(SEMI)SYMBOLIC MODELING OF LARGE LINEAR SYSTEMS: PENDING ISSUES

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OUTLINE

- ✓(Semi)symbolic versus numerical analysis.
- ✓Symbolic, semisymbolic and numerical links.
- ✓MATLAB algorithms
- ✓SNAP tool
- ✓Conclusion

(SEMI)SYMBOLIC VERSUS NUMERICAL ANALYSIS

Input data: Linear model of continuous-time (C-T) system, or
Linear model of discrete-time (D-T) system
+ identification of the required analysis aims.

Analysis outputs: The system functions as equations.
+ possible graphs or tables if required.

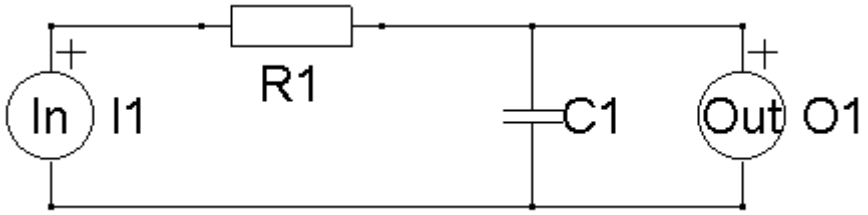
Forms of the analysis outputs:

symbolic form: math. formula which includes symbols of circuit parameters;

semisymbolic form: numerical values are instead of symbols, the complex frequency s or z (freq. domain) or the time variable t or k (time domain) is the only symbol in the formula;

numeric form: numerical results (poles and zeros, waveform points,..)

Example: RC cell



Voltage gain (open output)

s,t

```
_____________symbolic_____________

1
-----------------------------------
1
+s*( R1*C1 )


_____________semisymbolic_____________

Multip. Coefficient =  1.00000000000000E+0005

 1.00000000000000E+0000
-----------------------------------
 1.00000000000000E+0005
 1.00000000000000E+0000 * s
```

fraction line

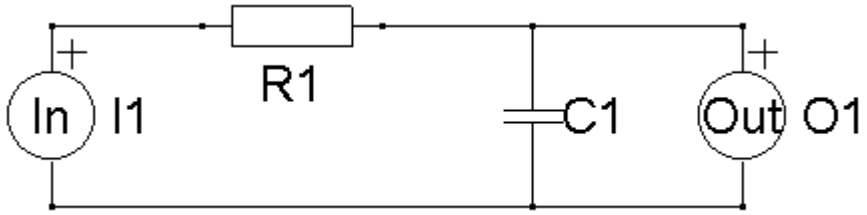
symbolic analysis:

$$K_V = \frac{1}{1+sR_1C_1}$$

semisymbolic analysis:

$$K_V = 1e5\,\frac{1}{1e5+s}$$

Example: RC cell




```
_________________zeros_____________________

none

_________________poles_____________________

-1.0000000000000E+0005


____________step response_________________

 1.0000000000000E+0000
-1.0000000000000E+0000*exp(-1.0000000000000E+0005*t)


____________pulse response_________________

 1.0000000000000E+0005*exp(-1.0000000000000E+0005*t)
```

