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Monte Carlo Reliability Analysis of Systems with a Human Operator

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Abstract: This paper deals with a description of systems with a human operator - the MMS systems and their reliability. Reliability of these systems is given by technical reliability and reliability of a human operator. Using a modern approach - the Monte Carlo reliability analysis for reliability analysis of systems MMS is main goal of this paper. This method is explained at a fundamental level with emphasis on its application to reliability analysis of MMS systems. The last part of this paper presents an example of MMS reliability analysis using Monte Carlo reliability analysis.

Keywords: Man-machine systems, Reliability, Human reliability, System reliability, Monte Carlo simulation, Mean Time between failures (MTBF).

1. INTRODUCTION

Nowadays, the technical systems are often quite complex and it is difficult to ensure their reliability. However, prediction of their reliability is very important. There are a lot of reliability analyses for technical systems and the issue of reliability analysis of technical system is fairly well mastered. The most widely used quantitative analyses are Analysis of Reliability Block Diagram (RBD), Fault Tree Analysis (FTA) or Markov Reliability Analysis (MA). However, the Monte Carlo statistical reliability analysis has recently become very popular (Ericson, 2005).

Many of today's systems are in cooperation with a human operator. These systems are called Man-Machine Systems (MMS). (Boril, Jalovecky, & Ali, 2012) The global reliability of these systems is influenced and degraded by a human factor. The evaluation of reliability parameters is bounded on human fail quantification. Reliability discipline called Human Reliability Assessment (HRA) deals with an influence of human errors on the reliability of technical systems.

In the following text is presented a new approach to reliability analysis of the MMS systems - using Monte Carlo reliability analysis. The Monte Carlo simulation was developed in the 1940s by scientists at the Los Alamos National Laboratory for modelling of the random diffusion of neutrons. The name refers to the Monte Carlo Casino in Monaco. (Alexander, 2003) Today, the Monte Carlo is used in a wide array of application. One of these applications is just the quantitative reliability analysis.

2. THE SYSTEMS WITH A HUMAN OPERATOR

The interaction between a human and the technical system called MMS is demonstrated in Fig. 1. Reliability of these systems is given by reliability of all consisting elements, i.e. reliability of technical system – Machine and human - Man reliability.

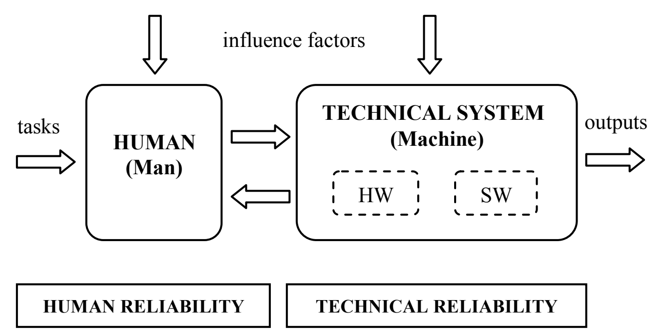

Fig. 1. MMS system and its reliability.

2.1 *The technical reliability*

Reliability of the technical systems is defined as a capability of a system to perform a required function. This capability (or attribute) constitutes a complex property of the given system and it is expressed via probabilistic variables, where the random variable is time to the first failure - t. These probabilistic variables are called reliability indicators and they are described using exponential distribution of probability most often (Smith, & Simpson, 2004).

The most widely used reliability indicators are probability of failure $Q(t)$, failure probability density $f(t)$, probability of reliability $R(t)$, Mean time between failure $MTBF$ and failure rate $\lambda(t)$. The calculation and relations of these indicators are described via followed equations in general (Ericson, 2005).

$$Q(t) = \int_0^t f(\tau)d\tau \qquad [-] \qquad (1)$$

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$$f(t) = \frac{dQ(t)}{dt} \qquad [\mathrm{h}^{-1}] \qquad (2)$$

$$R(t) = 1 - Q(t) \qquad [-] \qquad (3)$$

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - Q(t)} \qquad [\mathrm{h}^{-1}] \qquad (4)$$

$$MTBF = \int_0^\infty R(t)dt = \frac{1}{\lambda} \qquad [\mathrm{h}] \qquad (5)$$

The overall calculation of these indicators is carried out using quantitative reliability analyses, which are suitable tools to express total reliability – the value of the indicators, lifetime and other aspects.

