Probabilistic buckling analysis of thin-walled steel columns using shell finite elements

Zdeněk Kala, Jiří Kala and Abayomi Omishore

***Abstract*—**The article deals with the probabilistic analysis of the reliability of a compressed steel column with hollow thin-walled rectangular cross-section. The stochastic computational model is based on the non-linear finite element method and numerical simulation Latin Hypercube Sampling method. Initial geometric imperfections were introduced according to the local and global buckling modes. Statistical analysis of the load carrying capacity was performed for four variants of wall thicknesses. The influence of the nominal thickness of the wall of the hollow rectangular cross-section on the ultimate limit state was studied.

***Keywords*—**ANSYS, buckling, column, imperfection, reliability, stochastic, structure.

I. Introduction

Thin-walled slender members are frequently used as structural elements in the field of civil engineering [1]. The low self weight of steel structures is an important prerequisite for economical and reliable design of tall buildings, such as skyscrapers, high-rise buildings, masts, towers and structures with spans over 30 meters, such as roof structures of halls, stadiums, bridges and special roofing and ceilings. Thin-walled structural elements are highly valuable in the management of installations, such as, for the distribution of heat, water, electricity, sprinklers and ventilation [2]. Thin-walled steel box structures are utilized in combination with prestressed rods.

Variability of design solutions and the rich selection of optimum geometric and material parameters of members of thin-walled steel structures provides the prerequisites for efficiently designed load bearing elements to transfer loading as efficiently as possible. Material savings, however, come at a price, because the low weight of the structure could mean an increased failure probability due to loss of stability. It should be noted that slender thin-walled members cannot be used in plastic design [3]. Local buckling occurs before the so-called full-plastic moment is reached in bent thin-walled elements [4], [5]. Due to the interaction of local and global buckling loss of load carrying capacity could occur before the development of plastic zones in compressed thin-walled elements. Loss of stability of structural elements may or may not be fatal for the whole structure, but always presents a negative phenomenon, which must be prevented through detailed static and stability analysis of the structure.

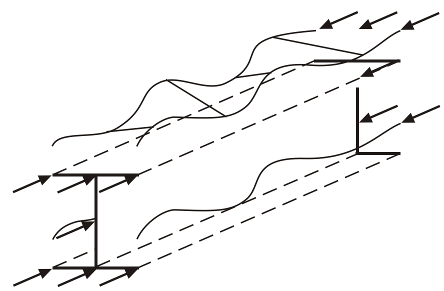

Fig. 1 Local buckling of thin-walled open I-section

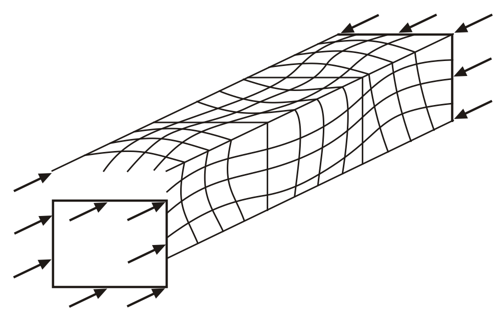

Fig. 2 Local buckling of thin-walled hollow rectangular section

Demonstrating the safety and reliability of thin-walled steel beam structures is normally based on static and stability analysis utilizing computer modelling of real structural systems using the finite element method [6]. The types of finite elements and methods of solution are selected according to the geometry of the thin-walled elements and the presumed failure mechanism [1]. The mode of failure and static resistance of the element depends primarily on whether the cross-section is open or closed. In open thin-walled compressed I-section columns the flange has a tendency to buckle before the web, which is supported on all sides, see Fig. 1. However, in closed sections, for e.g. hollow rectangular sections, both the flanges and webs act as internal elements and local buckling of these elements is dependent on their respective width to thickness

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Zdeněk Kala is with the Brno University of Technology, Faculty of Civil Engineering, Brno, 602 00 CZ (phone: +420-541147382; fax: +420-541147392; e-mail: kala.z@fce.vutbr.cz).

Jiří Kala is with the Brno University of Technology, Faculty of Civil Engineering, Brno, 602 00 CZ (phone: +420-541147392; fax: +420-541147392; e-mail: kala.j@fce.vutbr.cz).