no zeros

pole –1e5

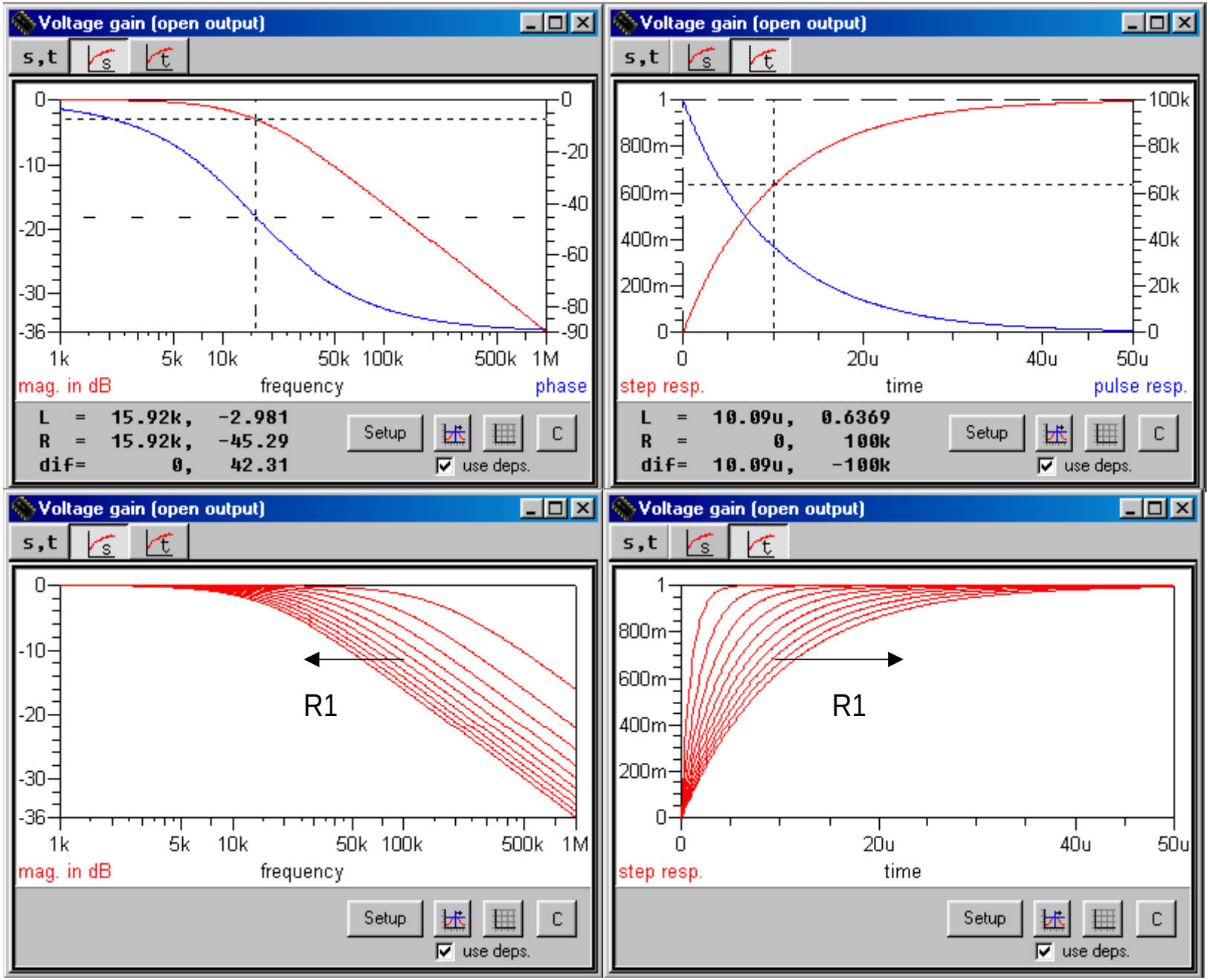
response to unity step

$h(t)=1-e^{-100000\,t}$

response to Dirac impulse

$g(t)=1e5e^{-100000\,t}$

RC cell – some results of numerical analysis



Why (semi)symbolic computation?