Analysis of Reliability Block Diagram – RBD, Fault Tree Analysis – FTA, Event Tree Analysis – ETA or Markov Reliability Analysis – MA are the most widely used quantitative analyses (Ericson, 2005), (Havlíková & Jirgl, 2013).

Each of these analyses is based on creating a reliability model and applying certain rules to calculate the indicators. The structure of most technical systems is relatively complex. Therefore, the reliability indicators calculation can be difficult. This problem solving is use of commercial software, e.g. BlockSIM, ITEM RBD, etc. However, costs are the main disadvantage of this solution.

Other way to prediction of reliability is use of statistical computer simulation tool, i.e. The Monte Carlo Reliability Analysis.

2.2 Human reliability assessment

Human Reliability Assessment – HRA is part of a reliability discipline, which studies a human performance in operating actions. Human reliability is usually defined as a probability that he/she will correctly perform some system-required activity during a given time period (if time is a limiting factor) without performing any extraneous activity that can degrade the system (Havlíková & Jirgl, 2013).

Human reliability indicators are similar to reliability indicators of technical systems. The log-normal probability distribution $LN(\mu, \sigma)$ is suitable for expression of human reliability. This distribution can be described by equation of failure probability density (6).

$$f(t) = \frac{1}{t\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{(\ln t - \mu)^2}{2\sigma^2}\right) \qquad [\mathrm{h}^{-1}] \qquad (6)$$

Here μ is mean (expected value) and σ is standard deviation of normal distribution of random variable $\ln(t)$.

However, the quantification of human reliability is usually expressed via $HEP(t)$ – Human Error Probability and $HSP(t)$ – Human Success Probability indicators. These indicators correspond to the probability of failure $Q(t)$ and probability of reliability $R(t)$ for technical systems. The $HEP(t)$ indicator is given by (7), the $HSP(t)$ by (8) (Havlíková & Jirgl, 2013).

$$HEP(t) = \int_{t=0}^{t_m} f(t)dt \qquad [-] \qquad (7)$$

$$HSP(t) = 1 - \int_{t=0}^{t_m} f(t)dt = 1 - HEP(t) \qquad [-] \qquad (8)$$

Description of reliability by the log-normal probability distribution $LN(\mu, \sigma)$ is quite difficult. Therefore, the approximation by Weibull distribution can be used. This distribution is three-parameter in general. However, the two-parameter Weibull distribution is used in practice. This distribution is defined by failure probability density (9).

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \cdot \exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right) \qquad [\mathrm{h}^{-1}] \qquad (9)$$

Here β is shape parameter and η is scale parameter. The value of β - shape parameter in the range (2 - 3) is suitable for approximation of log-normal distribution (Havlíková & Jirgl, 2013).

The $HEP(t)$ and $HSP(t)$ indicators are given by (10) and (11), in case of Weibull distribution using.

$$HEP(t) = \int_{t=0}^{t_m} f(t)dt = 1 - \exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right) \qquad [-] \qquad (10)$$

$$HSP(t) = \exp\left(-\left(\frac{t}{\eta}\right)^{\beta}\right) \qquad [-] \qquad (11)$$

Quantitative HRA methods are focused on translating the identified event or error into Human Error Probability – HEP. These techniques refer to databases of human tasks and associated error rates to expression of average error probability for a particular task. They are focused on identifying an event or error and the common result of task analysis or incident investigation. There are many quantitative HRA methods, e.g. THERP, ATHEANA, SLIM, etc. However, most of these methods are based on the complex model of human behavior and creating of failure/event tree for individual interactions in MMS. Then, these analyses can be quite difficult.

2.3 Reliability of MMS

As mentioned, reliability of MMS systems is given by reliability of technical system – Machine and human - Man reliability.