Abayomi Omishore is with the Brno University of Technology, Faculty of Civil Engineering, Brno, 602 00 CZ (phone: +420-541147131; fax: +420-541147392; e-mail: omishore.a@fce.vutbr.cz).

ratio. Local buckling occurs along the entire length of the member and a “checkerboard” wave pattern is developed on the member, see Fig. 2. Failure mechanisms and stress of compressed thin-walled elements are very sensitive to initial local and global imperfections, which reduce the load carrying capacity of the compressed elements [7]. The behaviour of compressed steel columns in structural systems is always dependent on initial imperfections [8].

In the presented article the thin-walled column with hollow rectangular cross-section is modelled using shell finite elements and the computational programme ANSYS. The load carrying capacity of the column is analysed using the geometric and physical non-linear solution. Randomness of initial imperfections is taken into consideration. The influence of change in the wall thickness of the hollow rectangular cross-section on the load carrying capacity of the column was studied. The article builds on the uncertainty quantification and validation of simulation experiments of slender steel structures [9], [10]. Reliability analysis is performed using the numerical simulation Latin Hypercube Sampling method [11], [12].

II. Compressed Thin-walled Column

The hollow rectangular cross-section comprises of walls mutually connected in the cross-section corners, see Fig. 3. During axial compression of a thin-walled cross-section the individual walls mutually influence each other and buckling occurs in the walls. The walls with higher slenderness buckle sooner, and buckling is then transferred to the walls with lower slenderness. Elastic built-in is provided to the walls by the longitudinal edge prior to local buckling in the wall with lower slenderness. The problem of columns integrated into a structural system is more complicated and it is necessary to take into consideration stiffness parameters of the column ends and the manner in which loading is transferred from the structure to the column during analysis.

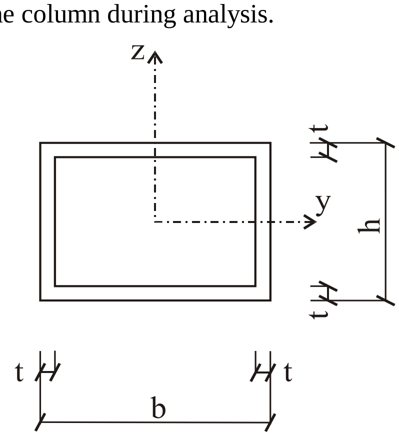

Fig. 3 Geometry of hollow rectangular cross-section

Thin-walled plate structures, which are subjected to axial compression, may have many different modes of buckling, see, e.g. [13]. Such structures are still able to sustain load after the local buckling mode. Due to local buckling there is a decrease in the stiffness of the section and a subsequent reduction in the load carrying capacity with regard to the non-locally buckled section. Global buckling mode results in the collapse of thin-walled structures. Destruction of the structure is accelerated by the the phenomenon of interaction between both buckling modes. The interaction of global and local buckling modes is a very interesting problem of great importance. The principles of the classical theory of elasticity, which presumes a non-deformable cross section, cannot thus be used for the analysis of limit states. During limit state the hollow rectangular cross-section of the thin-walled column is divided into the non-effective zones and the effective zones, see Fig. 4.

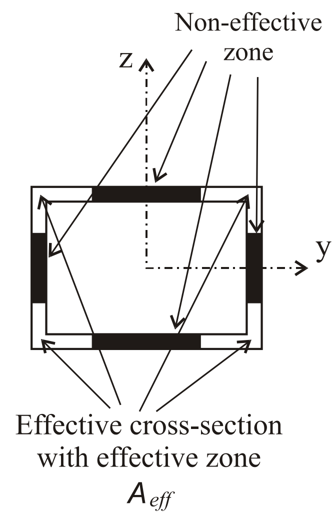

Fig. 4 Effective zone of hollow rectangular cross-section

In Fig. 4, the cross-sectional area that resists the loading is marked by the white colour. Only the zones close to the cross-section corners resist the loading, whilst the zones influenced by buckling loose their resistance. The deformation of member axis and the deformation of slender walls of the cross-section have a great influence on the ultimate limit state, therefore it is necessary to apply advanced non-linear models of finite elements for the solution of the load-carrying capacity. These models must always take into consideration the influence of initial imperfections, see Fig. 5.