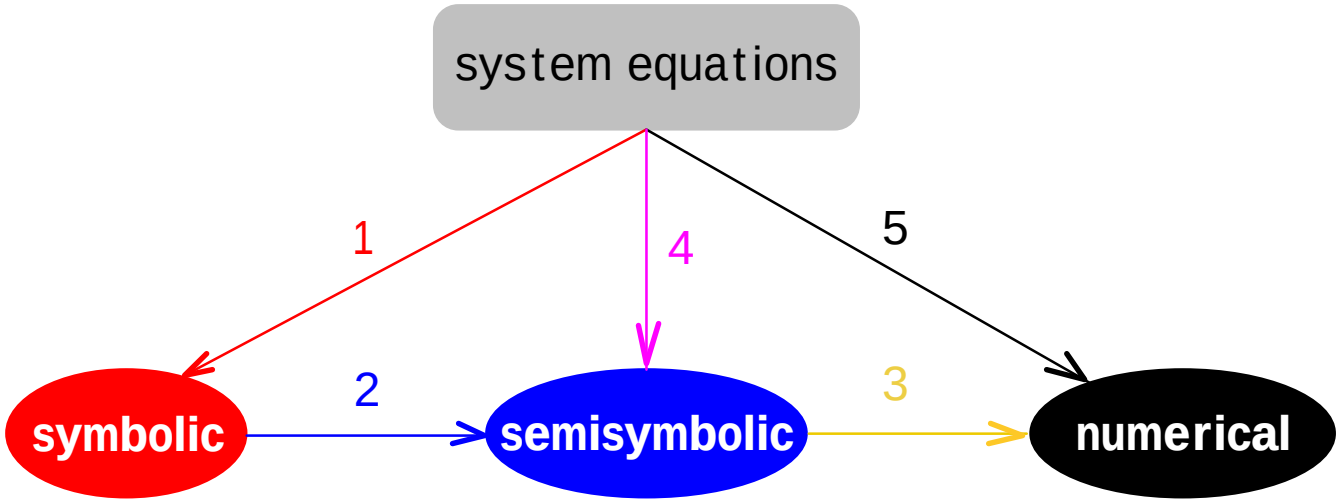
- ✓ Equations = source of more information than those from numerical results (they include them)
- ✓ From the equations, important connections between the system and its behaviour can be read
- ✓ Equations = important data for verification of system principle.
- ✓ Equations = important data for system optimization.

Why NOT (semi)symbolic computation?

- ☒ CPU time- and memory-expensive algorithms
- ☒ Serious numerical problems must be overcome in some cases.
- ☒ Complexity and non-transparency of symbolic result while analyzing large systems.