Human reliability is usually described by log-normal distributed indicators and technical systems by exponential distributed indicators. This fact should be considered. Calculation of reliability indicators can be relatively difficult and lengthy in some cases.

Solving of this problem can be using some simulation method, e.g. Monte Carlo statistical reliability analysis. This approach is namely able to work with different probability distributions components. Therefore, Monte Carlo statistical reliability analysis using is suitable for quantitative reliability analysis of whole MMS system, i.e. evaluation of some reliability indicators.

3. THE MONTE CARLO RELIABILITY ANALYSIS

This technique differs from other approaches in that it is not an analytical method but a statistical computer simulation tool. It utilizes a reliability model of the analyzed system. This model is based on mathematical description of individual components via probabilistic variables and knowledge about a system structure (Alexander, 2003).

Thus, the principle of this method is a statistical simulation of the modeled system behavior. A sufficiently large set of random numbers is needed and it is required a suitable random number generator, e.g. the MATLAB random number generator based on the Marsaglia's Random Number Generator algorithm. The random numbers from interval (0;1) are distributed with uniform distribution of probability. These numbers y_i must be transformed to numbers x_i with a required distribution with probability density $f(x)$ by (12) for each system component (Martinez-Velasco & Guerra, 2014), (Reinaldo A. González-Fernández, 2011).

$$y_i = \int_{-\infty}^{x_i} f(x)dx \quad [-] \qquad (12)$$

The random numbers x_i with a required distribution are given by solution of (12). Many experiments with a mathematical model of the analyzed system are realized after this transformation. Resulting mathematical model is function $Z(x_i)$ of random variables x_i with a distribution of probability $F(z)$. A result of n experiments is set of random numbers z_i with a distribution of probability $F_n(z)$. For sufficiently large count of experiments, where count of experiments $n \rightarrow \infty$, distribution $F_n(z)$ is very nearing to distribution $F(z)$. The results of this analysis are the reliability indicators of the analyzed system, i.e. probability of failure $Q(t)$, failure rate λ, mean time between failures *MTBF*, etc. Whole process (algorithm) of Monte Carlo reliability analysis is shown in Fig. 2 (Jirgl, Bradac, Stibor, & Havlikova, 2013).

A first step is initialization which includes a description of some reliability indicator and probability distribution of each element. Furthermore, a number of iteration N must be defined and all variables initialized.

In the next step - Generation of RN and transformation, the random numbers y_i are cyclically generated. They are transformed to the x_i values by (12). The transformation gives an equation in form $x_i = f(y_i)$. The number of generation-calculation of x_i depends on the number of system elements, i.e. if the system consists of j elements then j random numbers (y_{i1}, y_{i2}, ..., y_{ij}) are generated and their transformation into x_{i1}, x_{i2}, ..., x_{ij} is made.

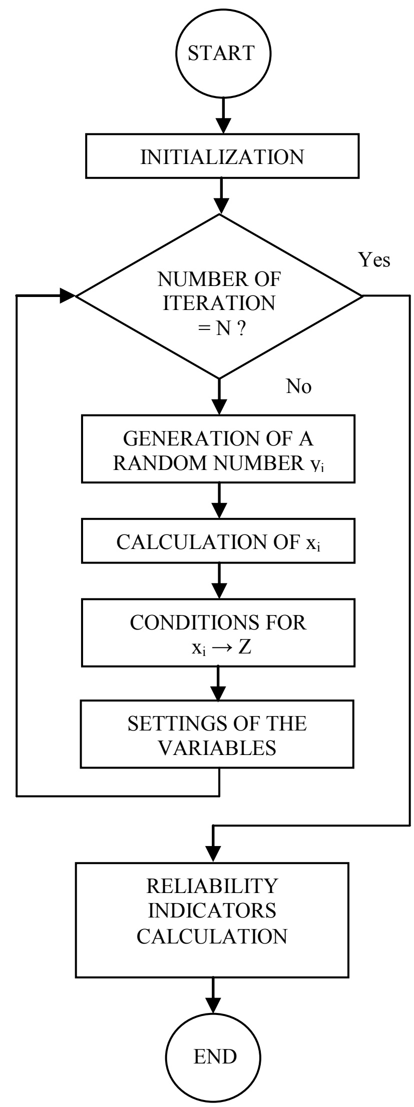

Fig. 2. The general Monte Carlo reliability analysis algorithm

Conditions for $x_i \rightarrow Z$ – these conditions are used to creating system mathematical model. There are two basic structures of reliability model. The first structure is a serial model, see in Fig.3.

