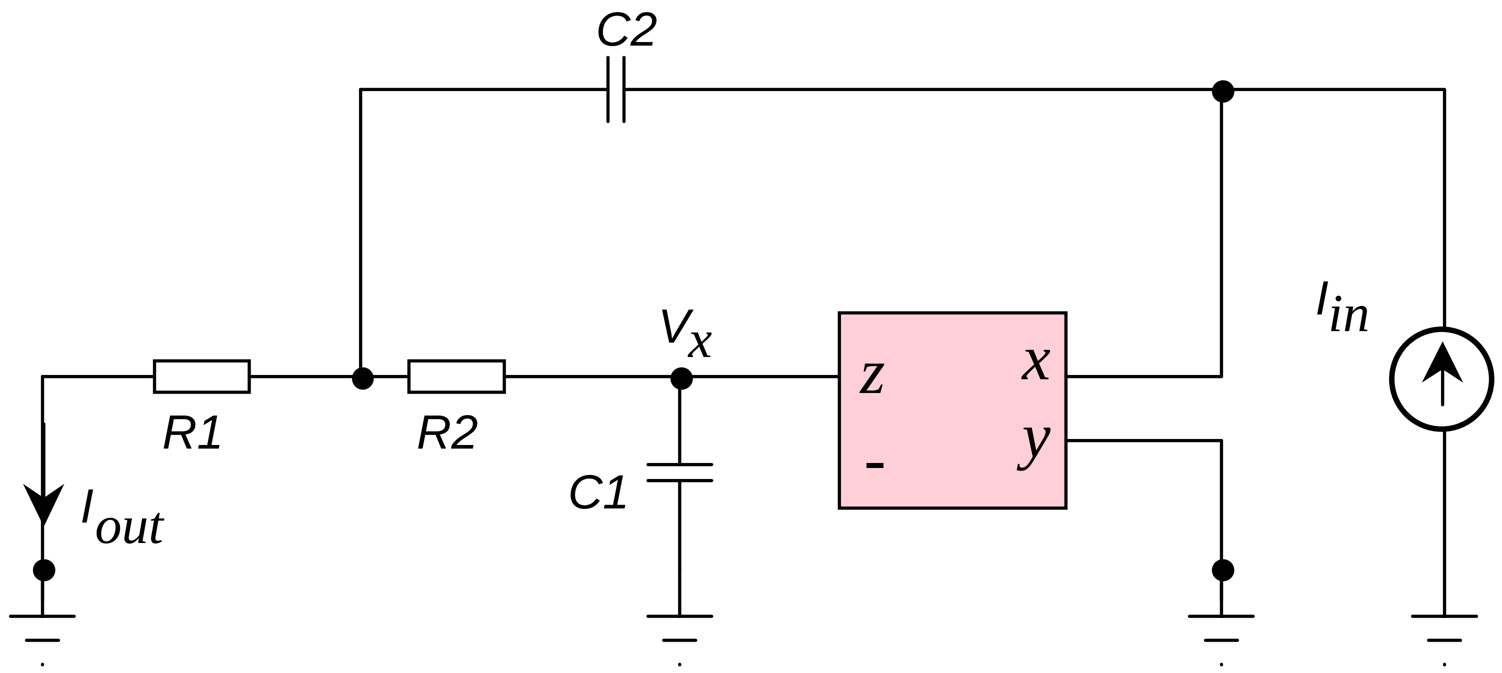


(d)

Proportional independent variation:

G1-G2-R5-R6, G3-C1, G4-C2

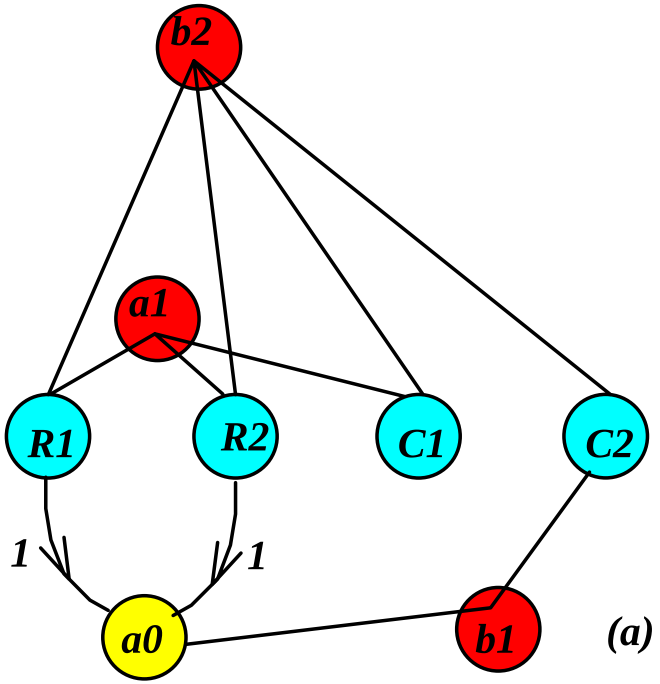
EXAMPLE 6: current-mode “Sallen-Key” filter



