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MATHEMATICAL MODELS FOR HUMAN PILOT MANEUVERS IN AIRCRAFT FLIGHT SIMULATION

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ABSTRACT

Mathematical models are presented in this paper to describe human maneuvers for aircraft flight simulation. Input parameters for the human pilot model (HPM), such as the course deviation indicator (CDI) and the heading change, are defined for the model, and are related mathematically to those in the proportional-integral-derivative (PID) controller for automatic control. Similarities are discussed between the parameters in HPM and those in the automatic control for better understanding of the significance of human factors and their effect on aircraft behavior. Examples for the HPM include aircraft instrument landing system (ILS) lateral and vertical control, heading change, and homing. The model is tested by using the high-fidelity flight simulation simulator JSBSim [1].

NOMENCLATURE

ail Actual aileron position.
ail_0 Desired aileron position.
α_{GS} Weight for glide slope CDI observation.
α_{LOC} Weight for localizer CDI observation.
β_{GS} Weight for the observation of descending angle change.
β_{LOC} Weight for the observation of heading angle change.
$\mathrm{CDI}_{\mathrm{GS}}$ Course deviation indicator for glide slope approach
$\mathrm{CDI}_{\mathrm{LOC}}$ Course deviation indicator for localizer approach
D Gain for derivative term.
δt Time increment for flight simulation.
δ_x Change of control variable.
dHeading Change of aircraft heading angle.
ele Actual elevator position.
ele_0 Desired elevator position.
γ_y Speed of the actuator movement.
h Aircraft altitude.
H Glide slope height at the aircraft horizontal location.
I Gain for integral term.
L_{GS} Distance from aircraft to glide slope unit.
L_{LOC} Distance from aircraft to localizer.
P Gain for proportional term.
φ_{ACF} Horizontal location angle of aircraft with respect to true north.
φ_{LOC} Horizontal direction angle of localizer beam with respect to true north.
ψ_{ACF} Horizontal location angle of aircraft with respect to east.
θ_{ACF} Elevation angle of aircraft with respect to horizontal plane.
θ_{GS} Glide slope elevation angle with respect to horizontal plane.
Σ Summation operator.
V Aircraft velocity along flight path.
V_v Aircraft vertical velocity.
x Control variable.
x_0 Desired control variable.
x, y, z Aircraft location in local coordinate system.
y Actuator position.
y_0 Desired actuator position.
$y_{\max}$ Maximum actuator position.
$y_{\min}$ Minimum actuator position.

INTRODUCTION

Large numbers of aircraft dynamic models are available and well documented for the simulation of aircraft flight since computers become popular. Among those, high-fidelity aircraft flight simulators include the JSBSim [1], X-Plane [2], and Microsoft Flight Simulator [3]. However, many applications that need the high-fidelity flight simulator and full animation, especially the Monte Carlo simulations for the safety studies on aircraft landings [4], require computer pilot models.

While the pilot models have been extensively studied [5–8], those models in the closed-loop control are difficult to implement since the aircraft dynamics is often unknown [9]. When modeling a human pilot maneuver, it is important to identify the signif-

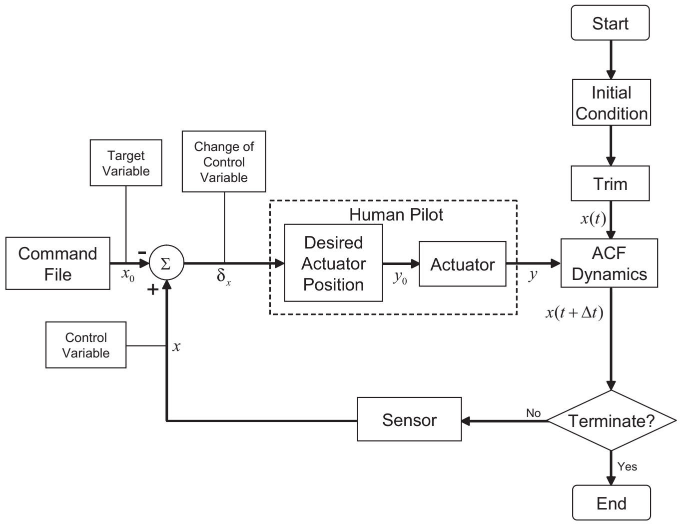

Figure 1. SYSTEM DESIGN OF HUMAN PILOT MODEL.

icant factors in as simple a model as possible that still captures major behavior and complexity.