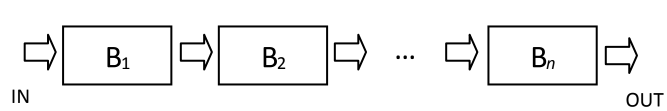

Fig. 3. A serial reliability diagram

System success requires that all blocks must operate successfully at the same time. System failure occurs if either one or more blocks fail. If probability of reliability of the each block B_i is $R_i(t)$, then the probability of reliability of the whole serial system $R_s(t)$ is given by (13).

$$R_s(t) = \prod_{i=1}^{n} R_i(t) \quad [-] \qquad (13)$$

The second structure - parallel form is shown in Fig. 4. These systems are called systems with a redundancy. System success requires that either one (or more) must operate successfully. System failure occurs only if all blocks fail are failed at the same time (Jirgl, Bradac, Stibor, & Havlikova, 2013), (Ericson, 2005).

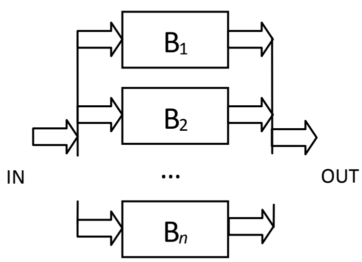

Fig. 4. A parallel reliability diagram.

If we assume that probability of failure of the each block B_i is $Q_i(t)$, then the probability of failure of the whole parallel system $Q_p(t)$ is given by (14).

$$Q_p(t) = \prod_{i=1}^{n} Q_i(t) \quad [-] \qquad (14)$$

The conditions for $x_i \rightarrow Z$ are $z_i = \min(x_{ij})$ in case of the serial system and $z_i = \max(x_{ij})$ in case of the parallel system. Here z_i is a mean time between failures for i-th iteration, i.e. $MTBF_i$. There are also the combinations of serial and parallel forms where the totally conditions must be composed from the basic conditions via the described rules.

In the step Settings of the variables, the variables for the calculation of reliability indicators are set. These variables are defined within initialization.

The last step is Calculation of reliability indicators. The reliability indicators of the whole system are calculated in this step.

The main advantages of Monte Carlo reliability analysis are possibility of calculation of reliability indicators of systems with different subsystems and also systems with a complex structure and possibility to calculation of reliability of systems with different distributions of probability. Therefore, this approach is suitable for quantitative reliability analysis of systems with a human operator - MMS systems.

4. THE EXAMPLE OF MMS RELIABILITY ANALYSIS

The aim of this example is a quantitative reliability analysis of complex technical system with a human operator - system MMS. The analyzed system is the system shown in Fig. 5.

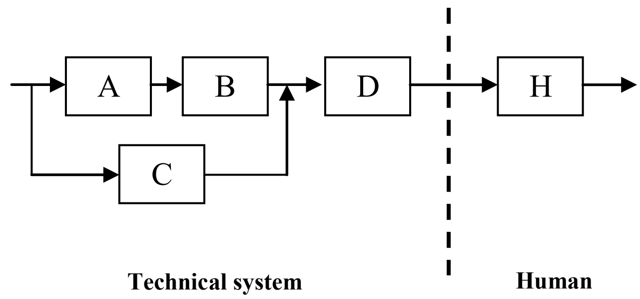

Fig. 5. Reliability diagram of the analyzed MMS system.