SAG, SBG, SDG → Simplification of symbolic results

Symbolic, Semisymbolic and Numerical links



operator (s, z)
symbolic formulae

NOT in time domain

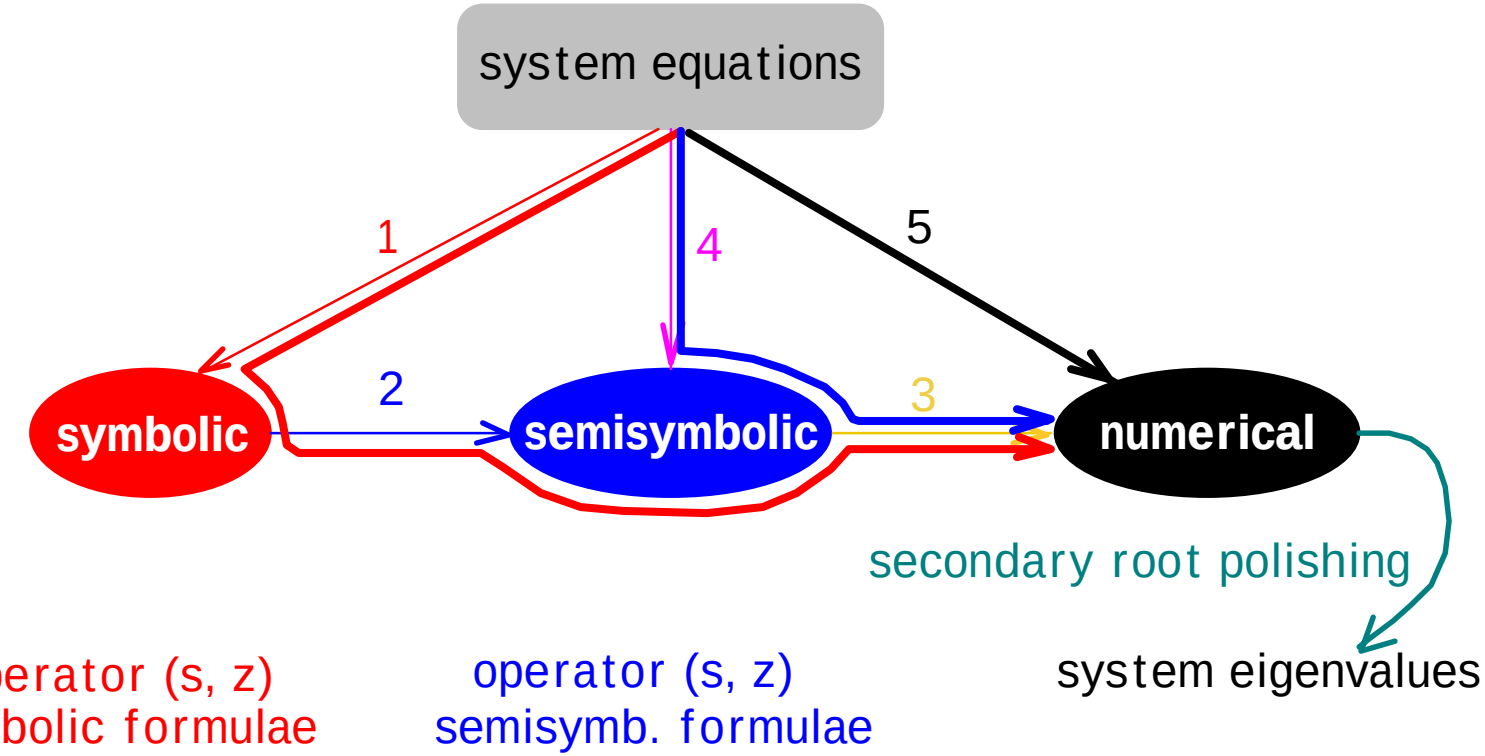
operator (s, z)
semisymb. formulae

time domain
semisymb. formulae

system eigenvalues
poles and zeros
frequency responses

unity step and other responses

Computing the system eigenvalues