This paper presents a generic HPM that is used in the closed-loop control for the high-fidelity aircraft dynamic model – JSBSim [1]. The input to the HPM is the change of control variable (e.g., the difference between the actual altitude and the desired altitude for leveling control) and the output is the actuator position (e.g., elevator or aileron position). Parameters are used to capture the "human factors" in the HPM, including the gains of the proportional, integral, and derivative terms that ensure the stability of the aircraft reaction and the simplicity of the model. Using the ILS localizer and glide slope approaches as examples, this paper shows that alternative physical parameters such as the heading error and the vertical velocity available on the cockpit can be used as the derivative terms of the localizer and glide slope CDIs, respectively. These derivative terms are not directly available for human pilots.

SYSTEM DESCRIPTION OF HUMAN PILOT MODEL

As shown in Fig. 1, the HPM takes the change of control variable (δ_x) as the input to "compute" the desired actuator position (y_0). The actuator is "moved" to the position y through the actuator model which is connected to the ACF dynamics. The generic mathematical algorithms that describes the HPM are shown below.

Denoting $x(t)$ and $x_0(t)$ as the control variable and predetermined target variable, respectively, the change of control variable is written as

$$\delta_x = x(t) - x_0(t). \tag{1}$$

It is assumed that the HPM "computes" the desired actuator position based on the sum of the strength, the integration, and the rate of the change of control variable, i.e.,

$$y_0(t) = P \cdot \delta_x(t) + I \cdot \int_0^t \delta_x(t')dt' + D \cdot \dot{\delta}_x(t), \tag{2}$$

where P, I, and D are gains and are often treated as random numbers in the numerical simulation to describe various human pilots.

The movement of an actuator (e.g., throttle, aileron, and elevator) for one time-step δt is modeled as the integration of actuator speed γ_y as:

$$y(t+\delta t) = y(t) - \text{sign}\left[y(t) - y_0(t)\right] \gamma_y \delta t, \tag{3}$$

where $y(0)$ is the initial actuator position. The actuator position is truncated and limited in the range of $[y_{\min}, y_{\max}]$, where "min" and "max" are minimum and maximum values, respectively. The pseudo code of the limiter is

$$\text{if } \ y(t) > y_{\max}, \text{ then } \ y(t) = y_{\max}, \tag{4}$$
$$\text{if } \ y(t) < y_{\min}, \text{ then } \ y(t) = y_{\min}. \tag{5}$$

For the HPM, the speed of the actuator movement γ_y varies for different HPMs. It is specified as a random number in the numerical simulation.

The sensor shown in Fig. 1 is modeled to be "perfect", meaning that the output from the sensor is exactly the same as the input.

ILS Localizer Approach

The control variable used by the HPM for ILS localizer approach is the CDI, the difference between the aircraft horizontal location angle (φ_{ACF}) in the local coordinate system and the localizer pointing angle (φ_{LOC}) as illustrated in Fig. 2. The E (east) and N (north) axis (or x and y axis) form a plane that passes through the localizer and is tangent to the ellipsoid earth surface defined by the WGS84 model [10]. When an aircraft is flying along a localizer centerline, the HPM uses the CDI as the major input to determine the desired aileron position (Here for simplicity only the aileron is considered). According to the PID control theory [11], the rate of the CDI is also important for aircraft stabilization and overshoot minimization. The following discussion will show that the heading angle could be (actually has been) used by the HPM to obtain the CDI rate information.