4.1 Description of the analyzed system and the task

The analyzed MMS system is a technical system consisting of 4 elements - A, B, C and D in cooperation with a human operator - H. The individual components of the technical system - A, B, C and D are described by the failure rates $\lambda_A = 5.10^{-4}$ h^{-1}, $\lambda_B = 5.10^{-5}$ h^{-1}, $\lambda_C = 3.10^{-5}$ h^{-1}, $\lambda_D = 9.10^{-6}$ h^{-1}. The exponential distribution of reliability is expected. Reliability of a human operator was described by graph of failure density of the human activity $f_H(t)$, see in Fig. 6. This graph was obtained via statistical description of a human failure based on observing of the long-term MMS interaction. The expecting probability distribution of human activity is log-normal distribution.

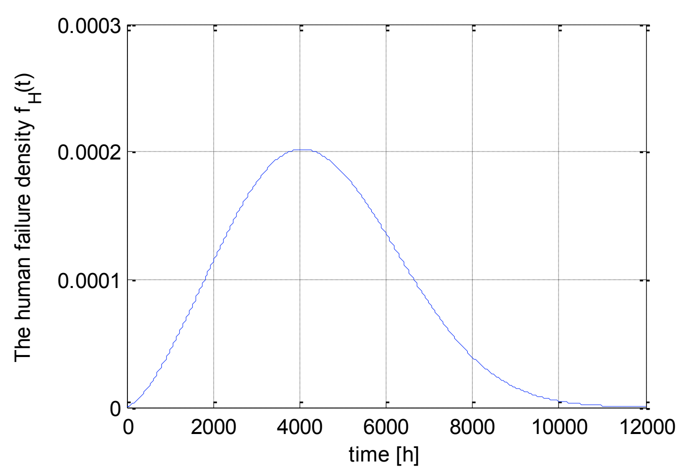

Fig. 6. The failure density graph of the human activity.

The task is evaluation of the reliability indicators of whole MMS system using Monte Carlo reliability analysis. The desired indicators are *MTBF*, probability of failure $Q(t)$ and probability of reliability $R(t)$ after 2000 hours.

4.2 Monte Carlo reliability analysis of given MMS

The Monte Carlo reliability analysis uses the algorithm shown in Fig. 2. There are described the individual steps of this algorithm applied to the concrete MMS system, see in Fig. 5.

Step 1: Initialization

Definition of reliability indicators of individual components and other required variables are expected in this step.

Reliability of individual components is described by failure rates in case of the technical system, i.e. $\lambda_A = 5.10^{-4}$ h^{-1}, $\lambda_B = 5.10^{-5}$ h^{-1}, $\lambda_C = 3.10^{-5}$ h^{-1}, $\lambda_D = 9.10^{-6}$ h^{-1}.

The human reliability is described by failure density function shown in Fig. 6, corresponding to the log-normal distribution of probability. An operation with a log-normal distribution is quite difficult, so it was approximated by two-parameter Weibull distribution. This approximation gives the failure density function in form of equation (9) where parameters β = 2.5 and η = 5000 h.

The number of iterations depends on the convergence of the algorithm. It must be a large number, e.g. N = 1000000.

Other important variables are variable for *MTBF* calculation $S = 0$ and variable for $R(t)$ and $Q(t)$ calculation $K = 0$.

Step 2: Generation RNs and transformation

The analyzed system includes 5 elements - A, B, C, D and H. Therefore, the process of y_i generation - transformation to x_i must be made five times, in case of each iteration i.

The exponential distribution of probability described by probability (failure) density function (15) with a parameter failure rate λ is assumed in case of the technical part consisting of elements A, B, C and D.

$$f(x) = \lambda \cdot \exp(-\lambda x) \qquad [\mathrm{h}^{-1}] \qquad (15)$$

The transformation of (15) via (12) is

$$y_i = \int_{-\infty}^{x_i} f(x)dx = \int_0^{x_i} \lambda \exp(-\lambda x)dx = 1 - \exp(-\lambda x_i)\text{, then}$$

$$x_i = -\frac{1}{\lambda} \cdot \ln(1 - y_i) \qquad (16)$$

The two-parameter Weibull distribution of probability described by probability (failure) density function (9) with parameters β - shape parameter and η - scale parameter is assumed in case of the description of human reliability H. The transformation of (9) via (12) is