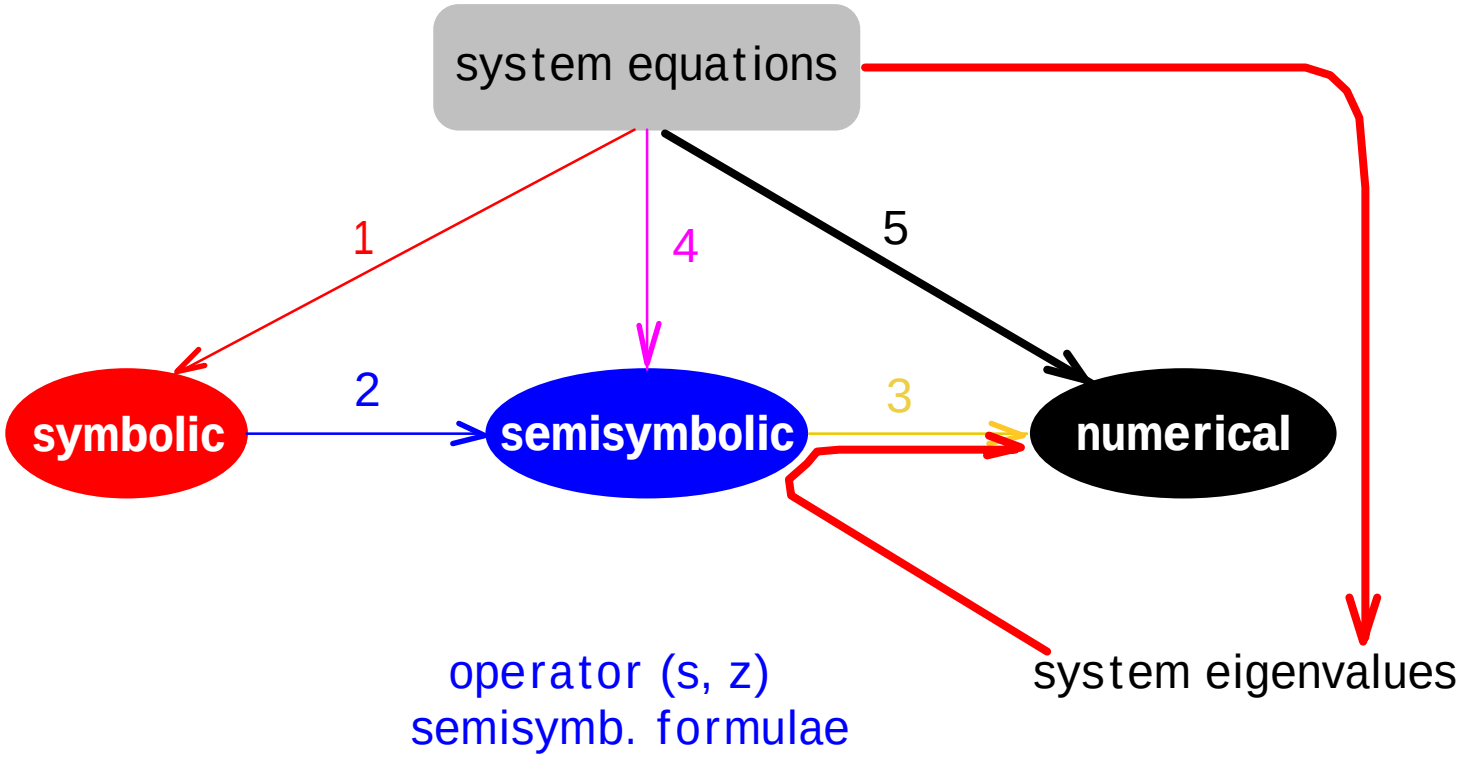


Numerical way (5): large circuits, problematic precision; QR, QZ,..

Semisymbolic way (4,3): moderate-size to large-size systems, problematic precision; FFT, Faddeyev algorithm (4), Laguer, method of accompanying matrix, “infinite precision” (3)

Symbolic way (1,2,3): small-size to moderate-size systems, excellent precision; ? (1), utilization of “infinite precision” (2,3).