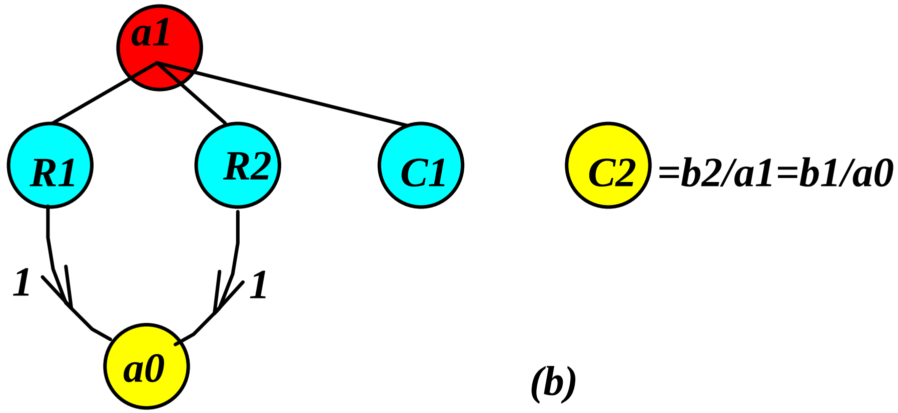
$$\frac{I_{out}}{I_{in}} = \frac{1}{1 + b_1 s + b_2 s^2}\,,\quad \frac{V_x}{I_{in}} = -\frac{a_0 + a_1 s}{1 + b_1 s + b_2 s^2}$$

$$a_0 = R_1 + R_2\,,\ a_1 = R_1 R_2 C_1\,,\ b_1 = \left(R_1 + R_2\right) C_2\,,\ b_2 = R_1 R_2 C_1 C_2$$

Objective: to find a law of element variation, which preserves both circuit functions.



(a)



(b)

Element variation:

C2 is fixed.

R1+R2 is fixed.

R1R2C1 is flexible.