Consider an aircraft located at distance x from the localizer centerline, and distance L_{LOC} from the localizer. The CDI, defined as the horizontal offset angle of the aircraft with respect to the localizer centerline, is approximated as

$$\text{CDI}_{\text{LOC}} \approx \frac{x(t)}{L_{LOC}(t)}, \tag{6}$$

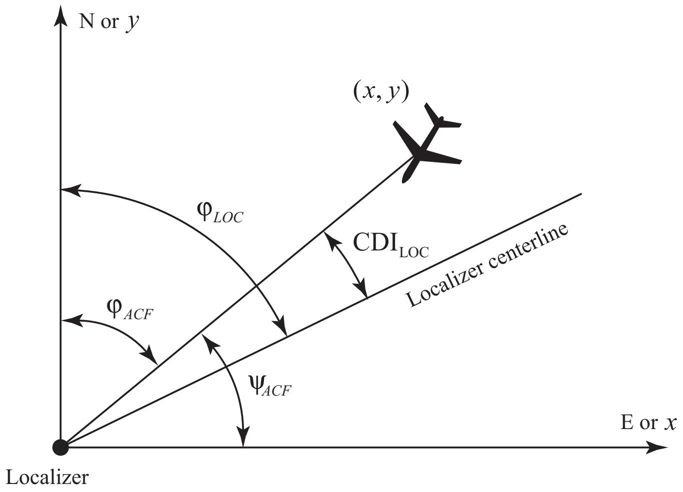

Figure 2. GEOMETRY OF ILS LOCALIZER APPROACH.

where $x(t) << L_{LOC}(t)$. The derivative of $\mathrm{CDI_{LOC}}$ is found as

$$\frac{d\mathrm{CDI_{LOC}}}{dt} = \frac{\dot{L}_{LOC}(t)}{L_{LOC}(t)}\left(\frac{\dot{x}(t)}{\dot{L}_{LOC}(t)} - \frac{x(t)}{L_{LOC}(t)}\right). \tag{7}$$

In the right side of Eqn. (7), $\dot{x}(t)/\dot{L}(t)$ is the heading angle in terms of the localizer centerline, $x(t)/L(t)$ is the CDI, and $\dot{L}(t)$ is the aircraft speed toward the localizer. Thus Eqn. (7) is rewritten using Eqn. (6) as

$$\frac{d\mathrm{CDI_{LOC}}}{dt} = \frac{V(t)}{L_{LOC}(t)}\left(\mathrm{dHeading} - \mathrm{CDI_{LOC}}\right). \tag{8}$$

Assume that the desired aileron position for the horizontal control by the HPM is modeled as the sum of the proportion and the derivative terms (the integration term is not shown here but could be added):

$$\mathrm{ail}_0 = P \cdot \mathrm{CDI_{LOC}} + D \cdot \frac{d\mathrm{CDI_{LOC}}}{dt}. \tag{9}$$

Then using Eqn. (8), one obtains

$$\mathrm{ail}_0 = \left[P - D \cdot \frac{V(t)}{L_{LOC}(t)}\right]\mathrm{CDI_{LOC}} + \left[D \cdot \frac{V(t)}{L_{LOC}(t)}\right]\mathrm{dHeading}. \tag{10}$$

Redefine the coefficients

$$\begin{cases} \alpha_{LOC} = P - D \cdot \dfrac{V(t)}{L_{LOC}(t)}, \\ \beta_{LOC} = D \cdot \dfrac{V(t)}{L_{LOC}(t)}, \end{cases} \tag{11}$$

Eqn. (10) is simplified as

$$\mathrm{ail}_0 = \alpha_{LOC} \cdot \mathrm{CDI_{LOC}} + \beta_{LOC} \cdot \mathrm{dHeading}, \tag{12}$$

where α_{LOC} and β_{LOC} are parameters that quantify the HPM observations of the CDI and the heading angle error with respect to the localizer centerline.

If $\alpha_{LOC} = 0$ and $\beta_{LOC} \neq 0$, then the aileron position is proportional to the change of heading angle, indicating that the change of heading is zero when the control process is stabilized and the aileron position is centralized. Thus this process is actually the "turning to the heading" and may happen when the HPM only looks at the compass to fly the aircraft for ILS landing.

If $\alpha_{LOC} \neq 0$ and $\beta_{LOC} = 0$, then the aileron position is proportional to the CDI. This will cause the aircraft to oscillate across the localizer centerline because of overshoot. This behavior will occur when the HPM only looks at the CDI to control the ILS landing.