Initial imperfections cause bending of the member as a whole and buckling of its walls immediately at the onset of loading with both phenomena proceeding in permanent interaction. Stress of slender walls changes during loading. The stress changes from zero to the values initiating and developing the plastic zones and permanent deformations. The internal side of the cross-section is compressed more than the external one, where buckling propagates at a lower speed. The load carrying capacity of the member is always lower than the critical load carrying capacity. The load carrying capacity of its walls is frequently substantially higher (due to the so-called post-critical load carrying capacity reserve) than the critical loading.

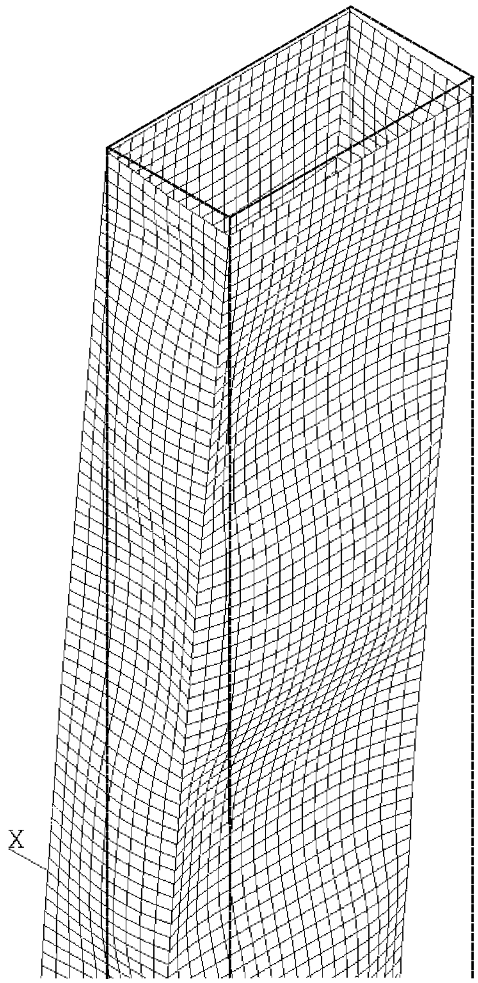

Fig. 5 FE model with elements SHELL 181

III. Calculation Model

The problem of stability loss was studied on a thin-walled member of length L=10 m with imperfections. The nominal plate thickness was considered in four variants $t \in \{10, 14, 18, 22\}$ mm. The nominal cross-section width b=1 m and nominal cross-section height h=0.5 m were considered, see Fig. 3.

The programme ANSYS was used to analyse the problem. With regard to the symmetry of the analysed member and to the computational demand, only one half of the member length was modelled. The plane of symmetry in x=5m was considered to constrain translation in the x direction, and to constrain rotations around axes y and z. Hinged boundary conditions were considered on both ends of the member, i.e. x=0 and x=L. This boundary condition enables free rotation and fixed translation.

Modelling of the column geometry was performed using a four-node shell element provided in [6] as SHELL 181, which makes solution of material and geometrically non-linear problems feasible. Provision was made in calculation for the non-linear material behaviour by means of a bi-linear kinematics model without stiffening.

Finite element meshes were selected in such a manner to enable description of local buckling of walls using the model, see Fig. 5 and Fig. 6. The column was modelled using 2012 finite elements with a total of 2050 nodes. The model has 12176 degrees of freedom. The non-linear finite element analysis of the load carrying capacity was performed using the incremental Euler method and the Newton-Raphson method [6]. The size of loading step was automatically adjusted within the interval 0.00001-0.1 of the sought after load carrying capacity. The load carrying capacity was defined as the loading at which the determinant of the structure’s tangential stiffness matrix equals zero with prescribed accuracy.