GENERALIZATION

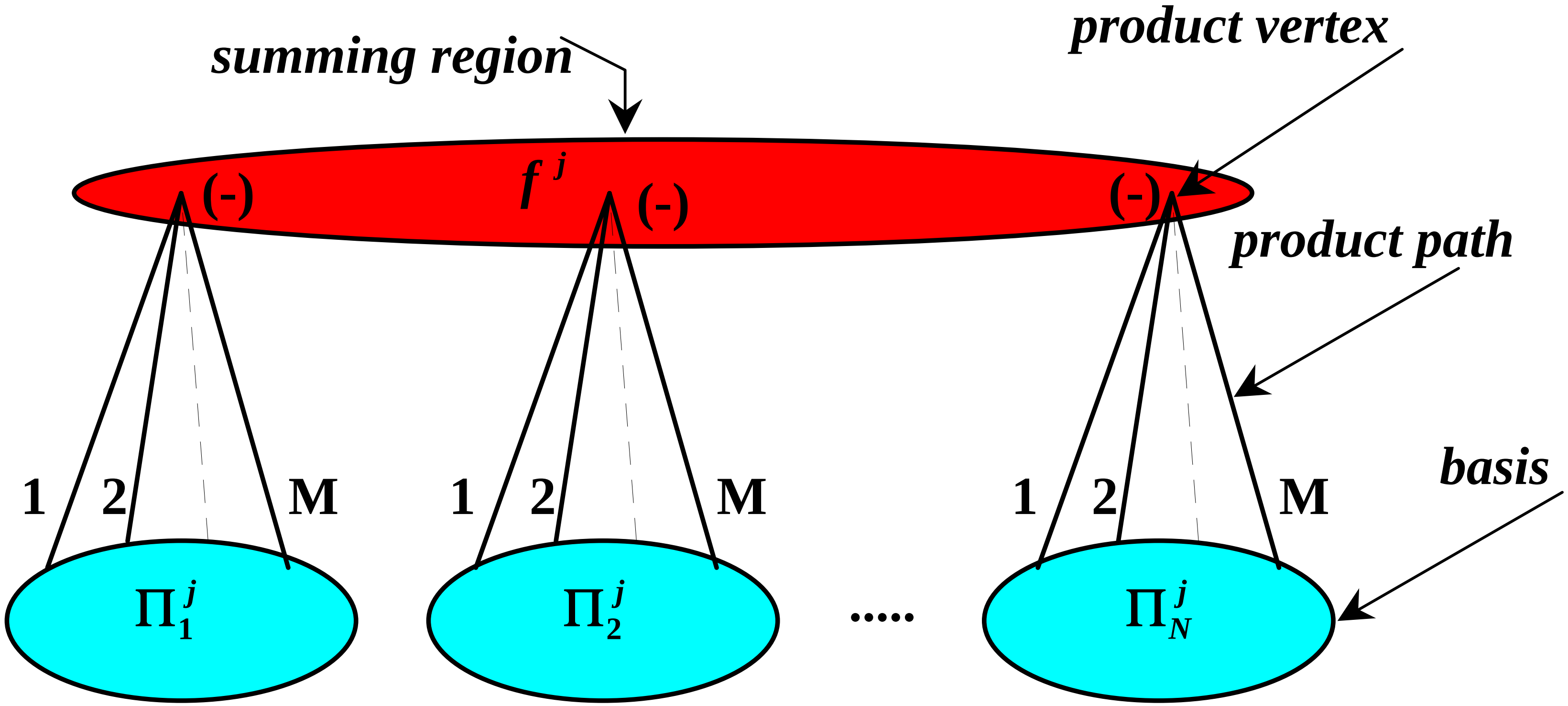
Coupling (Vertex) graphs - definition

Let circuit functions depend on m parameters $P=[p_1, p_2,.., p_m]$. These parameters are coupled by n coupling conditions of the type of

$$F^{j}(P)=f^{j},\ j=1, 2, .., n,$$

where f^{j} are constants (for example, transfer function coefficients).

$$F^{j}=\sum_{k=1}^{N}\left(\pm\Pi_{k}^{j}\right)=f^{j},\ j=1, 2, .., n.$$



N=1: graph with *single vertex* (or the *single graph*).

N>1: graph with *multiple vertex* (or *multiple graph*), which is sunk into the so-called *summing region*.

Algorithmic utilization of vertex graphs

- Algorithmic generation of required circuit functions in the symbolic form.
- Definition of coupling conditions according to the optimization objective.
- Interpretation of the vertex graph in the data structure.
- Algorithmic simplification of the vertex graph to so-called minimum form, or to the causal graphs.
- Utilization of the minimum graph for final optimization.

Applications

Some of the element parameters are changed (e.g. by replacement the element with a different type). The problem is to find a minimum number of other parameters, whose variation will fully compensate the effect of the original change on the monitored filter properties (full compensation), or to find other parameters, whose variation will compensate the effect of the original change in a defined way (partial compensation).

- Optimization of element ratios, problems of element replacement, compensation of real properties, ...

It is necessary to find a minimum number of parameters whose proportional variation (see later) results in the required control of filter parameters.

- Optimization of dynamic range of active filters: Let us have a multiple *OpAmp* active filter, where we have to change the voltage level at a certain output while preserving the voltages in remaining nodes.
- Algorithms of controlling the filter parameters.

Problems solved

1. Proposition of the (non)existence of VERTEX graph transformation to separated causal graphs.
2. Proposition of the graph regularity/singularity.
3. proposition of the number of freedom of graph which is compound of subgraphs with the known number of freedom.
4. Proposition of the optimum choice of graph basis.
5. System of rules of successive graph simplification.
6. General computer algorithm of graph evaluation.

SUMMARY

The method described enables the design of algorithms of finding mutual couplings between the element parameters of a complicated linear dynamic system. These couplings are defined by the features of required circuit functions. Typical examples of application are the optimization of element ratios and of the dynamic range of filters, and the compensation of the influence of undesirable changes in element parameters on the total circuit performance.

This poster can be downloaded from

http://www.vabo.cz/stranky/biolek

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