When $\alpha_{LOC} = \beta_{LOC}$ and the aircraft is stabilized with the zero aileron position ($\mathrm{ail}_0 = 0$), $\mathrm{CDI_{LOC}}$ and dHeading will be equal with opposite signs. This is actually "homing" because the aircraft flies with the same heading while it remains the same offset angle from the runway centerline.

To obtain a desired flight path for ILS localizer approach, the parameters should be $\alpha_{LOC} > \beta_{LOC} > 0$.

ILS Glide Slope Approach

The control variable used by the HPM for ILS glide slope approach is the CDI, the difference between the aircraft elevation angle (θ_{ACF}) in the local coordinate system and the glide slope elevation angle (θ_{GS}, usually 3.0 degrees) as illustrated in Fig. 3. The U (or z) axis is perpendicular to the "horizontal plane" formed by the E and N axes shown in Fig. 2. The "horizontal axis" is the projection of the glide slope onto the "horizontal plane". In addition, the information of the CDI rate must

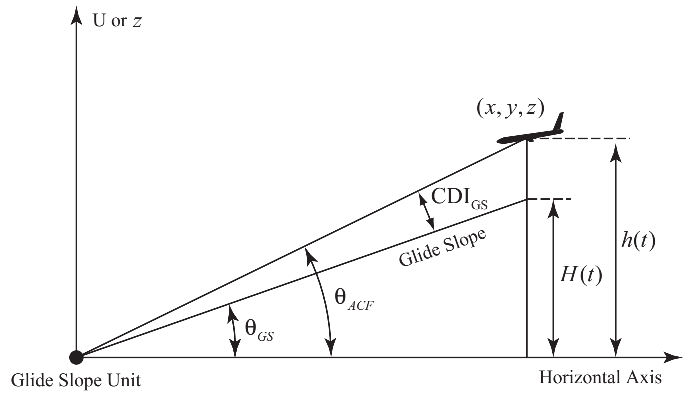

Figure 3. GEOMETRY OF ILS GLIDE SLOPE APPROACH.

also be used in order to stabilize the aircraft and minimize overshoot [11]. The discussion bellow shows that the CDI rate can be obtained from aircraft vertical speed.

Consider the aircraft altitude to be $h(t)$ at time t, $H(t)$ to be the corresponding altitude of the glide slope, and L_{GS} is the distance from the touchdown, the CDI of the glide slope is approximated as

$$\mathrm{CDI_{GS}} \approx \frac{h(t)-H(t)}{L_{GS}(t)}, \tag{13}$$

where $|h(t)-H(t)| << L_{GS}(t)$. The derivative of $\mathrm{CDI_{GS}}$ is found as

$$\frac{d\mathrm{CDI_{GS}}}{dt} = \frac{\dot{h}(t)-\dot{H}(t)}{L_{GS}(t)} - \frac{h(t)-H(t)}{L_{GS}^2(t)}\dot{L}_{GS}(t). \tag{14}$$

Define the following parameters:

$$\begin{cases} \dot{h}(t) = V_v(t), \\ \dot{H}(t) \approx V(t)\sin\theta_{GS}, \\ \dot{L}_{GS}(t) = V(t), \end{cases} \tag{15}$$

where $V_v(t)$ is the vertical speed, θ_{GS} is the glide slope elevation angle (usually 3°), and $V(t)$ is the aircraft speed along the flight path, Eqn. (14) is rewritten as

$$\frac{d\mathrm{CDI_{GS}}}{dt} = -\frac{V(t)}{L_{GS}(t)}\left[\mathrm{CDI_{GS}} - \left(\frac{V_v(t)}{V(t)} - \sin\theta_{GS}\right)\right]. \tag{16}$$