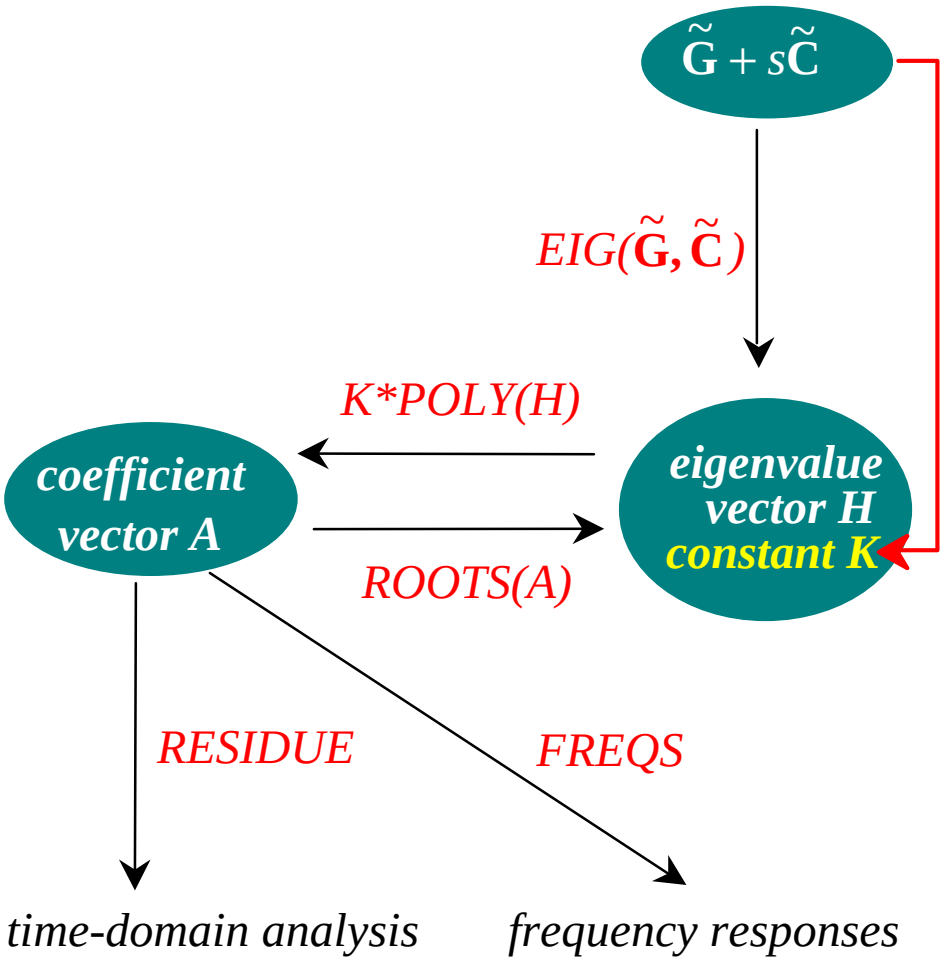
Computing the time responses



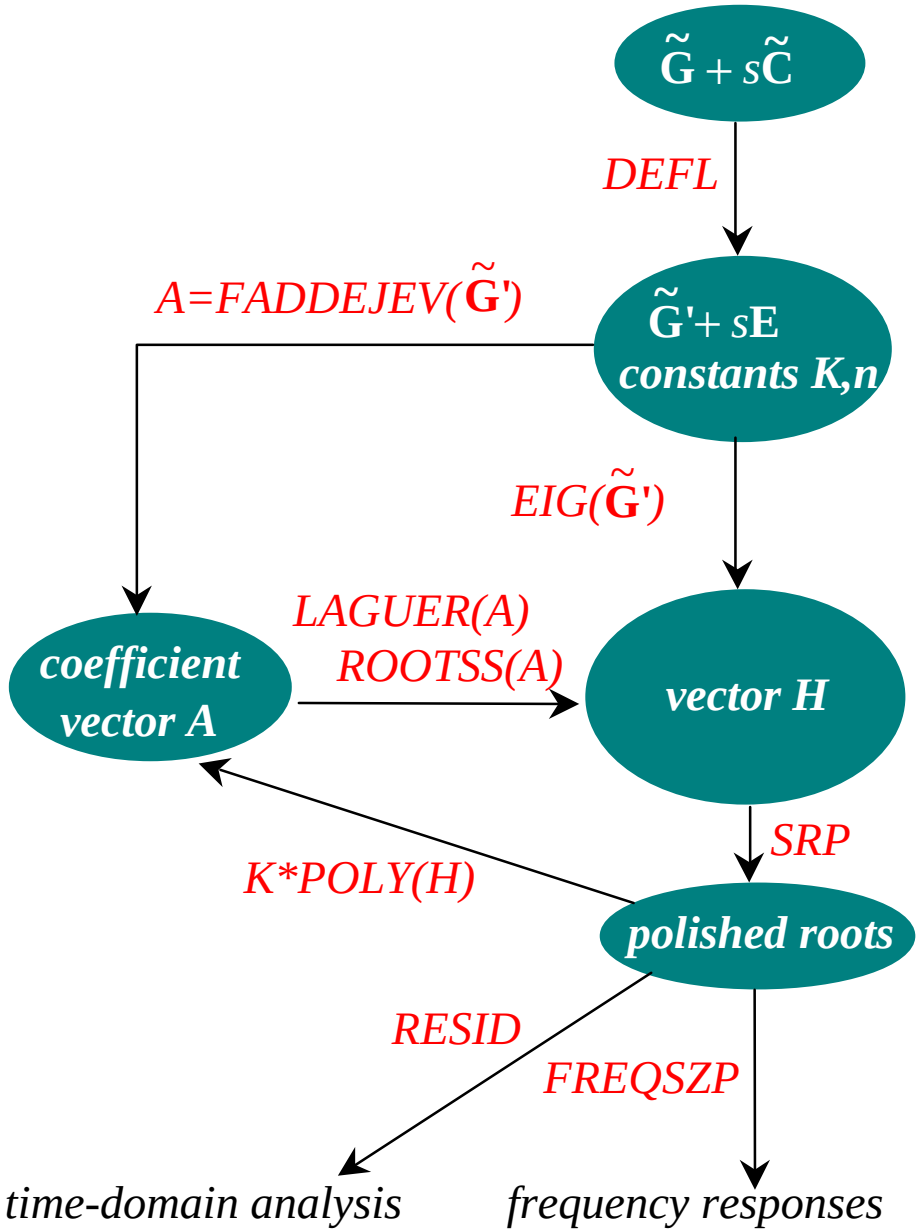
Numerical way (5): large circuits, good precision; classical integration formulas

Semisymbolic way: moderate-size to large-size systems, precision depends on computing eigenvalues; partial fraction expansion, “infinite precision” (3)