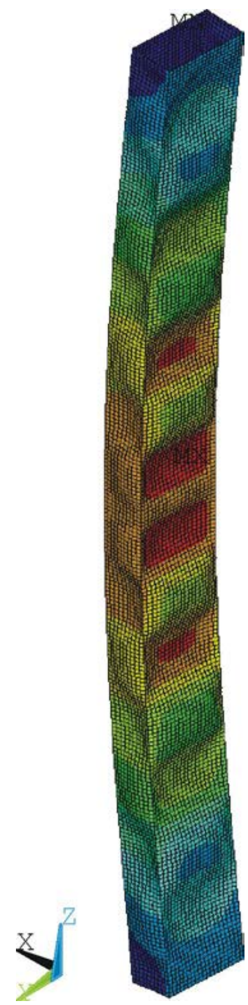

Fig. 6 Global and local imperfections

IV. Stochastic Calculation Model

Finite element methods and optimization computer techniques are a common part of the structural design of reliable and economical structures [14]. Modelling and numerical simulation should always be based on experimental research [15]. Numerical simulation of loading tests on computers based on experimental research substitute actual loading tests, which are economically very demanding [16]. A possible method for the evaluation of the reliability of the column is to consider the load carrying capacity as a random output variable. The outputs of the computational model can be evaluated as random variables if direct probabilistic methods [17] or advanced numerical simulation methods, such as Monte Carlo [11], [12], are used. Great attention should be paid to initial imperfections, which have a significant effect on the ultimate limit state.

Initial imperfections are generally random quantities, the statistical characteristics of which would be obtained from the measurement of a large number of members. Performing a higher number of tests on real members, i.e. with regard to imperfection, would be demanding economically and in production. The measurement of a small number of samples would yield insufficient data for reliable evaluation of the statistical results with regard to the expected variance and heterogeneity of imperfections.

The first random quantity considered in the analysis was the yield strength. The yield strength of steel grade S235 was expressed using a histogram [18] with mean value of 285.74 MPa, standard deviation 23.57 MPa, and with small, practically negligible, skewness.

The second imperfection that could have a substantial effect on the load carrying capacity is the initial curvature of the member. The shape of curvature was introduced in the form of a half-wave of the sine function within the interval, for both the initial deflection in the plane of the primary bending in the direction of the *z*-axis (random amplitude e_{01}), and for buckling in the direction of the *y*-axis (random amplitude e_{02}). Gaussian probability density function with mean value of zero, i.e. a perfectly straight member, was introduced for amplitudes e_{01}, e_{02}. The standard deviation for the two random quantities was introduced as $S_{e01}=S_{e02}=6$ mm according to the rule $2S_X$, see, e.g. [18].

Initial buckling of the member walls was introduced in the model as scaled eigenmodes obtained a priori from an elastic buckling analysis, see Fig. 5. It was considered as a random quantity e_{07} with mean value $m_{e07}= 0$, and standard deviation $S_{e07}= 1.67$ mm.

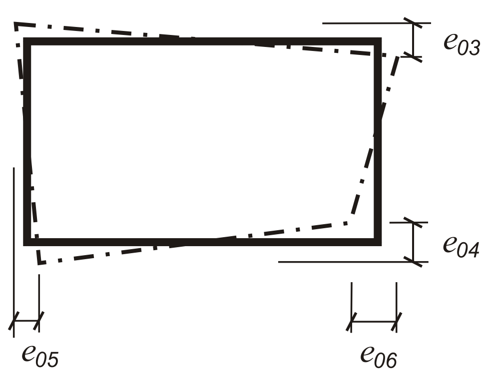

Fig. 7 Out-of-squareness imperfections

Another imperfection that has an influence on the load carrying capacity is the initial out-of-squareness of walls with box cross section, see Fig. 7. Normal distributions with mean values equal to 0 mm and standard deviation $S_{e03}=S_{e04}=1.25$ mm were assumed for random quantities e_{03}, e_{04}. Similarly, normal distributions with mean values of 0 mm and standard deviation $S_{e05}=S_{e06}=0.625$ mm were assumed for quantities e_{05}, e_{06}. The nominal value of wall thickness *t*=12mm was considered as the mean value of the Gaussian probability density function. The coefficient of variation of the wall thickness was equal to 0.07 in all cases [18].

V. Stochastic Analysis of Resistance

The load carrying capacity *R* is the output random quantity. The random realizations of the load carrying capacity were evaluated using the Latin Hypercube Sampling (LHS) method [11], [12]. 400 runs of LHS method were used.

A. Statistical analysis

Statistical analysis of the load carrying capacity was evaluated for four variants of the wall thickness, see histograms in Fig. 8, 9, 10, 11. Each histogram was processed using 400 runs of the LHS method.