Assume that the desired elevator position for the HPM is modeled as the summation of the proportional, integration, and derivative terms:

$$\mathrm{ele}_0 = P\cdot\mathrm{CDI_{GS}} + I\cdot\int_0^t \mathrm{CDI_{GS}}(t')dt' + D\cdot\frac{d\mathrm{CDI_{GS}}}{dt}, \tag{17}$$

one obtains from Eqn. (16):

$$\begin{aligned} \mathrm{ele}_0 =& \left[P - D\cdot\frac{V(t)}{L_{GS}(t)}\right]\mathrm{CDI_{GS}} \\ &+ \left[\frac{D}{L_{GS}(t)}\right][V_v(t) - V(t)\sin\theta_{GS}] \\ &+ I\cdot\int_0^t \mathrm{CDI_{GS}}(t')dt'. \end{aligned} \tag{18}$$

Redefine the coefficients

$$\begin{cases} \alpha_{GS} = P - D\cdot\dfrac{V(t)}{L_{GS}(t)}, \\ \beta_{GS} = \dfrac{D}{L_{GS}(t)}, \end{cases} \tag{19}$$

Eqn. (18) is simplified as

$$\begin{aligned} \mathrm{ele}_0 =& \alpha_{GS}\cdot\mathrm{CDI_{GS}} + \beta_{GS}\cdot[V_v(t) - V(t)\sin\theta_{GS}] \\ &+ I\cdot\int_0^t \mathrm{CDI_{GS}}(t')dt', \end{aligned} \tag{20}$$

where α_{GS} and β_{GS} are parameters that specify quantitatively the HPM observations of the glide slope $\mathrm{CDI_{GS}}$ and the vertical velocity $V_v(t)$.

The behavior of aircraft flight with glide slope is different from the flight with localizer because of the integration term. To obtain a desired path on the glide slope, the parameters should be determined by numerical experiment for $\alpha_{GS} > \beta_{GS} > 0$.

NUMERICAL EXAMPLE

The ILS approach of B747 to runway 08L on the George Bush Intercontinental Airport (KIAH) is simulated with the JSBSim and the HPM described in this paper. The key "human factors", α_{LOC} and β_{LOC} for the ILS localizer approach, are assigned to different values to represent various aircraft flight behavior, including heading, homing, damping, and oscillating. Instead of directly retrieving the CDI rate which is difficult for human pilot to achieve, the change of heading and vertical speed are used as alternative parameters which are available on the cockpit to obtain the CDI rate information.

Figure 4 shows the trajectories of B747 with different values for α_{LOC} and β_{LOC}. The aircraft initially located at latitude of −95.57356532°, longitude of 30.05671811°, and altitude of 3,000 ft, or about 12 nmi from the runway threshold and 3.38 nmi from runway centerline. The approaching speed is about 150 knots. For all trajectories, the parameters α_{GS} and β_{GS} for glide slope approach are the same as shown in the caption of the figure.

When $\alpha_{LOC} = 0$ and $\beta_{LOC} = 0.03$, indicating that the pilot only looks at the compass to monitor the heading angle during the landing, the aircraft will fly parallel to the runway with the trajectory marked by "**Heading**", but will not merge to the runway centerline.

In the situation when $\alpha_{LOC} = 0.03$ and $\beta_{LOC} = 0.03$, the CDI and the change of heading angle remain opposite signs and cancel out each other, so that the desired aileron position becomes zero and the aircraft keeps its state unchanged. Therefore the aircraft flies straight to the localizer like "**Homing**". It also implies that the pilot pays equal attention to the CDI and the compass.

The desired trajectory (marked as "**Damping**" in the figure) is achieved when $\alpha_{LOC} = 0.03$ and $\beta_{LOC} = 0.01$ ($\alpha_{LOC} > \beta_{LOC} > 0$ as required). This situation is interpreted as that the pilot pays about 3 times more attention to the CDI as the compass.

If the pilot pays too much attention to the CDI than the compass, namely $\alpha_{LOC} >> \beta_{LOC}$, then the aircraft will be subject to "**Oscillating**" as shown in the figure where $\alpha_{LOC} = 0.03$ and $\beta_{LOC} = 0.005$.

CONCLUSION

This paper demonstrates the human pilot model with three key "human factors": proportion, integration, and rate of change of control variables. The *proportion* factor denotes the sensitivity of the human pilot to the control error. The *integration* indicates the human pilot's memory for the control error, and the *rate* is interpreted as the strength of human pilot's reaction to the rate of change of the control variable.