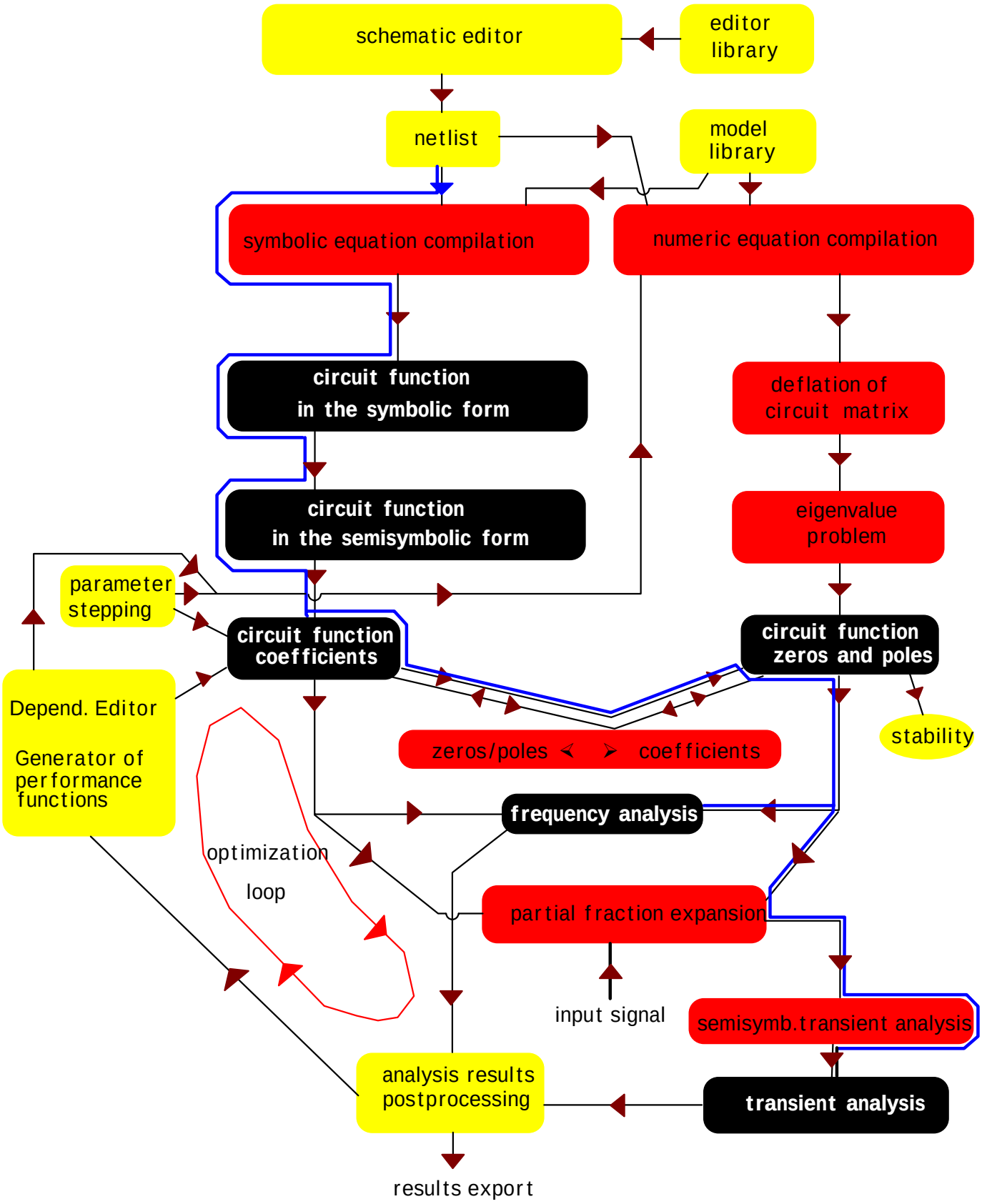
Standard MATLAB algorithms



New MATLAB algorithms



SNAP tool
(Symbolic and Numeric Analysis Program)



CONCLUSIONS

Algorithms with improved accuracy = combination of symbolic, semisymbolic, and numerical approaches + techniques of “infinite precision”.

To solve in future:

- rational arithmetic
- “infinite precision” arithmetic
- topological methods of matrix deflation
- solving the eigenvalue problem by means of arithmetic above
- solving the polynomial roots from symbolic results by means of “infinite precision” arithmetic.