$$y_i = \int_{-\infty}^{x_i} f(x)dx = \int_0^{x_i} \left[\frac{\beta}{\eta} \left(\frac{x}{\eta} \right)^{\beta - 1} \exp\left(-\left(\frac{x}{\eta} \right)^{\beta} \right) \right] dx = $$

$$= 1 - \exp\left(-\left(\frac{t}{\eta} \right)^{\beta} \right) \text{, then}$$

$$x_i = \eta \cdot \left(-\ln(1 - y_i) \right)^{\frac{1}{\beta}} \qquad (17)$$

Therefore, realization of this step consists in random numbers y_{Ai}, y_{Bi}, y_{Ci}, y_{Di} and y_{Hi} generating and transforming into x_{Ai}, x_{Bi}, x_{Ci}, x_{Di} and x_{Hi} via (16) and (17).

Step 3: Conditions for $x_i \rightarrow$ Z

The elements A and B are in serial form of reliability model with one redundancy - C. The element D is in series with this combination. The last element representing a human operator H is in series with the whole described model of the technical system. The conditions for $x_i \rightarrow$ Z are in form (18).

$$z_i = \min\left(\max\left(\min(x_{Ai}, x_{Bi}), x_{Ci}\right), x_{Di}, x_{Hi}\right) \qquad (18)$$

Step 4: Settings of the variables

Settings of the variables for $R(t)$, $Q(t)$ and *MTBF* calculation is occurred in this step. The current z_i is compared to defined time t = 2000 h. Success means $z_i > t$. In this case, the value of variable K is increased by 1 ($K = K$+1). The value of variable S is increased by z_i value ($S = S + z_i$).

Step 5: Calculation of the reliability indicators

The required reliability indicators $R(t)$, $Q(t)$ and *MTBF* are calculated after N iterations based on values of the program variables K and S. The values of these indicators are given by (19), (20) and (21).

$$R(t) = \frac{K}{N} \qquad [-] \qquad (19)$$

$$Q(t) = 1 - R(t) \qquad [-] \qquad (20)$$

$$MTBF = \frac{S}{N} \qquad [\mathrm{h}] \qquad (21)$$

The whole algorithm of Monte Carlo reliability analysis was implemented in MATLAB r2013b.

The values of required reliability indicators, i.e. probability of failure $Q(t)$ and probability of reliability $R(t)$ after 2000 h and *MTBF* of the analyzed system, obtained via Monte Carlo approach after N iterations are:

$$R(2000h) = 0.854$$

$$Q(2000h) = 0.146$$

$$MTBF = 4076h$$

The values of these indicators were verified using BlockSIM software. The results obtained using this software are comparable to the results obtained via Monte Carlo algorithm. They are:

$$R(2000h) = 0.855$$

$$Q(2000h) = 0.145$$

$$MTBF = 4072h$$

5. CONCLUSIONS

The main goal of this paper was to introduce the Monte Carlo reliability analysis and dmonstrate an example of its application. The aim of the example was reliability analysis of system with a human operator – MMS system. The task of this example was calculation of some reliability indicators of the given MMS system via Monte Carlo reliability analysis.

The Monte Carlo algorithm was implemented in MATLAB r2013b in this case. The values of Probability of failure $Q(t)$, probability of reliability $R(t)$ after 2000 h and *MTBF* of the analyzed system were obtained via this algorithm. These indicators describe the reliability of overall MMS system. They were verified using commercial software.

Of course, the indicators can be evaluated on the basis of knowledge of failure probability of individual elements and applying certain rules, i.e. (13), (14) and (1) – (5). However, this approach is relatively lengthy and difficult in some cases. Furthermore, calculation of some reliability indicators, e.g. *MTBF* by (5), can be also difficult. Use of commercial software can be other way to reliability analysis of these systems. Unfortunately, this software is mostly charged. The Monte Carlo reliability analysis is very useful tool in these cases. Its use is easy, fast and it gives relatively accurate results. The other advantage of Monte Carlo reliability analysis is possibility of using any programming languages for the algorithm implementation. Based on this, the described Monte Carlo algorithm provides available software suitable for reliability analysis of complex MMS systems.

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