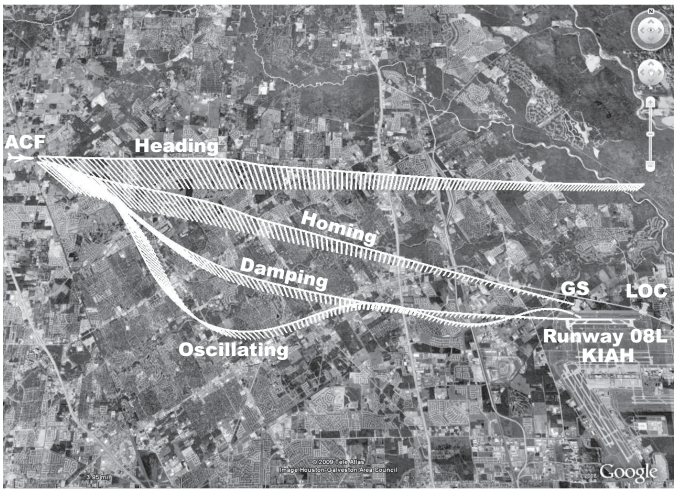

Figure 4. AIRCRAFT TRAJECTORIES FOR ILS LOCALIZER APPROACH WITH DIFFERENT HUMAN FACTORS. HEADING: $\alpha_{LOC} = 0, \beta_{LOC} = 0.03$. HOMING: $\alpha_{LOC} = 0.03, \beta_{LOC} = 0.03$. DAMPING: $\alpha_{LOC} = 0.03, \beta_{LOC} = 0.01$. OSCILLATING WITH SMALL DAMPING: $\alpha_{LOC} = 0.03, \beta_{LOC} = 0.005$. THE PARAMETERS FOR ILS GLIDE SLOPE CONTROL ARE $\alpha_{GS} = 1$, $\beta_{GS} = 0.05$, and $I = 0.001$.

Using both mathematics and numerical simulation, this paper also shows that the heading change and the vertical speed can be used to retrieve CDI rate. The CDI rate is needed for achieving necessary damping and stabilization but is difficult to obtain for a human pilot without much flight experience.

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REFERENCES

[1] Berndt, J. S., 2008. *JSBSim: An Open Source, Platform-Independent, Flight Dynamics Model in C++*, v1.0 ed. JSBSim Reference Manual.

[2] X-Plane by Laminar Research. On the WWW. URL `http://www.x-plane.com/`.

[3] Microsoft Flight Simulator. On the WWW. URL `http://www.microsoft.com/games/flightsimulatorX/`.

[4] McGovern, S. M., 2007. National Airspace System Stochastic Simulation Aircraft Model Validation Survey. Tech. Rep. DOT-RITA-RTV-4B-07-03-1.0, U. S. DOT Volpe National Transportation Systems Center, Cambridge, MA, May.

[5] McRuer, D., and Jex, H. R. "A review of quasi-linear pilot models". *IEEE Trans. on Human Factors in Electronics*.

[6] Hosman, R., and Stassen, H., 1999. "Pilot's perception in the control of aircraft motions". *Control Engineering Practice,* ***7***(11), November, pp. 1421–1428.

[7] Godwin, P., 2006. *Human Factors and Pilot Performance (Air Pilot's Manual S.)*. Air Pilot Publishing Ltd., London, UK.

[8] Foyle, D. C., 2008. *Human Performance Modeling Aviation*. CRC Press, New York.

[9] Johnson, E. N., 2002. Generic pilot and flight control model for use in simulation studies. Conference paper, AIAA Modeling and Simulation Technologies Conference and Exhibit, Monterey, CA, 5-8 August.

[10] WGS84, 2000. Department of Defense World Geodetic System 1984. Tech. Rep. NIMA TR8350.2, 3rd edition, National Imagery and Mapping Agency, Cambridge, MA, January. See also URL `http://earth-info.nga.mil/GandG/publications/tr8350.2/wgs84fin.pdf`.

[11] Tan, K.-K., Wang, Q.-G., and Hang, C. C., 1999. *Advances in PID Control*. Springer-Verlag, London, UK.