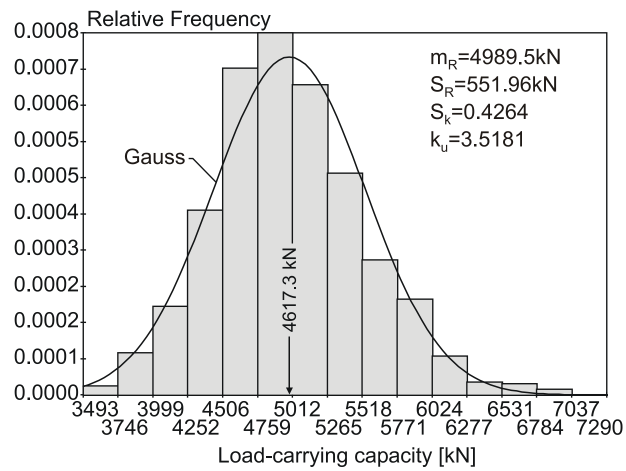

Fig. 8 Histogram of load carrying capacity, *t*=10 mm

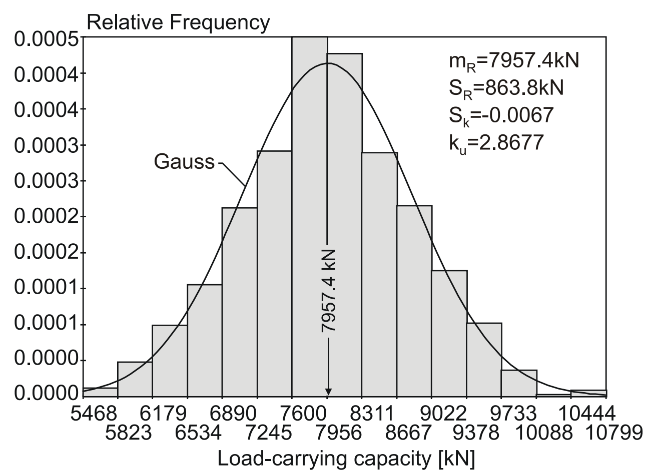

Fig. 9 Histogram of load carrying capacity, *t*=14 mm

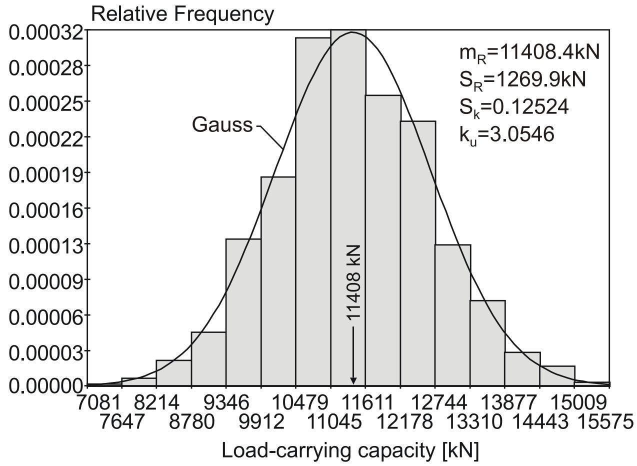

Fig. 10 Histogram of load carrying capacity, *t*=18 mm

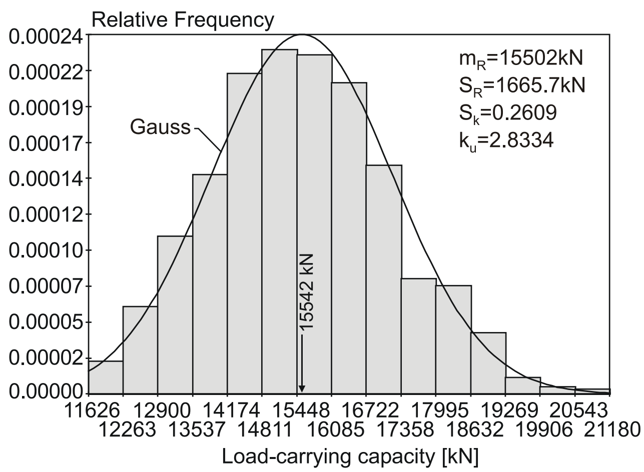

Fig. 11 Histogram of load carrying capacity, *t*=22 mm

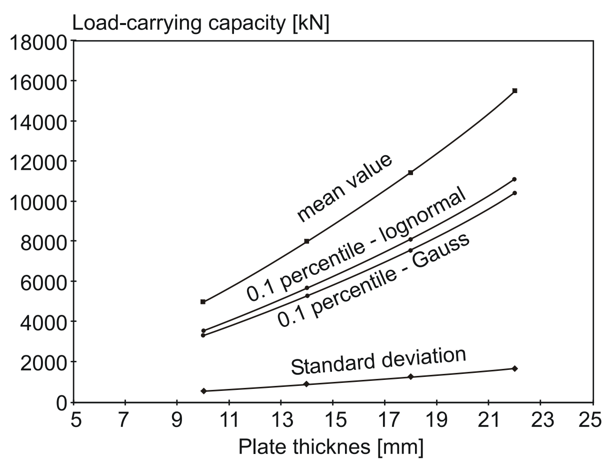

Fig. 12 Load carrying capacity vs web thickness

It is apparent from Fig. 12 that the curve of mean value is convex. The graph of the standard deviation of the load carrying capacity is slightly convex to linear. It is impossible to determine with certainty whether a similar trend is or is not present for skewness and kurtosis, as this trend may be burdened with statistical error given by the number of used simulation runs of the LHS method. According to standard EN1990 the design load carrying capacity can be evaluated as 0.1 percentile [19]. Design values of the load carrying capacity were evaluated as 0.1 percentiles upon approximation of the histograms with Gaussian and lognormal probability density functions. Both curves of the 0.1 percentiles are convex. The design values obtained using the Gaussian probability density function are lower than the design values obtained using the lognormal probability density function. Design values evaluated as 0.1 percentile are approximately thirty to thirty-five percent lower than the average values.

B. *Sensitivity analysis*

Sensitivity analysis is used to determine how change in an input is reflected in the result, output, of a computational model [20]. Sensitivity analysis can be used to identify the input random variable whose variability has the greatest effect on the output [21]. A known and used method of sensitivity analysis is based on the evaluation of the partial derivatives of the output with respect to input factors [22]. Other methods are, for e.g., scatter plots, regression analysis, variance-based methods, screening [20]. The selection of the appropriate method depends primarily on whether the sensitivity analysis is based on a sampling basis or not, see, e.g. [23], [24]. The use of sensitivity analysis in structural mechanics is expedient in situations where we need to set the parameters of virtual experiments in conditions in which the sensitivity of the outputs to the estimated factors is the greatest, see, e.g. [25] - [29].

Calculation of the correlation between the input random variables and model output is often used as the first method of identifying interdependencies of quantities. Evaluation is conditioned by the monotone dependence of output variable on the input random variable, therefore all realizations of the imperfections were considered by the absolute values. It can be seen from the sensitivity analysis that the load carrying capacity is sensitive to the variability of the plate thickness *t*, and to the variability of yield strength f_y. The initial curvature of the member axis e_{01}, e_{02} and geometrical imperfections e_{03}, e_{04}, e_{05}, e_{06}, e_{07} have little influence on the load carrying capacity. However, it should be noted that more general conclusions on reliability cannot be made on the basis of just one study. The sensitivity analysis should be supplemented with the so-called variance based sensitivity analysis, which is capable of identifying higher order interactions among input random quantities and the load carrying capacity [25].

VI. CONCLUSION

The statistical and sensitivity analysis of the ultimate limit state of a compressed steel column with rectangular thin-walled hollow cross-section was presented in the article. Obtained results of the sensitivity analysis show that the load carrying capacity is very sensitive to the variability of the plate thickness. Increasing the wall thickness increases the cross-sectional area, and simultaneously the stiffness of the slender walls against buckling increases, thereby increasing the stability of the column. The variability of other initial geometric imperfections had a smaller influence on the load carrying capacity. Statistical characteristics of the load carrying capacity were studied in dependence on change in the nominal plate thickness of the cross section. Dependence between the mean load carrying capacity and plate thickness was slightly non-linear, see the convex curve in Fig. 12. Curves of the design load carrying capacities evaluated as 0.1 percentile were also slightly non-linear.

The results of the statistical analysis obtained from the virtual simulations could be a useful complement to the results of real experiments and be used to verify the design procedures of thin-walled columns. If the theoretical results are verified experimentally, then discussion could be started on the acceptability of standardised European prescriptions, which utilize the so-called effective cross-sectional area for calculation.

Acknowledgment

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